When does public debt impair economic growth? A literature review in search of a theory

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When does public debt impair economic growth?
A literature review in search of a theory*

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Abstract
Beyond inconclusive empirical research, this paper examines the theoretical literature concerning public debt and economic growth finding no univocal and straightforward answer. No meaningful assessment of debt and its effect on growth at any point in time is possible without reference to the whole debt trajectory and the specific state of the economy along the trajectory. An orderly and consistent analysis may be developed along two coordinates of debt assessment: sustainability/unsustainability, and efficiency/inefficiency. In our view, research should concentrate on the study of specific conditions and cases, and abandon the pursuit of a general law.

Keywords: Public debt, Debt burden, Debt sustainability, Economic growth, Endogenous growth models.
JEL Codes: E62, H63, O40

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1 Introduction

Policymakers and economists have long been engaged in the debate about the evaluation of public debt, and its impact on economic activity and growth. The large fiscal imbalances created worldwide by the Great Recession of 2008-09 revived general interest in the issue, and moved the attention from the poor and developing countries to the developed ones. The epicentre became Europe, and the Euro Zone in particular, where the Great Recession was followed by acute sovereign debt crises between 2010 and 2012. As a result, many countries implemented, or were forced to implement, also in compliance with the Euro Zone fiscal rules, restrictive fiscal policies aiming at reducing their budget deficit and, eventually, their public debt.

If not dictated by immediate threats, fiscal consolidation, the so-called "austerity", was also prescribed as a requisite for reinstating sound growth conditions before prolonged fiscal stimuli to the economy became self-defeating as public debt was growing too high.

There should be little question that European economies share the need to reduce public deficits and debts from levels that, as confirmed by a growing strand of empirical literature (Reinhart and Rogoff 2010, Kumar and Woo 2012) are likely to be harmful for growth in the medium term [...] (Buti and Pench 2012, p.1)

Reinhart and Rogoff (2010), in a large panel of countries, identified a critical threshold of 90% of the debt-to-GDP ratio beyond which debt is harmful to growth. Several countries in the world, notably in the Euro Zone, were fast approaching that threshold or already were well beyond. The Reinhart-Rogoff finding has spurred a buoyant empirical research in search of the debt threshold above which growth is jeopardised by public debt. Hitherto results of these researches are inconclusive or controversial.

In the first place, the Reinhart-Rogoff work was criticized with regard to the implied causality (Irons and Bivens 2010), and then for some methodological and statistical problems (Herndon et al. 2013). Further works support the existence of critical debt-to-GDP ratios under various time and space observational fields (but there is no agreement on their level: see, among others, Pattillo et al. 2011, Baum et al. 2012, Checherita-Westpahl and Rother 2012). Some authors point out

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2 Their application of the Granger causality test have showed that debt does not cause growth and growth does not cause debt.

3 The analysis was spoiled by coding errors, selective exclusion of available data, and unconventional weighting of summary statistics.
the existence of a positive relationship between debt and growth above a certain threshold (Minea et al. 2012). A third group of studies do not completely deny the existence of a negative relationship between the two variables, but rather claim that a general threshold is unlikely to exist, and it provides no guidance towards the adoption of widespread policies of debt reduction (e.g. Bowdler and Esteves 2013, Pescatori et al. 2014). Country-specific characteristics, contingencies and events play a prominent role, thus prompting a branch of literature that attempts to comprehensively understand the debt-growth relationship and its determinants (Panizza and Presbitero 2014, Eberhardt and Presbitero 2015).

In retrospect, one may say that the empirical pursuit of the debt-to-GDP threshold harmful to growth lacks deeper foundational work. Why should we expect a negative public debt-growth relationship in the first place? If such a relationship exists, why should it take the specific form of a threshold of the debt-GDP ratio, and why should we expect this threshold to be equally valid across time and space?

To address these questions, we refocus attention on the possible theoretical underpinnings of the (negative) debt-growth relationship. In this paper we explore the rather extended range and vintages of theoretical explanations that can be found in the literature, which presents itself as a scattered, heterogeneous and rather coarse constellation of theories, models and case studies. With the aim to provide the reader with an effective guide in the search for explanation of the debt-growth relationship, the material is organised according to distinct, though possibly connected, channels through which debt can impinge on growth negatively: the effects of debt being kept on a solvency path and those due to debt being on an unsustainable path.

The first block, presented in section 2, concerns the evolution of the notion of "debt burden" in public finance where the relevant debt-growth channel goes through the consequences of policies aimed at consolidating debt and secure its solvency. Debt management essentially consists of choosing a sustainable future path of expenditures and taxation, hence the problem is how various forms of expenditure and taxation affect economic activity and growth (e.g. Zagler and Dürnecker 2003).

The second block, in section 3, gathers another specialised literature concerned with the consequences of expected, or effective, debt default. Of course, there are overlaps across this classification, which inevitably involves some degree of arbitrariness. Yet the debt-growth channels involved are quite different from the previous ones. In many cases, default is a policy choice of the government as a result of trading off the costs/benefits of default with those of solvency (e.g. Buiter...
and Rahbari 2013, and Reinhart and Rogoff 2009 for a historical overview). Default may be unexpected, anticipated or even self-generated by creditors; each case has its own specific impact on the economy quite different in nature than those mentioned above.

