DEM Working Papers

N. 2017/1

Land, Housing, Growth and Inequality

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ABSTRACT

This paper contains a growth model that incorporates productive assets, residential land and residential structures. Moreover, it accounts for the existence of two social classes: the capitalists, who invest both in productive assets and in housing but do not provide labor services, and the workers, who invest only in housing and decide on how much labor effort to provide. Within this formal setup, it is shown that the relative price of land grows in the long run at the same rate as the economy's GDP, while both the quantity of housing services and their price grow slower than it. Numerical examples show that i) shifting the taxation away from income and towards the property of land enhances long-term GDP growth and leads in the long-run to a more equalitarian (i.e. more favorable to the workers) income and wealth distribution, ii) a marginal increase in the fraction of investment expenditures in residential structures that is tax deductible reduces inequality in the distribution of income and wealth, iii) a change in agents' preferences that gives more weight in the utility function to residential services leads in the long run to a distribution of income and wealth that is more favorable to the capitalists, iv) changes in taxation or in preferences increasing the fraction of total investment devoted to the accumulation of residential wealth rather than to the accumulation of productive assets brings about a balanced growth path characterized by a higher wealth-income ratio. Moreover, the paper illustrates how endogenous fluctuations may be generated along the equilibrium trajectory converging to the balanced growth path, in a model where housing wealth—as well as residential land—is distinguished from productive capital and only fundamentals (initial endowments, preferences and technologies) drive the economy's dynamics.

Key words: Productive assets, Residential structures, Urban rents, Land value tax.

JEL Classification: H24, O18, O41, R31.

Acknowledgements: I am very grateful to Ned Phelps for the inspiring conversations during my staying as visiting scholar at the Center on Capitalism and Society at Columbia University. I also thank the Center for its warm hospitality. For valuable research assistance, I am indebted to Riccardo Degasperì.

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1. INTRODUCTION

David Ricardo (1817) gave solid foundations to the concern that land – being a non-reproducible factor in fixed supply – could act in the long run as a brake on economic growth and determine an increasingly unequal income distribution in favor of those who own a disproportionately large portion of it. This concern seemed to fade away when, as a consequence of the industrial revolution, the share of agriculture in national income gradually fell, thus drastically reducing the importance of land as a factor of production and source of value. Recently, a renewed interest in the role of land as a cause of widening inequalities in income and wealth has been aroused by the strong rise in residential land prices driving the remarkable increase in the value of housing and in the share of housing services in national income observed in many advanced economies during the last four decades. Such increase, indeed, explains a substantial portion of the rise in the wealth-income ratio and of the decline in the labor share of income that have accompanied the growing inequalities recorded in these countries in this period (see Figures 1, 2, 3, 4 and 5). In contrast, less attention has been dedicated to the contribution that the rapidly rising value of housing and residential land may have given to the slowing down of long-term growth affecting the advanced economies since the 1970s. Furthermore, even less effort has been devoted to offer a unifying and rigorous treatment of both these issues, namely the effects of the changing value of residential land and housing on economic growth and on inequality. This paper presents a dynamic model aimed at providing such a theoretical setup.

The remainder of the paper proceeds as follows. Section 2 motivates the paper and discusses some of the relevant literature. The building blocks of the model are presented in section 3. Section 4 discusses

the behavior of the economy along an equilibrium path and the impact on it of changes in tax rates and parameter values. Section 5 concludes. The mathematical derivations are contained in the Appendix.

FIGURE 1 Housing wealth/domestic income (at factor prices) ratios, 1970-2010

Source: Rognlie (2014).

FIGURE 2 Domestic wealth excluding housing/domestic income (at factor prices) ratios, 1970-2010

Source: Rognlie (2014).

FIGURE 3 Net domestic capital income excluding housing/domestic income (at factor prices) ratios, 1960-2010

Source: Rognlie (2014).

FIGURE 4 Net housing wealth income/factor price domestic income (at factor prices) ratios, 1960-2010

Source: Rognlie (2014).

FIGURE 5 Net Capital Income as Shares of Total Private Domestic Value Added in the U.S., Canada, Germany, France, U.K., Italy and Japan, 1948-2010*

* U.S., France and U.K. data cover full period; Japan data starts in 1955, Canada in 1960, Italy in 1990, Germany 1991. For Canada and Japan, the residential housing sector is the owner-occupied housing sector due to data limitations. For all other countries, it covers all housing. This is the unweighted average of shares across all seven countries. To control for the changing composition of the sample, it displays time fixed effects from a panel regression, normalized to equal the actual average at the start of the sample.

Source: www.brookings.edu/bpea-articles/deciphering-the-fall-and-rise-in-the-net-capital-share

2. MOTIVATIONS AND RELEVANT LITERATURE

The model presented here takes into account what – according to many critics – Piketty (2013) blurred, that is the distinction between wealth (inclusive of residential land and housing) and “productive assets” (equipment, machinery, plants, softwares...) (see, e.g., Rognlie 2014, 2015; Rowthorn 2014; Stiglitz 2015a). By neglecting this distinction, Piketty (2013) interprets the observed decline in the labor share of income as the joint result of a more than unitary elasticity of substitution between capital and labor, and of a persistently positive differential between the rate of return on capital and the rate of GDP growth

(which—according to Piketty—accounts for the increase in the capital-income ratio documented in his book). However, the evidence is at odds with the hypothesis that such an increase is mainly due to the accumulation of productive assets replacing labor, since it is explained by the rise in housing prices that occurred in most advanced economies (see Bonnet et al. 2014, and Figure 6). The importance of housing in Piketty and Zucman (2014)’s data mostly reflects the influence of changes in the real price of land (see, for the U.S., Figure 7). Similarly, one can check that a significant portion of the fall in the labor share of income taking place in the same period has to be attributed to the rise in the weight of housing services. With this regard, it is worth to emphasize that nowadays a large part of the value of these services is made up of imputed rents, i.e., the value of the services of houses inhabited by homeowners, especially because in most advanced countries the home ownership rate has increased considerably in the post-World War II period (see Table 1).

It is likely that the trends outlined above negatively affect long-run economic growth because of the crowding-out effect exerted on investment in (material and immaterial) productive assets by the high rate

FIGURE 6 House price indices (advanced economies)*

*Full sample=100. Figures are CPI-deflated and seasonally adjusted.

Source: Scatigna et al. (2014).

FIGURE 7 Indices of prices of residential land, of house prices and of replacement costs in the U.S. (Log Scale)

Source: Davis and Heathcote (2007)

TABLE 1 Home ownership rates (percent) in selected countries (1900-2010)

	Canada	Germany	France	Italy	Switzerland	U.K.	U.S.	Average
1900							47	
1910							46	
1920						23	46	
1930							48	
1940	57					32	44	
1950	66	39	38	40	37	32	47	43
1960	66	34	41	45	34	42	62	46
1970	60	36	45	50	29	50	63	48
1980	63	39	47	59	30	58	64	51
1990	63	39	55	67	31	68	64	55
2000	66	45	56	80	35	69	67	60
2010	69	45	58	82	37	64	65	60

Source: Jordà et al. (2014).

of return on housing investment,¹ which is raised by the expected capital gains obtainable thanks to the long-term increase in house prices. It is worth to notice that such increase has been paralleled in the last decades by a decline in the relative price of investment goods observed (see Figure 8), due to the tendency of technological progress to be faster in the production of investment goods than in the production of consumer goods and services. Given these opposite trends in the prices of housing and investment goods, any investment in plants, equipment, software and similar must generate a higher increment in expected profits in order to be undertaken. Furthermore, the rising share of total income consisting of rent paid or imputed to house owners has compressed also the labor's share of total income, with disincentive effects on labor market participation and investment in human capital. These disincentive effects are probably

FIGURE 8 Ratio between the price of investment goods and the price of consumer goods

Source: Twaites (2015).

¹ The hypothesis that the increase in the value of residential land that has driven up house prices displaces productive investment was explored by Stiglitz (2015b).

more relevant in recent years, when—as an effect of the ageing population and the decreasing fertility rate—a relatively large fraction of the workforce inherits or expects to inherit some real estate wealth, rather than in the past, when most households could become owner of a house only by buying it with their own labor income. Hence, one should expect that the higher wealth-to-labor income ratio brought about by the rising house prices has had some impact on the work attitudes of a sizeable portion of the middle and low-middle class, by raising their reservation wage, reducing the propensity to participate in the labor market of elderly, spouses, teenagers (in particular, postponing young people’s entry in the job market), and discouraging labor mobility.²

What are the structural features explaining the increase in the value of housing that has been observed globally in the last four decades? The main ones are probably five. The first one has to do with the fact—mentioned above—that land is a non-reproducible factor in fixed supply: in areas becoming increasingly congested with any kind of human activities because of economic growth and population increase, the relative price of the residential land tends to rise, driving up the market value of the houses built on it (the process of tertiarization further raises the value of urban areas by making agglomeration economies more important)³; the second features derives from the fact that, as income grows, families tend to spend an increasing fraction of it for living in larger and more comfortable houses, possibly localized in more attractive neighbourhoods, and possibly for having access to a second or a third property in some pleasant

² Phelps (1994) models economies in which a rise of the ratio between wealth per worker and wage depresses employment through its effects on labor supply. Blanchflower and Oswald (2013) document the positive link between home ownership in a geographical area and subsequent high unemployment in that area, suggesting that higher levels of home ownership reduce mobility, increase commuting times and reduce rates of business formation.

³ As argued by the so-called ‘new economic geography’, “firms agglomerate to benefit from ‘Marshallian externalities’ such as the spreading of knowledge among similar industries, a greater pool of labour to choose from or the ability to access indivisible goods such as conference venues or airports. Hence, when operating within proximity of each other, firms can save on transaction costs and enjoy greater productivity.” (Békés and Ottaviano, 2016: p. 29).

location; the third feature refers to the effects of the tighter regulations that in many areas restrict housing density, size and height of buildings etc.; the fourth feature amounts to the favorable tax treatment applied in most countries to residential properties (in particular, to owner-occupied houses), and the fifth feature is associated with the financial innovations (such as the “originate to distribute” model) that have made much easier and cheaper for a large number of households to get a mortgage for the purchase of a house (the so-called “Great Mortgaging”, see Jordà et al., 2014).

The model contained in this paper may account for the first four features, while the role that house prices and collateralized household borrowing play in business cycle phenomena and boom bust episodes like those of the 2000s is beyond its scope.⁴ Indeed, the paper aims at assessing how the incorporation of housing wealth and residential land in an endogenous growth setup may affect the dynamics of the economy and the distribution of income and wealth, net of the short-term fluctuations and additional volatility that money, credit and finance may determine. In this way, this paper can help explaining the important and often neglected influence that housing—and hence, public policy concerning housing—has on long-term inequality and growth. Furthermore, it can show how—once housing wealth and residential land are distinguished from productive capital—endogenous fluctuations may be generated in a model where only fundamentals (initial endowments, preferences and technologies) drive the economy’s dynamics.

The model presented here follows Davis and Heathcote (2007) in treating housing as a bundle of land and residential structures.⁵ However, it differs from their treatment of land since they assume that a fixed

⁴ For a recent survey of the literature on housing in macroeconomics see Piazzesi and Schneider (2016).

⁵ Notice that land’s net share of income is particularly large because land does not depreciate (all its income is net), while residential structures depreciate at a much lower rate than productive capital.

quantity of additional land is inelastically supplied each period (thus, the accumulated amount of land that is combined with residential structures tends to infinity as time goes to infinity), while in the present paper the economy-wide amount of land is fixed. The assumption that the quantity of land is fixed is more appropriate than Davis and Heathcote (2007)'s alternative assumption for studying how land has mattered for the recent performances of the advanced economies, especially in the light of the convincing hypothesis set forth by Knoll et al. (2014). Indeed, the latter point out that the rapid increase in residential land prices observed in the advanced economies between 1950 and 2012 can be attributed to the increasing scarcity of residential land, in sharp contrast with the period from the late 19th to the mid-20th century, when residential land prices remained approximately constant in advanced economies despite substantial population and income growth thanks to the transportation revolution. In fact, after the mid-19th century, faster, cheaper and more comfortable private and public transportation made available land for residential purposes in the outskirts of urban centers, which was previously practically inaccessible. The effect of the transportation revolution on the availability of residential land faded away by the mid-20th century, making plausible the claim that from then on the quantity of residential land has to be considered fixed—and therefore increasingly scarce—in the advanced economies. It is likely that this scarcity has been exacerbated in many of these economies by stricter zoning and land-use restrictions (see Glaeser et al., 2005). The limited supply of residential land can therefore be considered the main cause of the pronounced increase in residential land prices that has driven the rise in housing prices, while the increase in construction costs does not appear to have been a major driver of such a rise. The minor role of construction costs in determining the recent growing trend of housing prices is hardly reconcilable with Borri and Reichlin (2016), whose model explains this trend as mainly due to a sort of

Baumol's cost disease, namely to the systematically lower productivity growth exhibited by the construction sector relatively to the manufacturing sector.