One main problem emerging from these strands of literature is that a consistent theoretical framework for the debt-growth nexus is missing. Therefore, in section 4 we introduce a third block where the evolution of debt and its effect on growth are cast in an intertemporal horizon such that the whole debt trajectory and the specific state of the economy along the trajectory can be examined. We provide a guideline to this approach by means of a fiscal model of endogenous growth freely inspired to Barro (1990) and Diamond (1965). In Barro (1990) endogenous growth is sustained by productive public expenditure fully covered by taxation. To introduce debt, we adopt a sequential economy with two-period generations à la Diamond (1965) where public expenditure is financed by debt in the first period and the debt burden in covered by taxation in the second period. An advantage of this framework is that the debt-growth relationship can be analysed in four scenarios centred on the notions of efficiency/inefficiency, sustainability/unsustainability of debt, including the case of "voluntary" default.

Our conclusion, to be found in section 5, is that the literature that we have been able to examine and organise, on the one hand offers a rich variety of explanations and insights to researchers of the debt-growth relationship, but on the other it does not provide any one-way conclusion: the relationship may be negative, positive, or even no relationship may exist. Even less is theoretically founded the existence of a general debt-to-GDP threshold above which growth is consistently stifled. Each country's specific characteristics, circumstances, and events have an overwhelming importance that cannot be encapsulated in a single general law. Research should concentrate on the former and abandon the pursuit of the latter.

2 Public finance and the debt burden view

2.1 Debt burden and debt service

The idea that public debt may represent a burden for the economic system has distant origins in public finance. Its focus is on who and how should pay for debt, and with what consequences on the economy. Note that the concept of debt burden is broader than, and should not be confused with, the concept of debt sustainability (to be introduced later in section 3.1) though the two may be related. The impact
of debt repayment on the economy, according to different repayment strategies, is of course central to this analysis. The debt burden is usually identified and described by some indicators, and the most common involve the debt service: the interest-to-debt ratio, the interest-to-GDP ratio, the interest-to-taxation ratio, or the interest-to-export ratio. The amount of external debt with respect to the total outstanding debt is considered as a measure of the external burden, a relevant indicator when the focus is on the foreign creditors.

In the mid-1950s, Sun (1954) distinguished between three interrelated concepts of debt burden: psychological (subjective, and related to people’s confidence in the government debt policies and in the stability of the economy), financial (referred to the amount of taxes required to repay the principal and the interest charges), and real economic burden (related to a decrease in national income, a decline in production, etc.). Sun concluded that the effective burden of the public debt depends on the economic conditions, and that some principles must be respected in order to minimize it. The analysis turned to incentives in Meade (1958), who argued that a reduction in public domestic debt can improve economic incentives, but there could be a cost if the economic conditions worsen in the short-run.

Particularly relevant is the approach put forward by Bohn (1995, 1998). It is based on the fiscal policy reaction function that relates the primary balance, as the control variable, to outstanding debt in such a way that the latter is kept on a path converging to some finite level (see e.g. Greiner et al. 2007, Ghosh et al. 2013, Passamani et al. 2015 for applications to the Euro Zone countries). One main merit of this approach is that it allows for a relatively simple and measurable "fiscal effort" that should consistently be borne by the government (i.e. the relevant economic subjects) over time.

This can easily be seen by means of the standard dynamic equation of the debt/GDP ratio. If $D_t$ is the amount of debt at time $t$, the corresponding debt-to-GDP ratio is $d_t = D_t/Y_t$, and its change over the previous year is given by (approximately)

$$\Delta d_t = (i_t - g_{nt})d_{t-1} - b_t - \mu_t + v_t$$

where $i_t$ is the average interest payment (interest rate for short) on debt, $g_{nt}$ is the nominal growth rate of GDP, and $b_t$, $\mu_t$, $v_t$ are, respectively, the GDP-ratios of the fiscal primary balance, the monetisation of debt, and exogenous changes in the amount of debt. A primary surplus $b_t > 0$ generates a negative impulse to the debt/GDP ratio, and vice versa a primary deficit $b_t < 0$. The government's
commitment to debt control implies that \( b_t \) is targeted to achieving a specified path of \( d_t \), i.e. \( \Delta d^*_{t} \). Therefore,

\[
(2) \quad b^*_{t} = (i_t - g_{nt})d_{t-1} - \Delta d^*_t - \mu_t + v_t
\]

is the necessary fiscal effort in terms of primary surplus.

As long as \( v_t = 0 \), the above expression shows that the extent of the fiscal effort is determined by some key factors. The first is the debt-path target. For instance, the minimal requirement of debt stability, \( \Delta d^*_{t} = 0 \), implies that \( b^*_{t} \) is entirely dependent on the remaining factors. The reformulation of fiscal rules in the Euro Zone introduced with the so-called Six Pack and Fiscal Compact, state that \( \Delta d^*_{t} \) should be set on a decreasing path equal to 1/20th of the difference between the outstanding debt \( d_{t-1} \) and 60%. To the extent that \( \Delta d^*_{t} < 0 \), the fiscal effort should be greater.

The second factor that may determine a greater fiscal effort is the outstanding debt \( d_{t-1} \) (note therefore that in the Euro Zone it weighs twice). The third critical factor is the interest-growth gap \( (i_t - g_{nt}) \) – also known as "snowball effect". If high debt is accompanied by high interest payments and low growth, the resulting interest-growth gap magnifies the required fiscal effort. Finally, a fourth factor is the extent to which the government has access to debt monetization \( \mu_t \). As is well known, for the EZ governments \( \mu_t = 0 \).

The extent of the fiscal effort may underpin the notion that the debt burden (i.e. \( d_{t-1} \) in equation (2)) may depress growth provided that it is explained how the creation of a primary surplus depresses growth. We shall return to the point later in this section.

Before the financial and the debt crises in which many advanced countries have been entangled, the debt-growth relationship was of particular interest to the Heavily Indebted Poor Countries (HIPC): many works have analysed how the debt burden, or the "debt overhang", affects economic development, and whether the debt relief programs would have been useful and, in case, which was the best way to implement them. The debate around the HIPCs and debt relief programs goes beyond the scope of our work, however a few cases are worth mentioning.

Starting with the Costa Rica’s and the Mexico’s defaults at the beginning of the 80s, the growth of the debt burden for high indebted countries has been recognized as an issue. Kamin et al. (1989) gauged whether Argentina, Chile, Brazil, and Mexico had been better off in terms of GDP path for having borrowed considerable amounts before experiencing the debt crises, concluding that these countries were in fact worse off. Cunningham (1993) proposed one of the first empirical analysis investigating the effect of debt burden on economic growth, in which the debt
burden was measured as the rate of change in the long-run debt service to public and publicly guaranteed ratio. Its main empirical finding indicates that the debt burden negatively affects economic growth.

This conclusion is in line with the concept of debt overhang, initially proposed by Krugman (1988), according to whom the relationship between the sovereign debt and the national product is described by an inverted U relationship. Thus, above a certain level of the public debt, both the debtor and the creditor countries could find it convenient to forgive a part of the debt. In addition, the incentive to invest in the country may be reduced when public debt becomes “large”.

Some authors have focused mainly on the external debt component, which is often deemed to be the most relevant component as it implies a real transfer from the debtor to the creditor country. According to Karagol (2002, p.40), “foreign debt acts like a tax when the debt situation is such that an improvement in the economic performance of the indebted country has the side product of higher debt repayments”, and he found that, for the specific case of Turkey, external debt service has a negative short-run impact on economic growth. External debt, generated by large and persistent current account deficits, has also been indicated as the driver of the Euro Zone sovereign debt crises (e.g. Alcidi and Gros 2011, Gros 2013).

In summary, the debt burden view points out several channels whereby (high) public debt may directly or indirectly hamper economic growth and country’s development. In fact, this has been the situation in which the high indebted poor countries have been entangled, and for which initiatives of debt reduction or even debt forgiveness were widely proposed and studied in the last decades. However, this literature is essentially empirical and applied to historical cases, with no commitment to any particular theory of the debt-growth relationship. If anything, these studies are important because they add further strength to the warning that specific conditions play a crucial role that can hardly be absorbed into a general law.

2.2 Consolidation policies

A corollary of the burden view is that, sooner or later, the government will be forced to implement some consolidation policy, i.e. a fiscal restriction aimed at reducing debt and bringing it onto a sustainable path. The connection between this argument and the conclusion that, therefore, high debt impairs growth is less straightforward than one may think at first sight. The fiscal-effort equation (2) may provide simple and useful guidance.
To begin with, an analytical shift occurs from a stock to a flow approach, for the reason that a fiscal restriction is a change in the government budget (more surplus or less deficit) aimed at reducing the outstanding debt (or, more commonly, the debt-GDP ratio). Hence, the initial stock conditions should matter, which are, however, seldom specified.

In this perspective, the front line has long been represented by the traditional Keynesian vs. neoclassical models of fiscal expansions or restrictions, revolving around the extent of "crowding out" (in case of expansions) or "crowding in" (in case of restrictions) of private expenditure. Note that, generally, these models yield mirror effects in the two cases.

As is well known, Keynesian models (e.g. in the IS-LM class) predict that fiscal restrictions have a net negative effect on economic activity. Typically, the net negative effect results form the direct impact of the fiscal restriction on GDP via aggregate demand, amplified by the Keynesian multiplier, and the indirect, though incomplete, "crowding in" of private expenditure by way of a lower interest rate. This standard result, however, has a tenuous relation with the debt-growth issue as is embodied in equation (2).

First, the role of the debt stock is, at best, indirect and implicit in that a "large" debt will require a "large" restriction. In the basic IS-LM setup at least, the interest-rate effect is not related to the stock of debt, but to the stock money: with a fixed stock of money, the contraction of economic activity reduces the transaction demand for money which is satisfied by a lower interest rate. Second, the endogenous variable is current GDP, not its long-run growth rate, and it is not specified whether the fiscal restriction is permanent or transitory. This connection, however, may be provided by the notion of "hysteresis" such that a temporary demand-led downturn may be translated into a long-run loss of output capacity on the supply side (DeLong and Summers 2012, Fatàs and Summers 2018).

The traditional neoclassical approach is well-known for predicting a small or null effect of fiscal expansions owing to their "crowding-out" effect on private expenditure (see for instance Bernheim (1989) for a reassessment). It is less noticed that, since the mechanisms involved operate symmetrically, fiscal contractions are expected to have small or null effect on economic activity, with the consequence that the debt burden may not be per se conducive for lower growth.

In fact, this class of earlier neoclassical models is characterized by perfectly rational consumers with a finite lifespan, with access to a perfect capital market, where the consumption level is determined by a utility maximisation process. Analysis focuses on the amount of loanable funds in the capital market. In this
context, a fiscal surplus decreases lifetime consumption. However, provided that the economic resources remain fully employed, less consumption implies higher saving. The interest rate falls to keep investments equal to savings, crowding-in private expenditure.