In the model contained in this paper, land is owned and can be traded in each period by the two types of agents populating the economy (capitalists and workers). Consistently with the democratization of home ownership that occurred in the decades following the Second World War in most advanced economies, it is assumed that both types of agents invest in housing, own it and may rent it (or rent it out): housing enters both capitalists and workers' decisions in its dual role of a consumer durable and an asset. In contrast, the stylized fact that in these economies the property of productive assets is concentrated in the hands of a fraction of the population is captured by assuming that only capitalists invest in productive capital and own it.

The distinction between two types of agents allows to study the implications of the inclusion of land and residential structures within a growth model on the distribution of income and wealth, as well as the effects of changes in economic fundamentals and in tax policies concerning land and housing on inequality. This is a value added with respect to models with a unique representative agent, such as Hornstein (2009), and Grossman and Steger (2016). The latter is also characterized by exogenous technological progress and fixed labor supply, thus differing from the formal setup presented here, in which productivity gains are generated by the positive externalities of the learning process taking place when firms utilize their productive assets, and labor effort is the result of the workers' choice between work and leisure. This endogeneization of aggregate productivity advancements and labor supply is instrumental to analyze the impact on long-run growth of adding land and residential structures to the assets held by the agents. Deaton and Laroque (2001) conducted a similar analysis within an overlapping

generations model, where life-cycle retirement saving can be invested in both the accumulation of productive capital and the purchase of residential land, reaching the conclusion that land can crowd out productive capital and reduce welfare in the long run stationary equilibrium. However, given that they consider an unique representative agent living two periods, Deaton and Laroque (2001) provide no hint of the distributive implications of the inclusion of residential land into the model. This holds also for Twaites (2015)'s overlapping generations model, that shows how an (exogenously given) decline in the relative price of productive capital exerts a downward pressure on the interest rate, which reduces the user cost of housing and induces the households to accumulate more debt for buying housing, thus boosting house prices.

The model presented here permits to study the effects of a land value tax on economic growth and inequality. The best known proponent of such a tax was Henry George (1935). More recently, Feldstein (1977) showed that a land tax spurs the accumulation of productive capital, bringing about an increase in steady-state consumption and welfare. However, Calvo et al. (1979) demonstrated that Feldstein's positive effect of a rise in land taxation on capital formation disappears in a framework where the Barro-Ricardo equivalence theorem holds and the tax on land is fully capitalized in its price. Also Eaton (1988) found that, in a finite-lived small-open economy with full international capital mobility, a land tax hike has no effect on the domestic stock of productive capital, since it does not divert domestic saving away from land into productive capital, but rather it lowers the value of foreign investment and hence raises the steady-state welfare of nationals. In contrast, in the life-cycle small open economy with perfect capital mobility and endogenous labor supply modeled by Petrucci (2006), the consequences of land taxation critically depend on how the tax proceeds are used by the government (in particular, when the tax

proceeds are used to cut labor income taxes, land taxation raises domestic wealth and aggregate consumption).

As a matter of fact, one can observe in the advanced economies a variety of favorable tax treatments related to the purchase and possession of houses that do not exist for other forms of investment: exemptions from property tax for the owner's principal place of residence; zero taxation of imputed home rents; tax allowances against home-loan interest payments; house-building subsidies; home-loan subsidies and guarantees, often directed particularly to first-time buyers; rewards to home-loan providers for lending to first time home buyers with poor credit records, etc. The literature on the efficiency costs of these tax subsidies typically focused on the intra-asset distortion caused by overinvestment, and over-consumption, of housing services at a point in time. Skinner (1996) criticizes this literature by arguing that such static measures understate the true cost of these tax subsidies, which make housing a relatively more attractive asset, thus raising its price to the benefit of the current homeowners. Gervais (2002) finds that taxing imputed rents and cutting the deductibility of mortgage interest payments is welfare improving, since in this way one reduces the misallocation of households' savings due to higher (post-tax) return on housing capital relative to business capital. According to Cho and Francis (2011), the preferential tax treatment of housing induces agents to over-invest and over-consume housing. Hence, removing this preferential treatment would cause reallocation toward financial assets and non-housing consumption. Observing that housing takes a sizeable share of intergenerational bequests, Auerbach and Hassett (2015) state that cutting the tax benefits for owner-occupied housing in a progressive manner (together with a deregulation of land use) can be more effective to reduce inequality than the generalized wealth tax

advocated by Piketty. The model developed in the next sections extends the analysis provided by this literature to the effects of changes in the tax treatment of housing on long-run growth and inequality.

3. THE MODEL

We model a closed economy in discrete time with an infinite time horizon. This economy is populated by (dynasties of) workers and capitalists. Capitalists own productive assets (capital) and do not work; in contrast, workers supply labor and do not own productive assets.⁶ Both types of agents consume housing services and a (non-housing) consumption good.

Housing services are produced by combining land and residential structures: houses are combinations of land and residential structures. Both types of agents may own houses and may rent them (or rent them out): they may consume more (less) housing services than those provided by their own properties.

The total quantity of land existing in the economy is fixed. Land can be bought and sold.

In this economy there are also firms that produce a good used both for consumption and for investment. These firms hire labor and rent productive assets. We distinguish between investment in productive assets (equipment, machinery, non-residential structures...) on the one hand and investment in residential structures on the other hand. In this way, we capture the fact that these assets have different uses: a productive asset is entirely used for production, while residential housing is mostly a consumer durable and – to the extent that is used for production – it is for production at home that for the most part is not marketed.⁷

⁶ This classical dichotomy can be microfounded by assuming “that capitalists are on a corner of their labor supply decision due to their wealth, leisure being a normal good” (Judd, 1985: p.84), and that workers do not find valuable to invest in productive assets because of the information, agency and transaction costs associated with holding small amount of them.

⁷ In distinguishing between productive assets and residential assets, I follow Davis and Heathcote (2005).

The government raises taxes (on income and land) and distribute lump-sum tranfers to workers and capitalists.

Finally, agents are assumed to have rational expectations. Since there is no source of random disturbances, this amounts to assume that they have perfect foresight.

2.1 Firms

In each period t , there is a large number (normalized to be one) of identical firms producing the good Y_t according to the following technology:

$$Y_t = A_t K_t^{1-\eta} N_t^\eta, \quad 0 < \eta < 1, \quad (1)$$

where K_t , N_t , and A_t are, respectively, the capital stock, the labor input and the state of technology (total factor productivity) of the representative firm producing Y_t . Total factor productivity is a positive function of the capital installed in the economy: $A_t = K_t^\eta$.⁸ This assumption combines the idea that some learning-by-doing takes place whenever a firm utilizes its capital stock and the idea that knowledge and productivity gains spill over instantly across all firms (see Barro and Sala-i-Martin, 1995). Therefore, in accordance with Frankel (1962), it is supposed that although A_t is endogenous to the economy, each firm takes it as given, since a single firm's decisions have only a negligible impact on the aggregate stock of capital.⁹

The profits of the representative firm, π_t , are given by

$$\pi_t = Y_t - W_t N_t - R_t K_t, \quad (2)$$

⁸ Consistently with this formal set-up, one can interpret technological progress as labor augmenting.

⁹ This amounts to say that technological progress is endogenous to the economy, although it is an unintended by-products of firms' capital investment rather than the result of purposive R&D efforts.

where W_t is the wage and R_t is the capital rental rate in period t . The price of Y_t , which is treated as the numéraire of the system, is normalized to be one.

2.2 Workers

There is a large number (normalized to be n) of identical workers. Each of them owns a piece of land L_{wt} and some residential structure H_{wt} . By combining L_{wt} and H_{wt} , the representative worker generates a quantity $L_{wt}^\phi H_{wt}^{1-\phi}$, $0 < \phi < 1$, of housing services for consumption in period t . In each t , the representative worker may either like to consume more housing services than those generated by his/her own property, thus renting a quantity $h_{wt} > 0$ of housing services to be added to $L_{wt}^\phi H_{wt}^{1-\phi}$ for his/her own consumption, or s/he may like to consume less housing services than $L_{wt}^\phi H_{wt}^{1-\phi}$, thus renting out some of the housing services generated by his/her own property ($h_{wt} < 0$). Furthermore, s/he may decide either to buy an additional piece $s_{wt} \geq 0$ of land so as to increase the total amount of land that s/he will own in the next period, or to sell a piece ($s_{wt} < 0$) of land so as to decrease the total amount of land that s/he will own in the next period. Finally, in each t , the representative worker decides on investment I_{wt}^H in residential structures, on the level E_t of work effort (the maximum possible level is normalized to one), and on the amount C_{wt} of good Y_t to consume. Thus, his/her problem is to determine $\{h_{wt}\}_0^\infty$, $\{E_t\}_0^\infty$, $\{s_{wt}\}_0^\infty$, $\{I_{wt}^H\}_0^\infty$ and $\{C_{wt}\}_0^\infty$ in order to $\text{Max} \sum_{t=0}^{\infty} \rho_w^t u_w(L_{wt}, H_{wt}, h_{wt}, C_{wt}, E_t)$, $0 < \rho_w < 1$, subject to

$$T_{wt} + (1 - \tau)E_t W_t \geq C_{wt} + (1 - b_H \tau)I_{wt}^H + (1 - i_w \tau)P_t h_{wt} + Q_t s_{wt} + \zeta Q_t L_{wt},$$

$$0 < \tau < 1, 0 \leq b_H < 1, 0 \leq \zeta < 1, \quad (3)$$

$$H_{wt+1} = I_{wt}^H + (1 - \delta_H)H_{wt}, \quad 0 < \delta_H < 1, \quad H_{w0} \text{ given}, \quad (4)$$

$$L_{wt+1} = s_{wt} + L_{wt}, \quad L_{w0} \text{ given} \quad (5)$$

where the workers' period utility function $u_w(L_{wt}, H_{wt}, h_{wt}, C_{wt}, E_t)$ is given by

$$u_w(.) = \beta \ln(L_{wt}^\phi H_{wt}^{1-\phi} + h_{wt}) + \gamma \ln(1 - E_t) + (1 - \beta - \gamma) \ln(C_{wt}), \quad \beta > 0, \quad \gamma > 0, \quad \beta + \gamma < 1. \quad (6)$$

Notice that ρ_w is the workers' time-preference parameter, T_{wt} are the net lump-sum transfers that each worker receives from the government, τ is the income tax rate, b_H is the fraction of the investment expenditure in residential structures that can be deducted from taxable income, i_w is a dummy variable assuming value 0 if $h_{wt} > 0$ and value 1 if $h_{wt} < 0$ (it captures the fact that in most countries the income earned by renting out a house is taxed, while the rent paid for a house cannot be deducted from taxable income), P_t is the market price of one unit of housing services (the housing rental rate), Q_t is the market price of one unit of land, ζ is the rate at which the value of land is taxed, and δ_H is the rate at which residential structures depreciate.

2.3 Capitalists

There is a large number (normalized to be one) of identical capitalists. Each of them owns a piece of land $L - nL_{wt}$ (L is the fixed quantity of land existing in the economy) and some residential structure H_{ct} . By combining $L - nL_{wt}$ and H_{ct} , the representative capitalist generates a quantity $(L - nL_{wt})^\phi H_{ct}^{1-\phi}$ of housing services for consumption in period t . In each t , s/he may either rent a quantity $h_{ct} > 0$ of housing services to be added to $(L - nL_{wt})^\phi H_{ct}^{1-\phi}$ for his/her own consumption, or s/he may rent out some housing services ($h_{ct} < 0$). In each t , the representative capitalist has also to decide on the quantity s_{ct} of land to buy

($s_{ct} \geq 0$) or to sell ($s_{ct} < 0$), on investment I_{ct}^H in residential structures, on investment I_{ct}^K in productive assets, and on the amount C_{ct} of good Y_t to consume. Hence, his/her problem is to determine $\{s_{ct}\}_0^\infty$, $\{I_{ct}^H\}_0^\infty$, $\{I_{ct}^K\}_0^\infty$, $\{h_{ct}\}_0^\infty$ and $\{C_{ct}\}_0^\infty$ in order to

$$\text{Max} \sum_{t=0}^{\infty} \rho_c^t u_c(L_{wt}, H_{ct}, h_{ct}, C_{ct}), \quad 0 < \rho_c < 1, \text{ subject to}$$

$$T_{ct} + (1 - \tau)K_t R_t \geq C_{ct} + (1 - i_c \tau)P_t h_{ct} + (1 - b_H \tau)I_{ct}^H + (1 - b_K \tau)I_{ct}^K + Q_t s_{ct} + \zeta Q_t (L - nL_{wt}), \quad 0 \leq b_K < 1, \quad (7)$$

$$H_{ct+1} = I_{ct}^H + (1 - \delta_H)H_{ct}, \quad H_{c0} \text{ given}, \quad (8)$$

$$K_{t+1} = I_t^K + (1 - \delta_K)K_t, \quad 0 < \delta_K < 1, \quad K_0 \text{ given}, \quad (9)$$

$$L - nL_{wt+1} = s_{ct} + L - nL_{wt}, \quad (10)$$

where the capitalists' period utility function $u_c(L_{wt}, H_{ct}, h_{ct}, C_{ct})$ is given by

$$u_c(.) = \alpha \ln[(L - nL_{wt})^\phi H_{ct}^{1-\phi} + h_{ct}] + (1 - \alpha) \ln(C_{ct}), \quad 0 < \alpha < 1. \quad (11)$$

Notice that ρ_c is the capitalists' time-preference parameter, T_{ct} are the net lump-sum transfers that each capitalist receives from the government, b_K is the fraction of the investment expenditure in productive assets that can be deducted from taxable income, i_c is a dummy variable assuming value 0 if $h_{ct} > 0$ and value 1 if $h_{ct} < 0$, and δ_K is the rate at which productive assets depreciate.