Note two important caveats, however. The first is that the extent of crowding-out or crowding-in is largely seen as an empirical matter, and in some circumstances fiscal imbalances may retain some limited Keynesian effects on economic activity (Bernheim 1989). The second is that, as in the Keynesian counterparty, there is no explicit treatment of the debt stock and its effects on growth over time.

The modern neoclassical approach, that we can date back to the path-breaking works on "Ricardian Equivalence" by Barro (1974, 1989a, 1989b), lend further theoretical strength to the notion that budgetary adjustments along the government's intertemporal budget constraint are neutralised by compensatory adjustments on the private side. Barro (1979) proposed a specific model incorporating the Ricardian Equivalence and an empirical analysis supporting it, while in Barro (1989b, p.1) he concluded that the Ricardian Equivalence is a “good first-order approximation to reality”, supported also by the empirical evidence.

There are noteworthy differences with earlier neoclassical models. One is that households are characterized by a sort of “inter-generational altruism”, as if they were living infinitely. If current public expenditure is financed by resorting to public debt, the current generation will leave the necessary amount of money to the following generation in order to compensate the future increment in taxes. Thus, Ricardian Equivalence has also the strong implication that public deficits have no effects on the interest rate, provided that households firmly believe that the government’s intertemporal budget will be honoured. Hence, the hallmark of traditional neoclassical theory (and quite a popular argument about the negative effects of high indebtedness) is muted. Moreover, if creating a deficit is neutral on economic activity, creating the surplus when the debt service falls due will also be neutral (taxpayers have already hoarded the equivalent of taxation). Hence Ricardian Equivalence does not seem an appropriate support to the view that high debt reduces growth as a consequence of consolidation policies. However, various

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4 They may not be neutral on the supply side of the economy as long as intertemporal shifts between consumption and leisure are also considered.
aspects of this approach have raised doubts and debates also in the neoclassical camp fostering research of caveats and limitations to this theory.5

A variant in the same vein was introduced in the 1990s under the name of "expansionary fiscal consolidations", or "Non-Keynesian effects of fiscal policy", and it explicitly maintains that consolidation policies are not necessarily harmful for growth but, on the contrary, may have a positive effect. Giavazzi and Pagano (1990, 1995), and Alesina and Perotti (1995) were among the first who put forward and test this hypothesis empirically, followed more recently by Alesina and Ardagna (2010, 2013), and others. Their main conclusion, the fact that “even drastic fiscal adjustments are not associated with major recessions” (Alesina and Perotti, 1995, p.24), has recently been recovered in support of the Euro Zone austerity policies.6

Though framed within the modern intertemporal approach, the reasons why from Ricardian neutrality fiscal restrictions become expansionary remain unclear. A possibility may be that the public sector is not (no longer) on its intertemporal budget constraint. A simple device is an unexpected shock that raises outstanding debt. In that moment, the private agents discover that fiscal consolidation (the present value of future public surpluses) should be larger than previously expected. On the other hand, "Non-Keynesian" consolidation stories may in fact be due to several favourable Keynesian side conditions, regarding in particular the concomitant stance of monetary and exchange-rate policies (e.g. Favero et al. 2011, Perotti 2012, Blyth 2013, Part 3). That a fiscal contraction accompanied by expansionary monetary policy and exchange-rate depreciation may end up with a neutral, or net positive, effect on GDP has been well known ever since the basic Mundell-Fleming model (Foresti and Marani 2014).

As a matter of fact, this strand of literature was born, and has remained, essentially empirical being based on analyses of success stories of fiscal consolidations followed by fast recovery. Indeed, these analyses boil down to the choice of a growth-friendly consolidation design: large, front-loaded, with expenditure cuts rather than tax hikes (Alesina and Ardagna 2010, 2013, Carnot 2013). With reference to the fiscal-effort equation (2), the effect of $b^*_t$ on the growth

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5 As reported by Bernheim (1989), the Ricardian paradigm needs unrealistic assumptions to hold that make it implausible. Ricardian equivalence has also been tested in experimental laboratories by resorting, in general, to an overlapping generations design. Cadsby and Frank (1991) supported the validity of the Ricardian equivalence, but further developments found evidence of departures if more articulated experimental designs are employed (see Slate et al. 1995, Ricciuti and Di Laurea 2003).

6 See Blyth (2013, pp. 205 ff.) for a recollection of the evolution of this idea.
rate $g_{nt}$ depends not so much on its absolute dimension but on its composition. Public expenditure cuts do not impinge on the level of economic activity because households consume more in anticipation of less future taxes. The recommendation of a large and front-loaded consolidation relies on the argument that incomplete or delayed consolidation raises the present value of future fiscal surpluses; this is matched with less current spending and more hoarding by the private sector with a depressive effect on economic activity.

If the government fails to adopt the right consolidation design, then the conclusion may be that debt repayment has a negative effect on growth. Yet this conclusion also means that it is not the level of debt *per se* that can be pointed to as the determinant of future growth. Moreover, like the other approaches discussed so far, the focus is on short-run effects on economic activity, not on long-run determinants of growth.

Search for empirical support remains highly controversial as it intersects with the ever-lasting endeavour to estimate fiscal multipliers (Hebous 2011, Gechert et al. 2015). In parallel with a progressive revaluation of the Keynesian view, based on sizeable fiscal multipliers especially during downturns, some recent studies have cast doubts on the reliability of expansionary consolidations, and, more importantly, on their general value (IMF 2010). Perotti (2012) studies four individual episodes in different countries. He shows that all these episodes were in fact associated with an increase in growth but the explanation of why this occurred was to be found in the specific conditions of those countries. In line with this finding, Guajardo et al. (2016) argue that once changes in fiscal policy are motivated by a “desire to reduce the budget deficit and not by responding to prospective economic conditions”, there is little evidence of any expansionary effects. Jordà and Tylor (2016) also conclude that a fiscal consolidation is always associated with a fall in real GDP over a period of five years. The Euro Zone is a natural observational field of large fiscal consolidations, and the prevalent assessments yield negative effects on subsequent growth at least in the short to medium run (Berti et al. 2013, in’t Veld 2013, Beetsma et al. 2015).

If the fiscal consolidation has Keynesian effects, the debt/GDP ratio may *rise* instead of falling, as in fact happened throughout the Euro Zone. This effect therefore envisages a possible reverse causality between higher debt ratio and lower growth that should be taken account in empirical estimations. As shown by

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7 As to the wide debate on austerity see e.g. Corsetti (ed.) 2012, Buti and Carnot (2013), Tamborini 2015b.
Nuti (2013) and Tamborini (2013), this reverse causality occurs when the fiscal multiplier is greater than the reciprocal of the initial debt/GDP ratio.

Consider again the dynamic equation of the debt/GDP ratio (1), and assume $\mu_t = v_t = 0$. Let the interest rate be on its trend value $i$, and, $\phi = \Delta g_{nt}/\Delta b_t < 0$ be the Keynesian fiscal multiplier; this gives the deviation of nominal growth from trend $\Delta g_{nt}$, given a change in the primary balance ratio $\Delta b_t$. Hence $g_{nt} = g_n + \phi \Delta b_t$. This fact entails that the change in the debt ratio becomes

$$\Delta d_t = (i - g_n - \phi \Delta b_t) d_{t-1} - (b_{t-1} + \Delta b_t)$$

$$= [(i - g_n) d_{t-1} - b_{t-1}] -(1 + \phi d_{t-1}) \Delta b_t$$

The term in square brackets yields the debt path with unchanged fiscal effort. The point is that an increase in fiscal effort ($\Delta b_t > 0$) generates a negative impulse to the debt/GDP ratio only if $(1 + \phi d_{t-1}) > 0$, or $|\phi| < 1/d_{t-1}$. Hence, a combination of high outstanding debt and strong fiscal multiplier can eventually produce both less growth and higher debt/GDP ratio due to a reverse causality effect. Note that when the debt/GDP ratio is large, even a relatively small multiplier may produce this effect.

2.3 Fiscal space

Another particular, indirect debt-growth channel that can be found in this context relates to the so-called "fiscal space". This is a measure of the extent of fiscal expansion a government enjoys given its outstanding debt and some target or constraint on it (different indicators have been proposed by e.g. Aizenman and Hutchison 2012, Ghosh et al. 2013, Buti and Carnot 2016). This notion, indirectly, subscribes to the Keynesian view that fiscal expansions do have anti-cyclical properties.

The different indicators converge to the point that the higher the debt, the lower the fiscal space. Consequently, high debt inhibits the stabilisation capacity of the government. Persistent under-stabilisation may impinge on growth in the long run if other phenomena are considered such as hysteresis (DeLong and Summers 2012, Fatàs and Summers 2018).

Overall, one may argue that high debt may have a negative effect on economic activity in the short-to-medium run, and possibly on long-run growth, to the extent that an unanticipated fiscal restriction becomes necessary, or limits to anti-cyclical fiscal space are to be met. However, this conclusion is not univocal, and, somewhat paradoxically, it finds some support more from the Keynesian view, based on "large" fiscal multipliers, than from the neoclassical camp. Indeed, if Ricardian Equivalence holds, the result is fiscal super-neutrality: both fiscal expansions and
subsequent consolidations are neutral on aggregate economic activity at any point along the time path. If we observe a consolidation action, and this is not neutral, some condition(s) of Ricardian Equivalence should not hold.

3 Unsustainable debt, (self-fulfilling) expectations, and default

As previously explained, a critical point that is often blurred in the search for the debt-growth relationship is whether it goes through the fiscal policies that are adopted to service the debt or through the consequences of the debt being unsustainable and bound to default. This latter channel is examined here. A further distinction to be considered is between the negative effects on growth in the run-up of the debt crisis, and those due to the actual default.

3.1 Sustainability

A formal definition of sustainability, based on financial first principles, states that the outstanding value of debt should not exceed the discounted value of the current and future expected primary surpluses. This definition is fraught with several implementation problems (e.g. the choice of the appropriate discount rate, time horizon, and budget items) leading to controversial if not inconclusive judgements. Therefore, less demanding, empirically based, criteria have been put forward.

Liviatan (1984) proposed a “macro-absorption” approach for the sustainability of the debt burden, and classified the debt burden indicators into three categories: naive, simple, and composite. Naive indicators concern the total amount of debt and the related measures, such as the interest rate and the maturity; simple indicators involve the debt service and the related ratios, while composite indicators are weighted averages of naive and simple indicators, and aim at better identifying the debt burden and the approaching of a default. Further developments include, among others, Collignon and Mundschenk (1999), Arnone and Presbitero (2007), Fincke and Greiner (2012).

Major international agencies and institutions, notably the International Monetary Fund and the European Commission, have developed sophisticated methods of sustainability analysis (IMF 2012, European Commission 2014). They build on projections of macroeconomic, fiscal and financial variables that are sensitive to the kind of hypotheses used in the models. Every kind of assumption involves a certain degree of scrutiny so, to deal with uncertainty, assessments are
typically in the form of different scenarios upon various policy assumption. The saga of the Greek debt crisis witnesses how controversial the assessment of sustainability may be.