2.4 Government

In each t , the government balances its budget constraint:

$$T_{ct} + nT_{wt} = \tau[K_t R_t + nE_t W_t - P_t(i_c h_{ct} + i_w n h_{wt}) - b_H(I_{ct}^H + nI_{wt}^H) - b_K I_{ct}^K] + \zeta Q_t L. \quad (12)$$

Moreover, we assume that the net lump-sum transfers received by the workers is a fixed fraction ξ of the government's revenues (hence, the net lump-sum transfers received by the capitalists is a fraction $1-\xi$ of the government's transfers):

$$nT_{wt} = \xi \{ \tau [K_t R_t + nE_t W_t - P_t(i_c h_{ct} + i_w n h_{wt}) - b_H(I_{ct}^H + nI_{wt}^H) - b_K I_{ct}^K] + \zeta Q_t L \}, \quad 0 < \xi < 1. \quad (13)$$

2.5 Market-clearing conditions

In each t , market clearing in the labor market and in the market for productive assets requires, respectively,

$$nE_t = N_t \quad (14)$$

and

$$K_t^s = K_t^d, \quad (15)$$

where K_t^s are the productive assets supplied by the capitalists and K_t^d are the productive assets rent by the firms.

Market clearing in the market for the good Y_t requires

$$Y_t = C_{ct} + nC_{wt} + I_{ct}^H + nI_{wt}^H + I_{ct}^K. \quad (16)$$

Market clearing in the market for housing services and in the market for land requires, respectively,

$$n h_{wt} + h_{ct} = 0, \quad (17)$$

and

$$n s_{wt} + s_{ct} = 0. \quad (18)$$

Market clearing in these five markets determines the equilibrium values of the prices W_t , R_t , P_t and Q_t , where prices are in units of the numéraire good Y_t .

4. THE EQUILIBRIUM PATH OF THE ECONOMY

One can derive the system of four difference equations in E_t , $V_t \equiv \frac{H_{ct}}{H_{wt}}$, $F_t \equiv \frac{h_{wt}}{H_{ct}^{1-\phi}}$ and L_{wt} governing the equilibrium path of the economy from the conditions that firms, workers and capitalists must satisfy for optimization, from the government budget constraint and from the market-clearing conditions (see A1 in the Appendix).

3.1 Optimality

For optimality the workers must equalize in every t the increment in utility that they derive from the consumption of the additional quantity of C_{wt} that they can buy by spending the increase in the (after-tax) labor income obtainable by providing one more unit of effort to the increment in utility that they can obtain by using this additional income for consuming more housing services, and to the increment in utility that they can derive by non providing that unit of effort. Together with this intratemporal condition, the workers must also satisfy two intertemporal conditions. The first relates to the choice of how much land to buy (or sell) in t , and it amounts to equalize the increment in current utility obtainable by increasing consumption rather than devoting this income to the purchase of an additional unit of land to the increment in discounted future utilities brought about by the current purchase of an additional piece of land. The latter is the sum of the discounted increase in next-period utility due to the increment in the consumption of housing services obtainable in $t+1$ thanks to the possession of one additional piece of land and the discounted increase in next-period utility obtainable if the market value of this additional

piece of land were used at the end of $t+1$ (after having paid the tax on it) for boosting consumption. The second intertemporal condition relates to the choice of how much to invest in residential structures: it is optimal for the workers to equalize the increment in current utility obtainable by increasing consumption rather than investing in residential structures (thus enjoying the specific tax credit) to the increment in discounted future utilities brought about by this investment. The latter is the sum of the discounted increase in next-period utility due to the increment in the consumption of housing services obtainable in $t+1$ thanks to the possession of one additional unit of residential structures and the discounted increase in next-period utility obtainable if the market value of this additional unit of residential structures were used at the end of $t+1$ (after having been subject to depreciation) for boosting consumption.

The capitalists must satisfy similar conditions for optimality. However, they do not provide any work effort, and therefore in every t they must only equalize the increment in utility that they can obtain by spending an additional unit of income in the purchase of C_{ct} to the increment in utility that they can obtain by spending this additional unit of income in the purchase of housing services. Moreover, they must satisfy three intertemporal optimality conditions. The first two relate to the choice of how much land to buy (or sell) and how much to invest in residential structures, and they are similar to those satisfied by the workers. The third relates to the choice of how much to invest in productive assets: for optimality, they must equalize the cost—in terms of utility—of the reduction in current consumption necessary to buy an additional unit of productive assets (thus enjoying the specific tax credit) to the increment in discounted future utilities brought about by this investment. The latter is the sum of the discounted increase in next-period utility due to the increment in consumption obtainable in $t+1$ by renting out this additional unit of productive assets and the discounted increase in next-period utility obtainable if the

market value of this additional unit of productive assets were used at the end of $t+1$ (after having been subject to depreciation) for boosting consumption.

It is worth to emphasize that long-run GDP growth is critically dependent on the rate at which capitalists accumulate productive assets, which is influenced by the relative convenience of investing in land and in residential structures: for instance, the expectation of large capital gains associated to the possession of land may make this form of investment more attractive, thus inducing the capitalists to invest less in productive assets.

The accumulation of productive assets is also sensitive to the level of effort supplied by the workers, since—other things being equal—the expected return on investment in productive assets is raised by a higher level of work effort. In its turn, the latter can be influenced by the workers' housing wealth, which reflects the value of the land and residential structures owned by the workers.

Government policies affecting the relative convenience of investing in residential structures (for instance, by setting b_H , i.e., the fraction of investment expenditure in residential structures that is tax deductible) and possessing land (for instance, by setting ζ , i.e., the rate at which the value of land is taxed) can influence the distribution of income and wealth between workers and capitalists, and at the same time they can have effects on long-run GDP growth.

3.2 Balanced growth path

To study the long-run behavior of the economy, one can derive the balanced growth path (BGP) values of E_t , V_t , F_t and L_{wt} (see A2 in the Appendix). Along a BGP, workers' effort is constant at level E^* , workers' property of land is constant at L_w^* and capital's rental rate is constant at R^* (the asterisk denotes the BGP value of a variable). In contrast, the economy's GDP, Y_t , C_{wt} , C_{ct} , I_{ct}^H , I_{wt}^H , I_{ct}^K , K_t , H_{wt} , H_{ct} , W_t

and the price of land Q_t grow along a BGB at the fixed rate $g(E^*)-1$ (see A1 and A2 in the Appendix). In particular, notice that Q_t (the price of land in units of Y_t), H_{wt} (the stock of residential structures owned by the workers) and H_{ct} (the stock of residential structures owned by the capitalists) grow forever along a BGB at the same rate as the economy's GDP. Finally, the BGB rate of growth of the housing services h_{wt} and h_{ct} is $[g(E^*)]^{1-\phi}-1$, while their price P_t grows along a BGB at the fixed rate $[g(E^*)]^\phi-1$, which implies that in an economy displaying perpetual growth ($g(E^*)>1$) the value of housing services grows forever along a BGB at the same rate as the economy's GDP, while both the quantity of housing services and their price grow slower than it.

Some insights about the possible effects on BGB growth of changes in the government tax policy and in the values of critical parameters can be provided by numerical examples.

3.3 Effects on BGB growth and on the distribution of income and wealth of shifting the taxation away from income and towards the property of land (numerical example)

Let $\alpha=0.55$; $\beta=0.20$; $\gamma = 0.52$; $\eta=0.60$; $\xi = 0.45$; $\phi=0.20$; $\delta_H=0.03$; $\delta_K=0.10$; $\rho_C=0.96$; $\rho_W=0.87$; $L=30$; $b_H=0.20$; $b_K=0.05$, and $n=3$. Given these parameter values, one can compare the BGB values of the endogenous variables for two different pairs of values of τ , that is the fixed rate at which all types of income are taxed, and ζ , that is the fixed rate at which the value of land is taxed (see Table 2).

It is significant that the BGB associated with a larger ζ and a marginally smaller τ is characterized by a slightly higher rate of GDP growth, a more equalitarian income and wealth distribution, a lower wealth-income ratio, and a larger productive capital-income ratio. In the light of this numerical experiment, one can argue that shifting taxation away from income and towards a non-reproducible factor as land favors the investment in productive assets, whose net-of tax returns are depressed by a larger τ , thus boosting

long-run growth, and at the same time it makes the income and wealth distribution more favorable to those who own a smaller fraction of the total land existing in the economy.

TABLE 2. BGP values for ($\tau=0.45$ and $\zeta=0.0537$) and ($\tau=0.44$ and $\zeta=0.0681$)

Variable	if $\tau=0.45$ and $\zeta=0.0537$	if $\tau=0.44$ and $\zeta=0.0681$
GDP rate of growth [$g(E^*)-1$]	0.0807	0.0826
Capitalists' share of income (pre-tax & pre-transfer)	0.6765	0.5854
Capitalists' share of income (post-tax & post-transfer)	0.6843	0.6017
Capitalists' share of wealth	0.96	0.8911
Wealth-income ratio	2.6343	2.2878
Productive capital-income ratio	0.4960	0.5936

3.4 Effects on the distribution of income and wealth of changes in the values of the parameters b_H , α and β (numerical example)

Let $\alpha=0.55$; $\beta=0.20$; $\gamma=0.52$; $\eta=0.60$; $\xi = 0.45$; $\phi=0.20$; $\delta_H=0.03$; $\delta_K=0.10$; $\rho_C=0.96$; $\rho_W=0.87$; $\tau=0.44$; $\zeta=0.0681$; $L=30$; $b_H=0.20$; $b_K=0.05$, and $n=3$. Consider the equilibrium path of the economy associated with these parameter values as the benchmark case.¹⁰ Then, let $b_H=0.21$ (rather than $b_H=0.20$ as in the benchmark)–keeping all the remaining parameter values unchanged–and compute the associated BGP

¹⁰ Given these parameter values, one has: $E^*=0.33$; $V^*=16.881$; $F^*=0.1726$, and $L_w^* = 1.350$.

values of the endogenous variables. Do the same exercise with $\alpha=0.60$ (rather than $\alpha=0.55$) and $\beta=0.25$ (rather than $\beta=0.20$). Table 3 compares the BGP values of some relevant variables in these different cases.

TABLE 3. BGP values for different values of b_H , α and β

Variable	benchmark	if $b_H=0.21$	if $\alpha=0.60$	if $\beta=0.25$
GDP rate of growth [$g(E^*)-1$]	0.0826	0.0826	0.0826	0.0826
Capitalists' share of income (pre-tax & pre-transfer)	0.5854	0.5835	0.6021	0.7390
Capitalists' share of income (post-tax & post-transfer)	0.6017	0.6003	0.6171	0.7361
Capitalists' share of wealth	0.8911	0.8891	0.8988	0.9600
Wealth-income ratio	2.2878	2.2884	2.3621	2.9218

According to this numerical example, a larger b_H , namely a marginal increase in the fraction of investment expenditures in residential structures that is tax deductible, leads in the long-run to an income and wealth distribution that is more favorable to the workers, which—differently from the capitalists—invest only in houses. In contrast, a change in capitalists and/or workers' preferences that gives more weight in their respective utility functions to residential services (i.e., as a result, respectively, of a larger α and a larger β) brings about a BGP distribution of income and wealth that is more favorable to the capitalists. Indeed, the greater importance that the population attaches to the consumption of housing services relative to other consumer goods leads in the long run to a permanent increase in the value of land as a ratio of total wealth, thus giving a relative advantage to those who own a disproportionately large

fraction of total land. At the same time, larger b_H , α and β are associated with a BGP characterized by a higher wealth-income ratio, since they increase the fraction of total investment devoted to the accumulation of residential wealth rather than to the accumulation of productive assets. Notice also that changes in b_H , α and β do not affect the GDP rate of growth along the balanced growth path.

3.5 Transitional path (numerical example)

To study the transitional path, one can linearize the system of four difference equations in E_t , V_t , F_t and L_{wt} that governs the equilibrium trajectory of the economy around (E^*, V^*, F^*, L_w^*) (see A3 in the Appendix). Taking into consideration the parameter values $\alpha=0.55$; $\beta=0.20$; $\gamma=0.52$; $\eta=0.60$; $\xi = 0.45$; $\phi=0.30$; $\delta_H=0.03$; $\delta_K=0.12$; $\rho_C=0.96$; $\rho_W=0.87$; $\tau=0.45$; $\zeta=0.1045$; $L=30$; $b_H=0.20$; $b_K=0.05$, and $n=3$, one can check that the linearized system is saddle-path stable. The initial conditions for V_t and L_{wt} are given. For initial conditions such that at time 0 the capitalists' stock of residential structures as a ratio of the workers' stock of residential structures is higher than its BGP level ($V_0=(1.02)V^*=271.979$) and the amount of land owned by the workers is smaller than its BGP value ($L_{w0}=(0.98)L_w^*=0.1163$), Figures 9-14 display a continuous time approximation of the stepped fluctuations characterizing the dynamics of some variables of particular interest along the saddle path.

FIGURE 9 Productive capital investment-income ratio (rate of productive investment)

FIGURE 10 Productive capital-income ratio

One may tentatively divide this transitional dynamics into four phases. In the initial phase (approximately from $t=0$ to $t=30$), investment in productive assets is a large—although declining—fraction of total income (see Figure 9). As a result of this, the productive assets-income ratio is on the

rise (see Figure 10). At the same time, investment in residential structures (especially on the part of workers) is such that also the residential structures-income ratio grows in this phase (see Figure 11). It is not surprising, therefore, that at the end of this phase the economy's wealth-income ratio is larger than at the beginning (see Figure 12). Notice also that the capitalists' share of both wealth and income declines during phase 1, thus increasing the workers' share (see Figures 13 and 14). In phase two (approximately from $t=30$ to $t=60$), the value of productive assets starts declining as a ratio of total income (see Figure 10), while the other variables taken into consideration continue the trends displayed in the previous phase (in particular, the value of the residential structures keeps increasing as a ratio of total income). In phase three (approximately from $t=60$ to $t=80/90$), the capitalists' share of both wealth and income stops declining and starts moving slowly up toward its long-term equilibrium level. In phase four (approximately from $t=80/90$ on), the long rising trend of both the wealth-income ratio and the residential structures-income ratio comes to an end, while the rate of productive investment ceases its long decline.

The fluctuations characterizing the economy along the saddle path are endogenously generated by the fact that out of the BGP the price of land, the stock of productive assets and the stock of residential structures tend to move at different speeds, thus determining movements of the workers' effort level and of the rate of GDP growth around their BGP levels. However, in the numerical example illustrated above, market forces do not allow endogenous variables to diverge endlessly from their BGP values: typically, if the price of land grows faster than its BGP rate $g(E^*)-1$, soon or late this rapid increase slows down, since it cannot be supported by a sufficiently rapid increase in the price of housing services, whose demand is strictly dependent on the population's income.

FIGURE 11 Residential structures-income ratio

FIGURE 12 Wealth-income ratio

FIGURE 13 Capitalists' share of wealth

FIGURE 14 Capitalists' share of income (post-tax & post-transfer)

5. Conclusions

The growth model contained in this paper incorporates productive assets, residential land and residential structures. Moreover, it accounts for the existence of two social classes: the capitalists, who invest both in productive assets and in housing but do not provide labor services, and the workers, who invest only in housing and decide on how much labor effort to provide. Within this formal setup, it is shown that the relative price of land grows in the long run at the same rate as the economy's GDP, while both the quantity of housing services and their price grow slower than it.

Numerical examples show that i) shifting the taxation away from income and towards the property of land enhances long-term GDP growth and leads in the long-run to a more equalitarian (i.e., more favorable to the workers) income and wealth distribution, ii) a marginal increase in the fraction of investment expenditures in residential structures that is tax deductible reduces inequality in the distribution of income and wealth, iii) a change in capitalists and/or workers' preferences that gives more weight in their respective utility functions to residential services leads in the long run to a distribution of income and wealth that is more favorable to the capitalists (indeed, the greater importance that the population attaches to the consumption of housing services relative to other consumer goods tends to augment the value of land as a ratio of total wealth, thus giving a relative advantage to those who own a disproportionally large fraction of total land, i.e., the capitalists), iv) changes in taxation or in preferences increasing the fraction of total investment devoted to the accumulation of residential wealth rather than to the accumulation of productive assets brings about a balanced growth path characterized by a higher wealth-income ratio.

Finally, the present paper illustrates how endogenous fluctuations may be generated along the equilibrium trajectory converging to the balanced growth path, in a model where only fundamentals (initial endowments, preferences and technologies) drive the economy's dynamics and housing wealth—as well as residential land—are distinguished from productive capital. These fluctuations are fed by the fact that out of the balanced growth path the price of land, the stock of productive assets and the stock of residential structures tend to move at different speeds, thus determining movements of the workers' effort level and of the rate of GDP growth around their balanced growth path levels. However, along the equilibrium trajectory converging to the balanced growth path, market forces do not allow endogenous variables to diverge endlessly from their long-run equilibrium values: typically, if the price of land grows faster than its BGP rate, soon or late this rapid increase slows down, since it cannot be supported by a sufficiently rapid increase in the price of housing services, whose demand is strictly dependent on the population's income.

The formal setup developed here can be extended to the case in which agents' preferences are non-homothetic, namely to the case in which—as their income grows and relative prices do not change—agents prefer to devote a larger fraction of their income to the purchase of residential services rather than to the purchase of other consumer goods and services. Future work will be also dedicated to the calibration of the model, so as to assign values consistent with the evidence available for the advanced economies to the parameter values and the initial conditions.

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APPENDIX

A1 Derivation of the difference equations governing an equilibrium path

A1.1 From firms’ profit maximization and labor market equilibrium we obtain:

$$W_t = \eta(nE_t)^{\eta-1} K_t, \quad t=0,1,\dots, \quad (A1)$$

$$R_t = (1-\eta)(nE_t)^\eta, \quad t=0,1,\dots. \quad (A2)$$

A1.2 From the representative worker’s optimization problem we obtain:

$$\frac{(1-\beta-\gamma)}{C_{wt}} = \lambda_{wt}, \quad t=0,1,\dots, \quad (A3)$$

$$\frac{\gamma}{1-E_t} = (1-\tau)W_t \lambda_{wt}, \quad t=0,1,\dots, \quad (A4)$$

$$\frac{\beta}{L_{wt}^\phi H_{wt}^{1-\phi} + h_{wt}} = \begin{cases} P_t \lambda_{wt} & \text{if } h_{wt} \geq 0 \\ (1-\tau)P_t \lambda_{wt} & \text{otherwise, } t=0,1,\dots, \end{cases} \quad (A5)$$

$$\frac{\rho_w \beta \phi \left(\frac{H_{wt+1}}{L_{wt+1}} \right)^{1-\phi}}{L_{wt+1}^\phi H_{wt+1}^{1-\phi} + h_{wt+1}} + \rho_w Q_{t+1} (1 - \zeta) \lambda_{wt+1} = Q_t \lambda_{wt}, t=0,1,\dots, \quad (A6)$$

$$\frac{\rho_w \beta (1 - \phi) \left(\frac{L_{wt+1}}{H_{wt+1}} \right)^\phi}{L_{wt+1}^\phi H_{wt+1}^{1-\phi} + h_{wt+1}} + \rho_w (1 - \mathfrak{b}_H) (1 - \delta_H) \lambda_{wt+1} = (1 - \mathfrak{b}_H) \lambda_{wt}, t=0,1,\dots, \quad (A7)$$

where λ_{wt} is the (current value) multiplier of the Hamiltonian associated with the representative worker's problem.¹¹

A1.3 From the representative capitalist's optimization problem and the equilibrium condition in the market for housing services we obtain:

$$\frac{(1 - \alpha)}{C_{ct}} - \lambda_{ct} = 0, t=0,1,\dots, \quad (A8)$$

$$\frac{\alpha}{(L - nL_{wt})^\phi H_{ct}^{1-\phi} - nh_{wt}} = \begin{cases} (1 - \tau) P_t \lambda_{ct} & \text{if } h_{wt} \geq 0 \\ P_t \lambda_{ct} & \text{otherwise, } t = 0,1,\dots, \end{cases} \quad (A9)$$

$$\frac{\rho_c \alpha \phi \left(\frac{H_{ct+1}}{L - nL_{wt+1}} \right)^{1-\phi}}{(L - nL_{wt+1})^\phi H_{ct+1}^{1-\phi} - nh_{wt+1}} + \rho_c Q_{t+1} (1 - \zeta) \lambda_{ct+1} = Q_t \lambda_{ct}, t=0,1,\dots, \quad (A10)$$

$$\frac{\rho_c \alpha (1 - \phi) \left(\frac{L - nL_{wt+1}}{H_{ct+1}} \right)^\phi}{(L - nL_{wt+1})^\phi H_{ct+1}^{1-\phi} - nh_{wt+1}} + \rho_c (1 - \mathfrak{b}_H) (1 - \delta_H) \lambda_{ct+1} = (1 - \mathfrak{b}_H) \lambda_{ct}, t=0,1,\dots, \quad (A11)$$

$$\rho_c [R_{t+1} (1 - \tau) + (1 - \mathfrak{b}_K) (1 - \delta_K)] \lambda_{ct+1} = (1 - \mathfrak{b}_K) \lambda_{ct}, t=0,1,\dots, \quad (A12)$$

where λ_{ct} is the (current value) multiplier of the Hamiltonian associated with the representative capitalist's problem.¹²

A1.4 Let suppose from now on that $h_{wt} \geq 0$. Hence, from (A3) and (A5) we get

¹¹ For optimality, the representative worker must also satisfy the transversality conditions $\lim_{t \rightarrow \infty} \rho_w^t Q_t L_{wt} \lambda_{wt} = 0$ and

$\lim_{t \rightarrow \infty} \rho_w^t H_{wt} \lambda_{wt} = 0$.

¹² For optimality, the representative capitalist must also satisfy the transversality conditions $\lim_{t \rightarrow \infty} \rho_c^t Q_t (L - nL_{wt}) \lambda_{ct} = 0$,

$\lim_{t \rightarrow \infty} \rho_c^t H_{ct} \lambda_{ct} = 0$ and $\lim_{t \rightarrow \infty} \rho_c^t K_t \lambda_{ct} = 0$.

$$C_{wt} = \frac{(L_{wt}^\phi H_{wt}^{1-\phi} + h_{wt})(1-\beta-\gamma)P_t}{\beta}, t=0,1,\dots, \quad (A13)$$

while from (A8) and (A9) we get

$$P_t = \frac{\alpha C_{ct}}{[(L - nL_{wt})^\phi H_{ct}^{1-\phi} - nh_{wt}](1-\alpha)(1-\tau)}, t=0,1,\dots \quad (A14)$$

Finally, by substituting (A14) for P_t in (A13), we get

$$C_{wt} = \frac{(L_{wt}^\phi H_{wt}^{1-\phi} + h_{wt})(1-\beta-\gamma)\alpha C_{ct}}{\beta[(L - nL_{wt})^\phi H_{ct}^{1-\phi} - nh_{wt}](1-\alpha)(1-\tau)}, t=0,1,\dots \quad (A15)$$

A1.5 By using (A2), (A8) and (A12), one can obtain

$$\frac{C_{ct+1}}{C_{ct}} = g(E_{t+1}), t=0,1,\dots, \quad (A16)$$

$$\text{where } g(E_{t+1}) = \rho_c \left[\frac{(1-\eta)(1-\tau)(nE_{t+1})^\eta}{(1-\mathfrak{z}_K)} + 1 - \delta_K \right].$$

A1.6 By using (A8), (A11) and (A16), one can obtain

$$C_{ct} = H_{ct} d(F_t, L_{wt}, E_t), F_t \equiv \frac{h_{wt}}{H_{ct}^{1-\phi}}, t=1,2,\dots, \quad (A17)$$

$$\text{where } d(F_t, L_{wt}, E_t) = \frac{[(L - nL_{wt})^\phi - nF_t](1-\mathfrak{z}_H)(1-\alpha)[(1-\eta)(1-\tau)(nE_t)^\eta - (1-\mathfrak{z}_K)(\delta_K - \delta_H)]}{\alpha(1-\mathfrak{z}_K)(L - nL_{wt})^\phi(1-\phi)} ..$$