In the approach put forward by Bohn (1995, 1998), introduced above in section 2.1, the focus falls on the fiscal effort borne by the government, i.e. the primary-balance ratio over GDP necessary to achieve a target debt path (see equation (2)). In this regard, the composition of the effort (e.g. more taxes vs. less expenditure) may matter as suggested by the studies on the expansionary fiscal consolidations.

The critical point is that the evaluation of sustainability of debt is not simply a technical matter, but it also depends on the (political) assessment of the costs and benefits of the consolidation policies (or the costs and benefits of some degree of default), which leads to the political economy literature on the solvency/default choice of governments that we shall consider below.

3.2 Sovereign risk and the confidence channel

Macro-models with sovereign risk belonging to the so-called "New Keynesian Macroeconomics", or "New Neoclassical Synthesis", developed since the early '90s, may be regarded as adding a new, "micro-founded" channel to the debt burden view (examples are Corsetti et al. 2010, 2013; Buti and Pench 2012 provide a summary view with reference to the Euro Zone). They also offer a clearer identification of the departure point from fiscal expansions/consolidations neutrality in the standard Neo-Ricardian framework. That is to say, at some point in time, agents do not have full confidence that the government will honour its intertemporal budget.

The key feature of these models is the connection between the level of the debt-GDP ratio, its sustainability assessment on the part of investors in sovereign bonds, and how this is translated into a risk premium. In turn, the risk premium is transmitted to the long-term interest rate impinging upon private expenditure (for consumption and/or capital goods). As is the hallmark of this school, we find a composition of Keynesian and neoclassical features. It should be noted that the involvement of the interest rate is not due to the excess absorption of loanable funds (a flow concept that was involved in the Keynesian-neoclassical dispute about the "crowding-out" effect of fiscal imbalances), but to the increasing sovereign risk of debt (a stock concept).

According to Buti and Pench (2012), the key factors can be encapsulated in a formula of the "stock fiscal multiplier" like the following

\[
(4) \quad [1 - \text{confidence}] \div [1 + (\text{monetary policy}) + (\text{competitiveness}) - (\text{financial constraints})]
\]
Confidence relates to the forward-looking-ness of investors, i.e. their probability assessment of future default, and hence the demand for risk premium. Higher debt triggers lower confidence, higher risk premium and interest rate, and hence a dragging anchor on economic activity. Other factors that affect this "debt multiplier" relate to the Keynesian side of these models: the monetary policy stance (an accommodative stance helps reduce the interest rate and sustain aggregate demand) and competitiveness gains via real exchange rate depreciation (which also sustain the foreign component of aggregate demand). Finally, financial constraints, another typical New Keynesian feature, inhibit Ricardian neutrality and amplify the impact of the interest rate on aggregate demand.

Reasoning in reverse, one may obtain further conditions for expansionary fiscal consolidations. Assume that the economy is in a state of high "debt multiplier" as represented by expression (4). Accordingly, output is depressed. If the government enacts an unanticipated fiscal restriction that lowers the probability of future default, investors demand a lower risk premium. The domestic private sector enjoys a lower interest rate and anticipates the lower path of future taxes: both spur expenditure. The higher the confidence, the smaller the multiplier of the consolidation, or it may even turn into a positive net effect. Unsurprisingly, such a rich set of factors yield nuanced results, and, once again, results are conditional on the state of economy and other side elements (Corsetti et al. 2010, 2013).

A test of this channel of the debt-growth relationship is provided by Fragetta and Tamborini (2019) who estimate the impact of austerity (increases in the structural primary balance) on growth in a panel of European countries from 2010 to 2015, controlling for other possible explanatory factors that have been indicated in the literature together with, or instead of, austerity. The authors find that the negative effect of austerity on growth is always statistically significant together with changes in the debt/GDP ratio. However, they warn that since austerity was associated with increasing debt/GDP ratios, as explained above the reverse causality hypothesis cannot be excluded.

8 "In relatively extreme cases where fiscal strains are severe and monetary policy is constrained for an extended period, fiscal tightening may even exert an expansionary effect. That being said, fiscal retrenchment is no miracle cure. Indeed, all our simulations feature a deep recession even if tighter fiscal policy, under the aforementioned conditions, may stimulate economic activity relative to an even bleaker baseline (Corsetti et al. 2010, p. 41, italics added)." The confidence channel has also been tested, and partially supported, through laboratory experiments: see Geiger et al. (2016), Mittone and Tomaselli (2017).
3.3 Default as a government's choice

With regard to debt sustainability and default risk, it is necessary to remind that, as is clear from equation (1), a sovereign government can always achieve any desired debt-path target provided that it has full control over expenditure, taxation and possibly monetisation. The problem lies in the costs associated with these instruments. Therefore, default is better understood as a policy choice of the government trading off the costs of default with those of solvency (Gros 2012, Buiter and Rahbari 2013, Tomz and Wright 2013, Tamborini 2015).

A partial or total default on public debt, either explicit (a refusal to pay back the capital and a subsequent reduction in the outstanding amount of debt) or implicit (through high inflation rates – or even hyperinflation – that conspicuously diminish the real sovereign debt), is an extreme occurrence influenced also by the general financial and political situation as well as by agents' behaviour and expectations. Default may be unexpected, anticipated or even self-generated by creditors; each case has its own specific impact on the economy.