A1.7 By using (A1), (A3), (A4) and (A15), one can obtain

$$\frac{C_{ct}}{K_t} = m(V_t, F_t, L_{wt}, E_t), V_t \equiv \frac{H_{ct}}{H_{wt}}, t=0,1,\dots, \quad (A18)$$

$$\text{where } m(V_t, F_t, L_{wt}, E_t) = \frac{[(L - nL_{wt})^\phi - nF_t]\beta\eta(1-\alpha)(1-\tau)^2(1-E_t)}{\alpha\gamma(nE_t)^{1-\eta}(L_{wt}^\phi V_t^{\phi-1} + F_t)}.$$

Notice that it derives from (A17) and (A18) that

$$\frac{H_{ct}}{K_t} = \frac{m(V_t, F_t, L_{wt}, E_t)}{d(F_t, L_{wt}, E_t)} = \frac{(1-E_t)(1-\tau)^2\beta\eta(1-\mathfrak{z}_K)(L - nL_{wt})^\phi(1-\phi)(L_{wt}^\phi V_t^{\phi-1} + F_t)^{-1}}{\gamma(1-\eta)(1-\tau)(nE_t)^\eta - (1-\mathfrak{z}_K)(\delta_K - \delta_H)](1-\mathfrak{z}_H)(nE_t)^{1-\eta}}, t=0,1,\dots \quad (A19)$$

A1.8 By using (A3), (A6) and (A7), one can obtain

$$\begin{aligned}
& \frac{\beta\phi\left(\frac{H_{wt+1}}{L_{wt+1}}\right)^{1-\phi}}{L_{wt+1}^\phi H_{wt+1}^{1-\phi} + h_{wt+1}} + \frac{Q_{t+1}(1-\zeta)(1-\beta-\gamma)}{C_{wt+1}} = \\
& = Q_t \left[\frac{\beta(1-\phi)\left(\frac{L_{wt+1}}{H_{wt+1}}\right)^\phi}{(1-\mathfrak{z}_H)[L_{wt+1}^\phi H_{wt+1}^{1-\phi} + h_{wt+1}]} + \frac{(1-\beta-\gamma)(1-\delta_H)}{C_{wt+1}} \right], t = 0, 1, \dots
\end{aligned} \tag{A20}$$

A1.9 By using (A8), (A10) and (A16), one can obtain

$$\begin{aligned}
Q_t = & \frac{\rho_c \alpha \phi \left(\frac{H_{ct+1}}{L - nL_{wt+1}} \right)^{1-\phi} C_{ct}}{[(L - nL_{wt+1})^\phi H_{ct+1}^{1-\phi} - nh_{wt+1}](1-\alpha)} + \\
& + \frac{(1-\zeta)(1-\mathfrak{z}_K)Q_{t+1}}{(1-\eta)(1-\tau)(nE_{t+1})^\eta + (1-\mathfrak{z}_K)(1-\delta_K)}, t = 0, 1, \dots
\end{aligned} \tag{A21}$$

A1.10 By using (A16) and (A21) for substituting Q_t , (A20) can be rewritten as

$$\begin{aligned}
& \frac{(1-\zeta)Q_{t+1}(1-\beta-\gamma)}{C_{wt+1}} \left[1 - \frac{(1-\mathfrak{z}_K)(1-\delta_H)}{(1-\eta)(1-\tau)(nE_{t+1})^\eta + (1-\mathfrak{z}_K)(1-\delta_K)} \right] - \\
& - \frac{(1-\zeta)Q_{t+1}\beta(1-\phi)(1-\mathfrak{z}_K)L_{wt+1}^\phi V_{t+1}[L_{wt+1}^\phi + V_{t+1}^{1-\phi}F_{t+1}]^{-1}}{(1-\mathfrak{z}_H)[(1-\eta)(1-\tau)(nE_{t+1})^\eta + (1-\mathfrak{z}_K)(1-\delta_K)]H_{ct+1}} = - \frac{\beta\phi L_{wt+1}^{\phi-1}}{(L_{wt+1}^\phi + V_{t+1}^{1-\phi}F_{t+1})} + \\
& + \frac{\alpha\phi[(1-\eta)(1-\tau)(nE_{t+1})^\eta + (1-\mathfrak{z}_K)(1-\delta_K)]^{-1}(1-\mathfrak{z}_K)C_{ct+1}}{(L - nL_{wt+1})^{1-\phi}[(L - nL_{wt+1})^\phi - nF_{t+1}](1-\alpha)} X \\
& X \left[\frac{\beta(1-\phi)L_{wt+1}^\phi V_{t+1}}{(1-\mathfrak{z}_H)(L_{wt+1}^\phi + V_{t+1}^{1-\phi}F_{t+1})H_{ct+1}} + \frac{(1-\beta-\gamma)(1-\delta_H)}{C_{wt+1}} \right], t = 0, 1, \dots
\end{aligned} \tag{A22}$$

A1.11 By using (A15), (A17) and (A22), one can obtain

$$Q_t = \frac{H_{ct}q(V_t, L_{wt}, E_t)}{(1-\zeta)}, t=1, 2, \dots, \tag{A23}$$

$$\text{where } q(V_t, L_{wt}, E_t) = \frac{\phi(1-\mathfrak{z}_H)V_t L_{wt}^\phi [(1-\eta)(1-\tau)(nE_t)^\eta - (1-\mathfrak{z}_K)(\delta_K - \delta_H)]}{(1-\phi)(1-\mathfrak{z}_K)(L - nL_{wt})[(L - nL_{wt})^\phi V_t^{1-\phi}(1-\tau) - V_t L_{wt}^\phi]} +$$

$$+ \frac{\phi(1-\mathfrak{b}_H) \left\{ \frac{V_t^{1-\phi}(1-\tau)(1-\mathfrak{b}_K)(1-\delta_H)}{(L-nL_{wt})^{1-\phi}} - \frac{[(1-\eta)(1-\tau)(nE_t)^\eta + (1-\mathfrak{b}_K)(1-\delta_K)]}{L_{wt}^{1-\phi}} \right\}}{(1-\phi)(1-\mathfrak{b}_K)[(L-nL_{wt})^\phi V_t^{1-\phi}(1-\tau) - V_t L_{wt}^\phi]}.$$

A1.12 By dividing the period budget constraint of the representative worker by C_{ct} , one can obtain:

$$\Psi(E_{t+1}, V_{t+1}, F_{t+1}, L_{wt+1}, E_t, V_t, F_t, L_{wt}) = 0, t=1, 2, \dots, \quad (A24)$$

$$\begin{aligned} \text{where } \Psi(.) = & \frac{\xi\tau}{n} \left\{ \frac{[(nE_t)^\eta + b_K(1-\delta_K)]}{m(V_t, F_t, L_{wt}, E_t)} - \frac{b_K g(E_{t+1})}{m(V_{t+1}, F_{t+1}, L_{wt+1}, E_{t+1})} + \frac{\alpha n F_t}{(1-\alpha)(1-\tau)[(L-nL_{wt})^\phi - nF_t]} + \right. \\ & + \frac{b_H(1-\delta_H)}{d(F_t, L_{wt}, E_t)} \left(1 + \frac{n}{V_t} \right) - \frac{b_H g(E_{t+1})}{d(F_{t+1}, L_{wt+1}, E_{t+1})} \left(1 + \frac{n}{V_{t+1}} \right) \left. \right\} + \frac{\xi\zeta q(V_t, L_{wt}, E_t)L}{n(1-\zeta)d(F_t, L_{wt}, E_t)} + \frac{(nE_t)^\eta \eta(1-\tau)}{nm(V_t, F_t, L_{wt}, E_t)} \\ & + \frac{(1-\mathfrak{b}_H)(1-\delta_H)}{V_t d(F_t, L_{wt}, E_t)} - \frac{(1-\mathfrak{b}_H)g(E_{t+1})}{V_{t+1} d(F_{t+1}, L_{wt+1}, E_{t+1})} - \frac{\alpha[(1-\beta-\gamma)L_{wt}^\phi V_t^{\phi-1} + (1-\gamma)F_t]}{(1-\alpha)(1-\tau)\beta[(L-nL_{wt})^\phi - nF_t]} - \frac{q(V_t, L_{wt}, E_t)[L_{wt+1} - L_{wt}(1-\zeta)]}{(1-\zeta)d(F_t, L_{wt}, E_t)}. \end{aligned}$$

A1.13 By dividing the period budget constraint of the representative capitalist by C_{ct} , one can obtain:

$$\Omega(E_{t+1}, V_{t+1}, F_{t+1}, L_{wt+1}, E_t, V_t, F_t, L_{wt}) = 0, t=1, 2, \dots, \quad (A25)$$

$$\begin{aligned} \text{where } \Omega(.) = & (1-\xi)\tau \left\{ \frac{[(nE_t)^\eta + b_K(1-\delta_K)]}{m(V_t, F_t, L_{wt}, E_t)} - \frac{b_K g(E_{t+1})}{m(V_{t+1}, F_{t+1}, L_{wt+1}, E_{t+1})} + \frac{\alpha n F_t}{(1-\alpha)(1-\tau)[(L-nL_{wt})^\phi - nF_t]} + \right. \\ & + \frac{b_H(1-\delta_H)}{d(F_t, L_{wt}, E_t)} \left(1 + \frac{n}{V_t} \right) - \frac{b_H g(E_{t+1})}{d(F_{t+1}, L_{wt+1}, E_{t+1})} \left(1 + \frac{n}{V_{t+1}} \right) \left. \right\} - \frac{\xi\zeta q(V_t, L_{wt}, E_t)L}{(1-\zeta)d(F_t, L_{wt}, E_t)} + \frac{(nE_t)^\eta (1-\eta)(1-\tau)}{m(V_t, F_t, L_{wt}, E_t)} \\ & + \frac{(1-\mathfrak{b}_H)(1-\delta_H)}{d(F_t, L_{wt}, E_t)} - \frac{(1-\mathfrak{b}_H)g(E_{t+1})}{d(F_{t+1}, L_{wt+1}, E_{t+1})} + \frac{\alpha n F_t}{(1-\alpha)[(L-nL_{wt})^\phi - nF_t]} + \frac{q(V_t, L_{wt}, E_t)n[L_{wt+1} - L_{wt}(1-\zeta)]}{(1-\zeta)d(F_t, L_{wt}, E_t)} \\ & - 1 - \frac{(1-\mathfrak{b}_K)g(E_{t+1})}{m(V_{t+1}, F_{t+1}, L_{wt+1}, E_{t+1})} + \frac{(1-\mathfrak{b}_K)(1-\delta_K)}{m(V_t, F_t, L_{wt}, E_t)}. \end{aligned}$$

A1.14 From (A6), one can obtain:

$$\Gamma(E_{t+1}, V_{t+1}, F_{t+1}, L_{wt+1}, E_t, V_t, F_t, L_{wt}) = 0, t=1, 2, \dots, \quad (A26)$$

$$\begin{aligned} \text{where } \Gamma(.) = & \frac{\rho_w \beta \phi L_{wt+1}^{\phi-1} V_{t+1}^{\phi-1}}{L_{wt+1}^\phi V_{t+1}^{\phi-1} + F_{t+1}} + \frac{\rho_w (1-\alpha)\beta(1-\tau)q(V_{t+1}, L_{wt+1}, E_{t+1})[(L-nL_{wt+1})^\phi - nF_{t+1}]}{\alpha(L_{wt+1}^\phi V_{t+1}^{\phi-1} + F_{t+1})d(F_{t+1}, L_{wt+1}, E_{t+1})} \\ & - \frac{(1-\alpha)\beta(1-\tau)q(V_t, L_{wt}, E_t)[(L-nL_{wt})^\phi - nF_t]}{\alpha(L_{wt}^\phi V_t^{\phi-1} + F_t)d(F_t, L_{wt}, E_t)(1-\zeta)}. \end{aligned}$$

A1.15 From (A7), one can obtain:

$$\Lambda(E_{t+1}, V_{t+1}, F_{t+1}, L_{wt+1}, V_t, F_t, L_{wt}) = 0, t=1, 2, \dots, \quad (A27)$$

$$\text{where } \Lambda(.) = \frac{\rho_w \alpha (1-\phi) L_{wt+1}^\phi V_{t+1}^\phi d(F_{t+1}, L_{wt+1}, E_{t+1})}{(1-\alpha)(1-\tau)[(L-nL_{wt+1})^\phi - nF_{t+1}]} + \rho_w (1-\mathfrak{b}_H)(1-\delta_H) -$$

$$- \frac{(1-\mathfrak{b}_H)(L_{wt+1}^\phi V_{t+1}^{\phi-1} + F_{t+1})[(L-nL_{wt})^\phi - nF_t]g(E_{t+1})}{(L_{wt}^\phi V_t^{\phi-1} + F_t)[(L-nL_{wt+1})^\phi - nF_{t+1}]}.$$

Notice that (A24)-(A27) is a system of difference equations in E_t, V_t, F_t and L_{wt} that governs the equilibrium path of the economy.