The economic consequences of default may be severe but concentrated in time, whereas the benefits of freeing the economy from the burden of debt may unfold over time. Behavioural aspects can play a role in terms of agents’ belief about past debt efficiency, agents’ expectations about current sustainability, and agents’ confidence level about future debt repayments, though they are not deemed to be always founded. Notably, these aspects have the potential to be self-fulfilling, thus leading to the actual default that could otherwise have been avoided.

The historical literature about sovereign defaults is vast and it goes beyond the purpose of this work (see Reinhart and Rogoff 2009; a mainly quantitative and a mainly qualitative literature reviews are respectively provided by Stähler 2013 and Tomz and Wright 2013), but it may be useful to briefly recall the most recent case of the sovereign debt crisis in the Euro Zone between 2010 and 2012.

Throughout the euro's first decade, countries like Belgium, Italy, and Greece were characterized by high public debt-to-GDP ratios. Meanwhile, "emerging" and fast-growing countries like Spain and Ireland started from very low levels of public debt but rapidly rising levels of private debts. Initially, investors regarded sovereign debts in the Eurozone as substantially equivalent prompting a remarkable convergence of interest rates towards the German safe rate. They became increasingly worried about debt sustainability after the Papandreou government's disclosure of the huge deterioration of Greek public finances, and after the sharp rise in the debt/GDP ratios due to the financial crisis and bank bailouts in other countries (notably Spain and Ireland), leading to a dramatic
increase in the interest rates of the bonds of the so-called "periphery countries" (Greece, Ireland, Italy, Portugal, Spain). The New Keynesian models mentioned above have been designed to capture these events. Their negative debt-growth relationship is the result of the combined effects on consumption and/or investment of higher interest rates and the anticipation of fiscal consolidation or default (as already discussed, the conclusions about the growth effects of consolidation are instead ambivalent).

The concomitant flourishing of empirical studies has also raised a number of challenges. In the first place, the relationship between the level (or growth rate) of public debt, the rise of risk premia, and speculative attacks turns out to be of dubious nature. Others have instead pointed out that speculative attacks have been driven by analogy with the "original sin" of many developing countries which issue debt in a foreign currency (De Grauwe and Ji 2012, 2013). This argument brings an institutional factor to the forefront: sovereign debt in euros is "foreign denominated" for Euro Zone countries because the issuers do not have a central bank in control of the currency. A third problematic area concerns the transmission channels of risk premia across countries. In this regard, there is evidence that post-2009 spreads not only reflect country-specific fundamentals, but were also highly sensitive to "systemic risk" and other exogenous factors (Manganelli and Wolswijk 2009, Attinasi et al. 2009, Caceres et al. 2010). In particular, research has focused on "contagion", that is, the transmission of high spreads across countries via non-fundamental channels (Constancio 2012, Arghyrou and Kostunica 2012).

3.4 Self-fulfilling expectations

Finally, the issue of self-fulfilling default expectations should be considered in its connection with the determination of the level of debt, or of the debt burden, that may trigger the sequence of events that may force a government into heavy fiscal consolidation or default. It may be recalled that the ECB President Mario Draghi opened his famous "whatever-it-takes" speech by saying that

(...) we are in a situation now where you have large parts of the euro area in what we call a "bad equilibrium", namely an equilibrium in which you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios. So, there is a case for intervening, in a sense, to "break" these expectations (Draghi 2012, p. 4).

The introduction of self-fulfilling expectations in macroeconomic models dates back to the 1980s (Farmer 1993). In general, they also entail multiple equilibria, the selection of which depends of the state of expectations. In our context, the typical mechanism is one where, as default expectations arise, the cost of debt
solvency also rises (e.g. because of higher risk premium) thus making the government default decision more likely.

The seminal study in our context is represented by the influential paper by Calvo (1988), who recognised that expectations about debt default may determine the equilibrium that is reached by the economy. Many other applications followed up to nowadays, and it is not possible to cite all of them. The Euro Zone crisis has also prompted new contributions.

De Grauwe (2012) presents a model of exogenous expectations determining an area of threat of self-fulfilling speculative attacks. Gros (2012) introduces a political-economy model of investors' subjective expectations of default affecting the market interest rate and the likelihood to observe multiple equilibria. Moreover, creditors may find it profitable to forgive part of a country’s debt to avoid the default, in line with the debt overhang theory. By following the same methodology, Tamborini (2015) shows that multiple equilibria can arise as a consequence of investors’ heterogeneous beliefs about primary balance sustainability, where the risk premium is higher the larger is the share of "pessimistic" investors about the level of fiscal effort beyond which the government prefers default. This model clarifies how the level of debt, its burden and sustainability are largely conditioned by the distribution of investors' beliefs. When pessimistic investors prevail, fostering more ambitious consolidation plans is likely to be counterproductive as greater, instead of smaller, risk premium is the result, which triggers the reverse causality between higher debt/GDP ratio and lower GDP.

Empirical evidence of self-fulfilling speculative attacks is uneasy to collect. De Grauwe and Ji (2013), and Passamani et al. (2015) can be mentioned among the studies that detect the phenomena presented above. Overall, research on the EZ crisis delivers a problematic picture of the "confidence channel" of the debt-growth relationship as far as its the connection between the debt level and the "fundamentals" of its sustainability are concerned. On the other hand, whatever the motivations behind rising risk premia, there is also evidence that 1) high debt levels may be more easily involved in confidence crises, 2) market pressure is a major vehicle for large fiscal consolidations, 3) these are likely to have strong and prolonged negative effects on economic activity (e.g. Born et al. 2018, Tomaselli and Tamborini 2019).
4 Four scenarios for the debt-growth relationship

On the basis of the previous literature review, it is not possible to reach a univocal conclusion regarding the debt-growth relationship. Even less is theoretically founded the existence of a general debt-to-GDP threshold above which growth is consistently stifled. We now move to a further strand of literature, which sets the debt-growth relationship in a fully developed intertemporal framework in which the debt life and the output growth are examined jointly.