A2 Balanced growth path (BGP)

A2.1 By setting $E_{t+1} = E_t = E$, $V_{t+1} = V_t = V$, $F_{t+1} = F_t = F$ and $L_{wt+1} = L_{wt} = L_w$, equation (A27) becomes

$$\frac{\rho_w L_w^\phi V^\phi [(1-\eta)(1-\tau)(nE)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)]}{(1-\tau)(1-\mathfrak{b}_K)(L-nL_w)^\phi} + \rho_w (1-\delta_H) - g(E) = 0, \text{ from which one can obtain}$$

$$V = [\chi(E)]^\frac{1}{\phi} \frac{(L-nL_w)}{L_w}, \quad (\text{A28})$$

$$\text{where } \chi(E) = \frac{[g(E) - \rho_w (1-\delta_H)](1-\mathfrak{b}_K)(1-\tau)}{\rho_w [(1-\eta)(1-\tau)(nE)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)]}.$$

A2.2 By setting $E_{t+1} = E_t = E$, $F_{t+1} = F_t = F$, $V_{t+1} = V_t = V$ and $L_{wt+1} = L_{wt} = L_w$, equation (A26) becomes

$$\rho_w L_w^{\phi-1} V^{\phi-1} - \frac{\left[\frac{1}{(1-\zeta)} - \rho_w \right] (1-\tau)(L-nL_w)^\phi}{[(1-\eta)(1-\tau)(nE)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)][(L-nL_w)^\phi V^{-\phi} (1-\tau) - L_w^\phi]} \left\{ \frac{(1-\mathfrak{b}_K \tau)(1-\tau)(1-\delta_H)}{(L-nL_{wt})^{1-\phi} V^\phi} + \right.$$

$$\left. + \frac{[(1-\eta)(1-\tau)(nE)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)]L_w^\phi}{(L-nL_w)} - \frac{(1-\mathfrak{b}_K \tau)g(E)}{\rho_c V L_w^{1-\phi}} \right\} = 0, \text{ which—by using (A28) to substitute for } V\text{—can be}$$

rewritten as

$$e(E) = 0, \quad (\text{A29})$$

$$\text{where } e(E) = \rho_w [(1-\eta)(nE)^\eta (1-\tau) - (1-\mathfrak{b}_K \tau)(\delta_K - \delta_H)] - [g(E) - \rho_w (1-\delta_H)](1-\mathfrak{b}_K \tau) + \left[\frac{1}{(1-\zeta)} - \rho_w \right] \left\{ \frac{(1-\mathfrak{b}_K \tau)}{\rho_c} - \right.$$

$$\left. - \frac{[(1-\eta)(nE)^\eta (1-\tau) - (1-\mathfrak{b}_K \tau)(\delta_K - \delta_H)][\chi(E)]^\frac{1}{\phi}}{[g(E) - \rho_w (1-\delta_H)]} \right\} g(E).$$

Notice that any value of E satisfying (A29) and such that $g(E) > \rho_w(1 - \delta_H)$, say E^* , is a BGP value of E_t .

A2.3 By setting $E_{t+1} = E_t = E$, $F_{t+1} = F_t = F$, $V_{t+1} = V_t = V$ and $L_{wt+1} = L_{wt} = L_w$, equation (A24) becomes

$$\begin{aligned} & \frac{\xi\tau\alpha\gamma(L_w^\phi V^{\phi-1} + F)[(nE)^\eta - b_K g(E) + b_K(1 - \delta_K)]}{n(1 - \alpha)(1 - \tau)^2 \beta \eta (nE)^{\eta-1} (1 - E)} + \frac{\xi\tau\alpha F}{(1 - \alpha)(1 - \tau)} - \left(\frac{n}{V} + 1\right)[g(E) - 1 + \delta_H]X \\ & X \frac{\xi\tau\alpha(L - nL_w)^\phi (1 - \phi)(1 - \phi_K)b_H(1 - \phi_H)^{-1}}{n(1 - \alpha)[(1 - \tau)(1 - \eta)(nE)^\eta - (1 - \phi_K)(\delta_K - \delta_H)]} - \frac{\alpha[(1 - \beta)(1 - E) - \gamma](L_w^\phi V^{\phi-1} + F)}{(1 - \alpha)(1 - \tau)\beta(1 - E)} - \frac{\alpha F}{(1 - \alpha)(1 - \tau)} \\ & - \frac{\alpha(L - nL_w)^\phi (1 - \phi)(1 - \phi_K)(1 - \alpha)^{-1}}{[(1 - \tau)(1 - \eta)(nE)^\eta - (1 - \phi_K)(\delta_K - \delta_H)]} \left\{ \frac{[g(E) - 1 + \delta_H]}{V} - \frac{\xi q(V, L_w, E)}{(1 - \zeta)(1 - \phi_H)} \left(\frac{\xi L}{n} - L_w \right) \right\} = 0, \end{aligned}$$

from which (by using (A28) to substitute for V and by setting $E=E^*$) one can obtain

$$F = z(L_w, E^*), \quad (A30)$$

$$\begin{aligned} \text{where } z(\cdot) = & \left\{ \frac{\alpha L_w [\chi(E^*)]^\frac{\phi-1}{\phi} [(1 - \beta)(1 - E^*) - \gamma]}{\beta(1 - E^*)(1 - \tau)(L - nL_w)^{1-\phi}} - \frac{\xi\tau\alpha\gamma L_w [\chi(E^*)]^\frac{\phi-1}{\phi} [(nE^*)^\eta - b_K g(E^*) + b_K(1 - \delta_K)]}{n(1 - \tau)^2 \beta \eta (nE^*)^{\eta-1} (1 - E^*)(L - nL_w)^{1-\phi}} + \right. \\ & + \left[\frac{\xi\phi_H}{n(1 - \phi_H)} \left(\frac{nL_w [\chi(E^*)]^\frac{1}{\phi}}{(L - nL_w)} + 1 \right) + \frac{L_w [\chi(E^*)]^\frac{1}{\phi}}{(L - nL_w)} \right] \frac{\alpha(L - nL_w)^\phi (1 - \phi)(1 - \phi_K)[g(E^*) - 1 + \delta_H]}{[(1 - \tau)(1 - \eta)(nE^*)^\eta - (1 - \phi_K)(\delta_K - \delta_H)]} + \\ & + \frac{\xi\alpha\phi(\xi L - nL_w) \left[(1 - \tau)(nE^*)^\eta (1 - \eta) - [\chi(E^*)]^\frac{1-\phi}{\phi} (1 - \tau)(1 - b_K \tau)(1 - \delta_H) + (1 - \delta_K)(1 - b_K \tau) \right]}{(1 - \zeta)n[\chi(E^*)]^\phi [(1 - \tau)(1 - \eta)(nE^*)^\eta - (1 - \phi_K)(\delta_K - \delta_H)][\chi(E^*)]^{-1}(1 - \tau) - 1] (L - nL_w)^{1-\phi}} \\ & - \frac{\xi\alpha\phi(\xi L - nL_w)(L - nL_w)^{\phi-1}}{(1 - \zeta)n[\chi(E^*)]^{-1}(1 - \tau) - 1] \int \left\{ \frac{\xi\tau\alpha\gamma[(nE^*)^\eta - b_K g(E^*) + b_K(1 - \delta_K)]}{n(1 - \tau)^2 \beta \eta (nE^*)^{\eta-1} (1 - E^*)} + \frac{\xi\tau\alpha}{(1 - \tau)} - \frac{\alpha(1 - \gamma - E^*)}{\beta(1 - \tau)(1 - E^*)} \right\}^{-1}}. \end{aligned}$$

A2.4 By setting $E_{t+1} = E_t = E$, $F_{t+1} = F_t = F$, $V_{t+1} = V_t = V$ and $L_{wt+1} = L_{wt} = L_w$, equation (A25) becomes

$$\frac{nF}{(1 - \alpha)} + \frac{(1 - \xi)\tau\alpha\gamma(L_w^\phi V^{\phi-1} + F)[(nE)^\eta - b_K g(E) + b_K(1 - \delta_K)]}{(1 - \alpha)(1 - \tau)^2 \beta \eta (nE)^{\eta-1} (1 - E)} + \frac{(1 - \xi)n\tau\alpha F}{(1 - \alpha)(1 - \tau)} - (L - nL_w)^\phi - \left(\frac{n}{V} + 1\right)X$$

$$\begin{aligned}
& X \frac{[g(E) - 1 + \delta_H](1 - \xi)\tau\alpha(L - nL_w)^\phi(1 - \phi)(1 - \mathfrak{b}_K)b_H}{(1 - \alpha)(1 - \mathfrak{b}_H)[(1 - \tau)(1 - \eta)(nE)^\eta - (1 - \mathfrak{b}_K)(\delta_K - \delta_H)]} - \frac{\zeta\alpha(L - nL_w)^\phi(1 - \phi)(1 - \mathfrak{b}_K)(\xi L - nL_w)q(V, L_w, E^*)}{(1 - \zeta)(1 - \mathfrak{b}_H)(1 - \alpha)[(1 - \tau)(1 - \eta)(nE)^\eta - (1 - \mathfrak{b}_K)(\delta_K - \delta_H)]} \\
& - \frac{\alpha(1 - \mathfrak{b}_K)[g(E) - 1 + \delta_H](L - nL_w)^\phi(1 - \phi)}{(1 - \alpha)[(1 - \tau)(1 - \eta)(nE)^\eta - (1 - \mathfrak{b}_K)(\delta_K - \delta_H)]} - \frac{(L_w^\phi V^{\phi-1} + F)(1 - \mathfrak{b}_K)[g(E) - 1 + \delta_K]}{(\alpha\gamma)^{-1}\beta\eta(1 - \alpha)(1 - \tau)^2(1 - E)(nE)^{\eta-1}} + \frac{\alpha\gamma(L_w^\phi V^{\phi-1} + F)(1 - \eta)nE}{(1 - \alpha)(1 - \tau)\eta\beta(1 - E)} = 0, \quad \text{from}
\end{aligned}$$

which (by using (A28) to substitute for V and by setting $E=E^*$) one can obtain

$$F = p(L_w, E^*), \quad (\text{A31})$$

$$\begin{aligned}
\text{where } p(.) = & \left\{ \left[\frac{(1 - \xi)\mathfrak{b}_H}{(1 - \mathfrak{b}_H)} \left(\frac{nL_w[\chi(E^*)]^{\frac{1}{\phi}}}{(L - nL_w)} + 1 \right) + 1 \right] \frac{(L - nL_w)^\phi \alpha(1 - \phi)(1 - \mathfrak{b}_K)[g(E^*) - 1 + \delta_H]}{[(1 - \tau)(1 - \eta)(nE)^\eta - (1 - \mathfrak{b}_K)(\delta_K - \delta_H)]} \right. \\
& - \frac{\zeta\alpha\phi(\xi L - nL_w) \left[(1 - \tau)(nE^*)^\eta(1 - \eta) - [\chi(E^*)]^{\frac{1-\phi}{\phi}}(1 - \tau)(1 - \mathfrak{b}_K\tau)(1 - \delta_H) + (1 - \delta_K)(1 - \mathfrak{b}_K\tau) \right]}{(1 - \zeta)[\chi(E^*)]^{\frac{1}{\phi}}[(1 - \tau)(1 - \eta)(nE^*)^\eta - (1 - \mathfrak{b}_K)(\delta_K - \delta_H)][[\chi(E^*)]^{-1}(1 - \tau) - 1](L - nL_w)^{1-\phi}} \\
& + \frac{\zeta\alpha\phi(\xi L - nL_w)(L - nL_w)^{\phi-1}}{(1 - \zeta)[\chi(E^*)]^{-1}(1 - \tau) - 1} - \frac{(1 - \xi)\tau\alpha L_w[\chi(E^*)]^{\frac{\phi-1}{\phi}}[(nE^*)^\eta - \mathfrak{b}_K g(E^*) + \mathfrak{b}_K(1 - \delta_K)]}{(1 - \tau)^2\beta\eta(nE^*)^{\eta-1}(1 - E^*)(L - nL_w)^{1-\phi}} \\
& + \frac{\alpha\gamma L_w(1 - \mathfrak{b}_K)[\chi(E)]^{\frac{\phi-1}{\phi}}[g(E^*) - 1 + \delta_K]}{(1 - \tau)^2\beta\eta(nE^*)^{\eta-1}(1 - E^*)(L - nL_w)^{1-\phi}} + \frac{(1 - \alpha)}{(L - nL_w)^{-\phi}} - \frac{\alpha\gamma(1 - \eta)L_w nE^*[\chi(E^*)]^{\frac{\phi-1}{\phi}}}{\beta\eta(1 - E^*)(1 - \tau)(L - nL_w)^{1-\phi}} \left\{ \frac{(1 - \xi)n\tau\alpha}{(1 - \tau)} \right. \\
& \left. + \frac{(1 - \xi)\tau\alpha\gamma[(nE^*)^\eta - \mathfrak{b}_K g(E^*) + \mathfrak{b}_K(1 - \delta_K)]}{(1 - \tau)^2\beta\eta(nE^*)^{\eta-1}(1 - E^*)} + \frac{\alpha\gamma nE^*(1 - \eta)}{\eta\beta(1 - E^*)(1 - \tau)} + n - \frac{\alpha\gamma(1 - \mathfrak{b}_K)[g(E^*) - 1 + \delta_K]}{\beta\eta(1 - \tau)^2(1 - E^*)(nE^*)^{\eta-1}} \right\}^{-1}.
\end{aligned}$$

A2.5 By setting $z(L_w, E^*) = p(L_w, E^*)$, one can solve for the BGP value of L_w :

$$L_w^* = l(E^*), \quad (\text{A32})$$

$$\text{where } l(E^*) = \left\{ \left[\frac{\alpha(1 - \phi)(1 - \mathfrak{b}_K)[g(E^*) - 1 + \delta_H]}{[(1 - \tau)(1 - \eta)(nE)^\eta - (1 - \mathfrak{b}_K)(\delta_K - \delta_H)]} \left[\frac{\tau(1 - \xi)b_H}{(1 - \mathfrak{b}_H)} + 1 \right] + \frac{\zeta\alpha\phi\xi}{(1 - \zeta)[\chi(E^*)]^{-1}(1 - \tau) - 1} \right. \right.$$

$$\begin{aligned}
& - \frac{\zeta \alpha \phi \xi \left[(1-\tau)(nE^*)^\eta (1-\eta) - [\chi(E^*)]^{\frac{1-\phi}{\phi}} (1-\tau)(1-b_K \tau)(1-\delta_H) + (1-\delta_K)(1-b_K \tau) \right]}{(1-\zeta)[\chi(E^*)]^\frac{1}{\phi} [(1-\tau)(1-\eta)(nE^*)^\eta - (1-b_K)(\delta_K - \delta_H)][[\chi(E^*)]^{-1}(1-\tau) - 1]} + 1 - \alpha \left\{ \frac{\alpha n E^* (1-\eta)}{\eta \beta (1-E^*)(1-\tau)} + \right. \\
& + \frac{(1-\xi) \tau \alpha \gamma [(nE^*)^\eta - b_K g(E^*) + b_K (1-\delta_K)]}{(1-\tau)^2 \beta \eta (nE^*)^{\eta-1} (1-E^*)} + \frac{(1-\xi) n \tau \alpha}{(1-\tau)} + n - \frac{\alpha \gamma (1-b_K)[g(E^*) - 1 + \delta_K]}{\beta \eta (1-\tau)^2 (1-E^*)(nE^*)^{\eta-1}} \Big\}^{-1} - \\
& - \left\{ \frac{\zeta \alpha \phi \xi \left[(1-\tau)(nE^*)^\eta (1-\eta) - [\chi(E^*)]^{\frac{1-\phi}{\phi}} (1-\tau)(1-b_K \tau)(1-\delta_H) + (1-\delta_K)(1-b_K \tau) \right]}{(1-\zeta)n[\chi(E^*)]^\frac{1}{\phi} [(1-\tau)(1-\eta)(nE^*)^\eta - (1-b_K)(\delta_K - \delta_H)][[\chi(E^*)]^{-1}(1-\tau) - 1]} - \frac{\zeta \alpha \phi \xi (1-\zeta)n^{-1}}{[[\chi(E^*)]^{-1}(1-\tau) - 1]} + \right. \\
& + \frac{\xi \mathfrak{b}_H \alpha (1-\phi)(1-b_K)[g(E^*) - 1 + \delta_H](1-b_H)^{-1}}{n[(1-\tau)(1-\eta)(nE^*)^\eta - (1-b_K)(\delta_K - \delta_H)]} \Bigg\} \left\{ \frac{[(nE^*)^\eta - b_K g(E^*) + b_K (1-\delta_K)]}{n(1-\tau)^2 \beta \eta (nE^*)^{\eta-1} (1-E^*)(\xi \tau \alpha \gamma)^{-1}} - \frac{\alpha(1-\gamma-E^*)}{\beta(1-E^*)(1-\tau)} + \right. \\
& + \frac{\xi \tau \alpha}{(1-\tau)} \Bigg\}^{-1} \left\{ \left\{ \frac{\zeta \alpha \phi (1-\zeta)^{-1}}{[[\chi(E^*)]^{-1}(1-\tau) - 1]} + \left[\frac{\xi \mathfrak{b}_H}{(1-b_H)} \left([\chi(E^*)]^\frac{-1}{\phi} - 1 \right) + [\chi(E^*)]^\frac{-1}{\phi} \right] \frac{(1-\phi)\alpha(1-b_K)[g(E^*) - 1 + \delta_H]}{[(1-\tau)(1-\eta)(nE^*)^\eta - (1-b_K)(\delta_K - \delta_H)]} - \right. \right. \\
& - \frac{\zeta \alpha \phi \left[(1-\tau)(nE^*)^\eta (1-\eta) - [\chi(E^*)]^{\frac{1-\phi}{\phi}} (1-\tau)(1-b_K \tau)(1-\delta_H) + (1-\delta_K)(1-b_K \tau) \right]}{(1-\zeta)[\chi(E^*)]^\frac{1}{\phi} [(1-\tau)(1-\eta)(nE^*)^\eta - (1-b_K)(\delta_K - \delta_H)][[\chi(E^*)]^{-1}(1-\tau) - 1]} + \frac{\alpha[(1-\beta)(1-E^*) - \gamma]\chi(E^*)}{[\chi(E^*)]^\phi \beta (1-\tau)(1-E^*)} - \\
& - \frac{\xi \tau \alpha \gamma [(nE^*)^\eta - b_K g(E^*) + b_K (1-\delta_K)]}{[\chi(E^*)]^\frac{1-\phi}{\phi} n(1-\tau)^2 \beta \eta (nE^*)^{\eta-1} (1-E^*)} \Bigg\} \left\{ \frac{\xi \tau \alpha \gamma [(nE^*)^\eta - b_K g(E^*) + b_K (1-\delta_K)]}{n(1-\tau)^2 \beta \eta (nE^*)^{\eta-1} (1-E^*)} + \frac{\xi \tau \alpha}{(1-\tau)} - \frac{\alpha(1-\gamma-E^*)}{\beta(1-E^*)(1-\tau)} \right\}^{-1} + \\
& + \left\{ n(1-\alpha) + \left[\frac{(1-\xi)\mathfrak{b}_H}{(1-b_H)} \left(1 - [\chi(E^*)]^\frac{-1}{\phi} \right) + 1 \right] \frac{n\alpha(1-\phi)(1-b_K)[g(E^*) - 1 + \delta_H]}{[(1-\tau)(1-\eta)(nE^*)^\eta - (1-b_K)(\delta_K - \delta_H)]} + \frac{\alpha \gamma (1-\eta) n E^* [\chi(E^*)]^\frac{\phi-1}{\phi}}{\beta \eta (1-E^*)(1-\tau)} + \right.
\end{aligned}$$

$$\begin{aligned}
& - \frac{\zeta \alpha \phi n \left[(1-\tau)(nE^*)^\eta (1-\eta) - [\chi(E^*)]^{\frac{1-\phi}{\phi}} (1-\tau)(1-b_K \tau)(1-\delta_H) + (1-\delta_K)(1-b_K \tau) \right]}{(1-\zeta)[\chi(E^*)]^{\frac{1}{\phi}} [(1-\tau)(1-\eta)(nE^*)^\eta - (1-b_K)(\delta_K - \delta_H)][[\chi(E^*)]^{-1}(1-\tau) - 1]} + \frac{\zeta \alpha \phi n (1-\zeta)^{-1}}{[[\chi(E^*)]^{-1}(1-\tau) - 1]} \\
& - \frac{\alpha \gamma (1-b_K)[g(E^*) - 1 + \delta_K][\chi(E^*)]^{\frac{\phi-1}{\phi}}}{(1-\tau)^2 \beta \eta (nE^*)^{\eta-1} (1-E^*)} + \frac{(1-\xi) \tau \alpha \gamma [(nE^*)^\eta - b_K g(E^*) + b_K (1-\delta_K)][\chi(E^*)]^{\frac{\phi-1}{\phi}}}{(1-\tau)^2 \beta \eta (nE^*)^{\eta-1} (1-E^*)} \left\{ \frac{(1-\xi) n \tau \alpha}{(1-\tau)} + \right. \\
& \left. + \frac{(1-\xi) \tau \alpha \gamma [(nE^*)^\eta - b_K g(E^*) + b_K (1-\delta_K)]}{(1-\tau)^2 \beta \eta (nE^*)^{\eta-1} (1-E^*)} + \frac{\alpha \gamma n E^* (1-\eta)}{\eta \beta (1-E^*)(1-\tau)} + n - \frac{\alpha \gamma (1-b_K)[g(E^*) - 1 + \delta_K]}{\beta \eta (1-\tau)^2 (1-E^*)(nE^*)^{\eta-1}} \right\}^{-1}.
\end{aligned}$$

Notice that the BGP value of V_t and F_t are given, respectively, by

$$V^* = [\chi(E^*)]^{\frac{1}{\phi}} \frac{(L - nL_w^*)}{L_w^*} \quad (A33)$$

and

$$F^* = z(L_w^*, E^*) = p(L_w^*, E^*). \quad (A34)$$

A2.6 Along a BGP, total income is given by

$$\begin{aligned}
K_t (nE^*)^\eta + P_t [(L - nL_w^*)^\phi H_{ct}^{1-\phi} + n(L_w^*)^\phi H_{wt}^{1-\phi}] &= \frac{H_{ct} (1-b_H) [(1-\eta)(1-\tau)(nE^*)^\eta - (1-b_K)(\delta_K - \delta_H)]}{(1-b_K)(1-\phi)(1-\tau)(L - nL_w^*)^\phi} X \\
& X \left\{ \frac{\eta E^* [(L_w^*)^\phi (V^*)^{\phi-1} + F^*]}{(1-E^*)(1-\tau)\beta \eta} + (L - nL_w^*)^\phi + n(L_w^*)^\phi (V^*)^{\phi-1} \right\}. \quad (A35)
\end{aligned}$$

Notice that (A35) includes the imputed rent for self-owned housing.

Along a BGP, capitalists' (pre-tax & pre-government transfers) income is given by

$$\begin{aligned}
\frac{K_t (nE^*)^\eta}{(1-\eta)^{-1}} + \frac{P_t (L - nL_w^*)^\phi}{H_{ct}^{\phi-1}} &= \frac{H_{ct} [(1-\eta)(1-\tau)(nE^*)^\eta - (1-b_K)(\delta_K - \delta_H)]}{(1-b_H)^{-1} (1-b_K)(1-\phi)(1-\tau)(L - nL_w^*)^\phi} X \\
& X \left\{ \frac{\eta E^* [(L_w^*)^\phi (V^*)^{\phi-1} + F^*]}{(1-\eta)^{-1} (1-E^*)(1-\tau)\beta \eta} + (L - nL_w^*)^\phi \right\}. \quad (A36)
\end{aligned}$$

Notice that (A36) includes the imputed rent for self-owned housing.

Along a BGP, capitalists' (post-tax & post-government transfers) income is given by

$$\frac{H_{ct} [(1-\eta)(1-\tau)(nE^*)^\eta - (1-b_K)(\delta_K - \delta_H)]}{(1-b_H)^{-1} (1-b_K)(1-\phi)(1-\tau)(L - nL_w^*)^\phi} \left\{ \frac{[(L_w^*)^\phi (V^*)^{\phi-1} + F^*]}{[\eta E^* (1-\eta)]^{-1} (1-E^*)\beta \eta} + (L - nL_w^*)^\phi - \tau \xi n F^* \right\} +$$

$$\begin{aligned}
& + H_{ct} b_H \tau \zeta [g(E^*) - 1 + \delta_H] + \frac{H_{ct} b_K \tau \zeta [(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)] \gamma (1-\mathfrak{b}_H) [g(E^*) - 1 + \delta_K]}{(1-\mathfrak{b}_K)(1-\phi)(L - nL_w^*)^\phi (1-E^*)(nE^*)^{\eta-1} (1-\tau)^2 \beta \eta (L_w^*)^\phi (V^*)^{\phi-1} + F^*} \quad (A37) \\
& - \frac{\zeta H_{ct} \phi}{(1-\zeta)} \left\{ \frac{[(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)]}{(L - nL_w^*)(L_w^*)^{-\phi} (V^*)^{-1}} + \frac{(1-\mathfrak{b}_K)(1-\tau)(1-\delta_H)}{(L - nL_w^*)^{1-\phi} (V^*)^{\phi-1}} - \frac{(1-\eta)(1-\tau)}{(L_w^*)^{1-\phi} (nE^*)^{-\eta}} - \frac{(1-\delta_K)(1-\mathfrak{b}_K)}{(L_w^*)^{1-\phi}} \right\} X \\
& X \frac{(1-\mathfrak{b}_H)(\zeta L - nL_w^*)[(1-\phi)(1-\mathfrak{b}_K)]^1}{[(L - nL_w^*)^\phi (V^*)^{1-\phi} (1-\tau) - V^* (L_w^*)^\phi]} - (1-\zeta) \mathfrak{H}_{ct} \left\{ \frac{[g(E^*) - 1 + \delta_H]}{(nb_H)^{-1} V^*} - \right. \\
& \left. - \frac{(1-\mathfrak{b}_H)[(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)] nE^* \gamma (L_w^*)^\phi (V^*)^{\phi-1} + F^*}{(1-\mathfrak{b}_K)(1-\phi)(L - nL_w^*)^\phi (1-E^*)(1-\tau)^2 \beta \eta} \right\}
\end{aligned}$$

Notice that (A37) includes the imputed rent for self-owned housing.

A2.7 Along a BGP, the stock of productive capital, total wealth and capitalists' wealth are given, respectively, by

$$K_t = \frac{H_{ct} (1-\mathfrak{b}_H) \gamma [(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)] (L_w^*)^\phi (V^*)^{\phi-1} + F^*}{(1-\mathfrak{b}_K)(1-\phi)(nE^*)^{\eta-1} (1-E^*)(1-\tau)^2 \beta \eta (L - nL_w^*)^\phi}, \quad (A38)$$

$$\begin{aligned}
K_t + Q_t L + H_{ct} + nH_{wt} &= \frac{H_{ct} (1-\mathfrak{b}_H) \gamma [(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)] (L_w^*)^\phi (V^*)^{\phi-1} + F^*}{(1-\mathfrak{b}_K)(1-\phi)(nE^*)^{\eta-1} (1-E^*)(1-\tau)^2 \beta \eta (L - nL_w^*)^\phi} + \\
& + H_{ct} + \frac{nH_{ct}}{V^*} + \frac{H_{ct} \phi (1-\mathfrak{b}_H) [(1-\zeta)(1-\phi)(1-\mathfrak{b}_K)]^1 L}{[(L - nL_w^*)^\phi (V^*)^{1-\phi} (1-\tau) - V^* (L_w^*)^\phi]} X \quad (A39)
\end{aligned}$$

$$X \left\{ \frac{[(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)]}{(L - nL_w^*)(L_w^*)^{-\phi} (V^*)^{-1}} + \frac{(1-\mathfrak{b}_K)(1-\tau)(1-\delta_H)}{(L - nL_w^*)^{1-\phi} (V^*)^{\phi-1}} - \frac{(1-\eta)(1-\tau)}{(nE^*)^{-\eta} (L_w^*)^{1-\phi}} - \frac{(1-\delta_K)(1-\mathfrak{b}_K)}{(L_w^*)^{1-\phi}} \right\}$$

and

$$\begin{aligned}
K_t + Q_t (L - nL_w^*) + H_{ct} &= \frac{H_{ct} (1-\mathfrak{b}_H) \gamma [(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)] (L_w^*)^\phi (V^*)^{\phi-1} + F^*}{(1-\mathfrak{b}_K)(1-\phi)(nE^*)^{\eta-1} (1-E^*)(1-\tau)^2 \beta \eta (L - nL_w^*)^\phi} + \\
& + \left\{ \frac{[(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{b}_K)(\delta_K - \delta_H)]}{(L - nL_w^*)(L_w^*)^{-\phi} (V^*)^{-1}} + \frac{(1-\mathfrak{b}_K)(1-\tau)(1-\delta_H)}{(L - nL_w^*)^{1-\phi} (V^*)^{\phi-1}} - \frac{(1-\eta)(1-\tau)}{(nE^*)^{-\eta} (L_w^*)^{1-\phi}} - \frac{(1-\delta_K)(1-\mathfrak{b}_K)}{(L_w^*)^{1-\phi}} \right\} X \\
& X \frac{H_{ct} \phi (1-\mathfrak{b}_H) [(1-\zeta)(1-\phi)(1-\mathfrak{b}_K)]^1 (L - nL_w^*)}{[(L - nL_w^*)^\phi (V^*)^{1-\phi} (1-\tau) - V^* (L_w^*)^\phi]} + H_{ct}. \quad (A40)
\end{aligned}$$

A2.8 Along a BGP, the discounted sequence of utilities of the representative worker is given by

$$\sum_{v=0}^{\infty} \rho_w^v \left\{ \beta \ln \left[H_{ct+v}^{1-\phi} \left(\frac{(L_w^*)^\phi}{(V^*)^{1-\phi}} + F^* \right) \right] + \gamma \ln(1-E^*) + (1-\beta-\gamma) \ln \left[\frac{H_{ct+v} ((L_w^*)^\phi (V^*)^{\phi-1} + F^*) (1-\mathfrak{z}_H)}{(1-\phi)(1-\tau)\beta(1-\beta-\gamma)^{-1}(1-\mathfrak{z}_K)} \right] + \right. \\ \left. + (1-\beta-\gamma) \ln \left[\frac{(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{z}_K)(\delta_K - \delta_H)}{(L - nL_w^*)^\phi} \right] \right\},$$

which can be rewritten as

$$\frac{\gamma \ln(1-E^*)}{1-\rho_w} + \frac{(1-\gamma)}{1-\rho_w} \ln \left(\frac{(L_w^*)^\phi}{(V^*)^{1-\phi}} + F^* \right) + \frac{(1-\beta-\gamma)}{1-\rho_w} \ln \left[\frac{(1-\mathfrak{z}_H)(1-\beta-\gamma)}{(1-\phi)(1-\tau)\beta(1-\mathfrak{z}_K)} \right] + \\ \frac{(1-\beta-\gamma)}{1-\rho_w} \ln \left[\frac{(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{z}_K)(\delta_K - \delta_H)}{(L - nL_w^*)^\phi} \right] + \frac{(1-\beta-\gamma)}{1-\rho_w} \left\{ \ln(H_{ct}) + \frac{\rho_w \ln[g(E^*)]}{1-\rho_w} \right\}. \quad (A41)$$

Along a BGP, the discounted sequence of utilities of the representative capitalist is given by

$$\sum_{v=0}^{\infty} \rho_c^v \left\{ \alpha \ln \left[H_{ct+v}^{1-\phi} ((L - nL_w^*)^\phi - nF^*) \right] + (1-\alpha) \ln \left[\frac{H_{ct+v} ((L - nL_w^*)^\phi - nF^*) (1-\mathfrak{z}_H)(1-\alpha)}{\alpha(1-\phi)(1-\mathfrak{z}_K)} \right] + \right. \\ \left. + (1-\alpha) \ln \left[\frac{(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{z}_K)(\delta_K - \delta_H)}{(L - nL_w^*)^\phi} \right] \right\},$$

which can be rewritten as

$$\frac{\ln((L - nL_w^*)^\phi - nF^*)}{1-\rho_c} + \frac{(1-\alpha)}{1-\rho_c} \ln \left[\frac{(1-\mathfrak{z}_H)(1-\alpha)}{\alpha(1-\phi)(1-\mathfrak{z}_K)} \right] + \frac{(1-\alpha)}{1-\rho_c} \ln \left[\frac{(1-\eta)(1-\tau)(nE^*)^\eta - (1-\mathfrak{z}_K)(\delta_K - \delta_H)}{(L - nL_w^*)^\phi} \right] + \\ + \frac{(1-\alpha\phi)}{1-\rho_c} \left\{ \ln(H_{ct}) + \frac{\rho_c \ln[g(E^*)]}{1-\rho_c} \right\}. \quad (A42)$$

A3 Transitional path

By linearizing the system (A24)-(A27) around (E^*, V^*, F^*, L_w^*) , one obtains the following linearized system:

$$\begin{bmatrix} E_{t+1} - E^* \\ V_{t+1} - V^* \\ F_{t+1} - F^* \\ L_{wt+1} - L_w^* \end{bmatrix} = \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix} \begin{bmatrix} E_t - E^* \\ V_t - V^* \\ F_t - F^* \\ L_{wt} - L_w^* \end{bmatrix}, \quad (A43)$$

where

$$d_{11} \equiv - \frac{[\Gamma_{V_{t+1}} (\Omega_{L_{wt+1}} \Psi_{F_{t+1}} - \Omega_{F_{t+1}} \Psi_{L_{wt+1}}) + \Gamma_{F_{t+1}} (\Psi_{L_{wt+1}} \Omega_{V_{t+1}} - \Omega_{L_{wt+1}} \Psi_{V_{t+1}}) + \Gamma_{L_{wt+1}} (\Omega_{F_{t+1}} \Psi_{V_{t+1}} - \Omega_{V_{t+1}} \Psi_{F_{t+1}})] d_{41}}{\Gamma_{V_{t+1}} (\Psi_{F_{t+1}} \Omega_{E_{t+1}} - \Psi_{E_{t+1}} \Omega_{F_{t+1}}) + \Gamma_{F_{t+1}} (\Psi_{E_{t+1}} \Omega_{V_{t+1}} - \Omega_{E_{t+1}} \Psi_{V_{t+1}}) + \Gamma_{E_{t+1}} (\Omega_{F_{t+1}} \Psi_{V_{t+1}} - \Omega_{V_{t+1}} \Psi_{F_{t+1}})} - \\ - \frac{[\Gamma_{V_{t+1}} (\Omega_{E_t} \Psi_{F_{t+1}} - \Omega_{F_{t+1}} \Psi_{E_t}) + \Gamma_{F_{t+1}} (\Psi_{E_t} \Omega_{V_{t+1}} - \Omega_{E_t} \Psi_{V_{t+1}}) + \Gamma_{E_t} (\Omega_{F_{t+1}} \Psi_{V_{t+1}} - \Omega_{V_{t+1}} \Psi_{F_{t+1}})]}{\Gamma_{V_{t+1}} (\Psi_{F_{t+1}} \Omega_{E_{t+1}} - \Psi_{E_{t+1}} \Omega_{F_{t+1}}) + \Gamma_{F_{t+1}} (\Psi_{E_{t+1}} \Omega_{V_{t+1}} - \Omega_{E_{t+1}} \Psi_{V_{t+1}}) + \Gamma_{E_{t+1}} (\Omega_{F_{t+1}} \Psi_{V_{t+1}} - \Omega_{V_{t+1}} \Psi_{F_{t+1}})},$$

[illegible]

Notice that all partial derivatives are evaluated at (E^*, V^*, F^*, L_w^*) .

The solution to (A43) is

$$E_t - E^* = G_1 j_{11} \mu_1^t + G_2 j_{12} \mu_2^t + G_3 j_{13} \mu_3^t + G_4 j_{14} \mu_4^t, \quad (A44)$$

$$V_t - V^* = G_1 j_{21} \mu_1^t + G_2 j_{22} \mu_2^t + G_3 j_{23} \mu_3^t + G_4 j_{24} \mu_4^t, \quad (A45)$$

$$F_t - F^* = G_1 j_{31} \mu_1^t + G_2 j_{32} \mu_2^t + G_3 j_{33} \mu_3^t + G_4 j_{34} \mu_4^t, \quad (A46)$$

$$L_{wt} - L_w^* = G_1 j_{41} \mu_1^t + G_2 j_{42} \mu_2^t + G_3 j_{43} \mu_3^t + G_4 j_{44} \mu_4^t, \quad (A47)$$

where $\begin{bmatrix} \mu_1 \\ \mu_2 \\ \mu_3 \\ \mu_4 \end{bmatrix}$ and $J \equiv \begin{bmatrix} j_{11} & j_{12} & j_{13} & j_{14} \\ j_{21} & j_{22} & j_{23} & j_{24} \\ j_{31} & j_{32} & j_{33} & j_{34} \\ j_{41} & j_{42} & j_{43} & j_{44} \end{bmatrix}$ are, respectively, the eigenvalues and the eigenvectors of the matrix

$D \equiv \begin{bmatrix} d_{11} & d_{12} & d_{13} & d_{14} \\ d_{21} & d_{22} & d_{23} & d_{24} \\ d_{31} & d_{32} & d_{33} & d_{34} \\ d_{41} & d_{42} & d_{43} & d_{44} \end{bmatrix}$, and $\begin{bmatrix} G_1 \\ G_2 \\ G_3 \\ G_4 \end{bmatrix}$ are constants whose values are determined by using the initial conditions. The

eigenvalues and the eigenvectors can be found by solving, respectively, $\text{Det}(D-M)=0$ and $DJ=JM$, where

$$M \equiv \begin{bmatrix} \mu_1 & 0 & 0 & 0 \\ 0 & \mu_2 & 0 & 0 \\ 0 & 0 & \mu_3 & 0 \\ 0 & 0 & 0 & \mu_4 \end{bmatrix}.$$