4.1 Debt in intertemporal perspective

An important contribution coming from public finance is that public debt should be evaluated along its whole-time path rather than at a specific point in time. Debt is created for a reason or purpose, then it unfolds its effects over time, and these effects may, or may not, contribute to debt repayment. All these aspects are to be considered when assessing the relationship between debt and growth, whereas the sheer measurement of the debt level at some point may be uninformative or misleading.

To be more specific, when tracking the debt/GDP ratio it should be first recognised that this is given by to the joint trajectories \( (D_{t-k}, \ldots, D_{t-1}), (Y_{t-k}, \ldots, Y_{t-1}) \), determined by the underlying sequence of budgetary policies and their consequences on the economy. The observed values \( D_t/Y_t \) may be the outcome of an efficient or inefficient trajectory, while subsequently they may turn out to be either sustainable or unsustainable.

In this literature, the debt trajectory can be considered efficient if the use of debt is consistent with its purpose in terms of general criteria of economic efficiency, and if it has no distortionary effects on social equity and social welfare. Efficiency implies sustainability \( \text{ex ante} \), the long-term solvency constraint represented by the condition that the present value of the future government deficits and surpluses converge to zero as \( t \) tends towards infinity. However, debt on an efficient trajectory may turn out to be unsustainable \( \text{ex post} \) owing to unforeseen events. Two other scenarios are possible: debt may be inefficient but sustainable, debt may both inefficient and unsustainable. These four scenarios have quite different implications in terms of growth, and, what is more important, the level of debt and the corresponding level of the debt-to-GDP ratio along the trajectory are irrelevant on their own.

An early example of this kind of analysis is the cyclical stabilisation role of fiscal policy theorised by Musaggrave (1959). If debt is created during a slump and repaid
during the recovery in order to equalise national income over the business cycle, both efficiency and sustainability (and intergenerational equity) are achieved. The time profile of debt may be quite different in different specific conditions.

Modigliani (1961) instead pointed out that an increase in the national debt (both internal and external) can be advantageous for the current generation, but it places a burden on future generations entailing a reduction in the available stock of private capital, thus causing a decrease in the future flows of goods and services. Analogous conclusions were reached by Bowen et al. (1960), according to whom, even if the repayment of the debt principal is continuously delayed, each current generation bears a burden represented by the taxes used to pay debt interests.

Another classic topic in this line of literature is the so-called "golden rule" of public finance (Musgrave 1964). This rule is the object of a long-standing branch of public finance which we need not examine here, whereas it is important to note how it fits in our four scenarios. As is well known, the rule states that the balance between current expenditure and revenues should be nil, while public debt is only allowed as a means to finance productive investment. Here the efficiency-sustainability criteria are even more transparent. Productive investment is realised as growth-enhancing debt-based expenditure, and efficiency requires the equality between marginal product and social cost. Sustainability should be guaranteed by equality between the marginal increase in revenues due to additional growth and the debt service. Equity lies in the fact that the generation paying for debt also enjoys a higher level of income.

In this context, Diamond (1965) was the first who studies the effects of debt on economic growth properly. Diamond divided debt into external debt (borrowed from foreign lenders) and internal debt (borrowed from domestic lenders). External debt has negative effects on growth in the long run because of the taxes needed to finance interest payments. Taxes are levied on domestic lenders whereas interests are paid to the foreign ones. Initially, taxes reduce consumers’ total lifetime income, and hence consumption. As a further consequence, taxes reduce savings and the capital stock. Internal debt, on the other hand, entails both effects as well as a reduction in the capital stock due to the fact that individuals substitute government debt for physical capital in their portfolios. Therefore, public debt crowds out private capital.

The 1990s have seen the emergence of discordant views. Some authors proposed models leading to opposite conclusions with respect to those of Diamond. For
instance, Dotsey and Mao (1994) introduced distortionary taxation,\(^9\) and in fact debt turned out to crowd-in investments. Ludvingson (1996) analysed deficit-financed fiscal policies in a forward-looking general equilibrium model and showed that the economy’s response to an increase in government expenditure depends on how it is financed. In particular, distortionary taxes may lead to a decline in output, consumption and investments, while deficits may increase output and consumption. Moreover, deficit-financed cuts in income taxation may increase investments although agents expect future taxes to be higher (due to the substitution between leisure and labour), a conclusion supported also by Lin (2000). Therefore, according to this branch of works, there are no crowding-out effects and the impact of government debt on growth can be positive.

In this perspective, endogenous growth theory deserves a specific treatment. This approach spread during the 1990s attempting to explain how long-term growth can be generated without relying solely on exogenous or "residual" technological changes as in the Solow foundational model. This strand of literature is relevant to our topic because, following the model proposed by Barro (1990), it examines how fiscal variables interact with the variables that generate endogenous growth. Interaction can be indirect (this is typically the case of taxation) or direct to the extent that public expenditure can sustain endogenous growth. This latter case is particularly important because it marks a shift of approach with respect to the neoclassical views examined above which typically see public expenditure as sheer consumption of resources.

The Barro model obtains a typical inverted U function of the relationship between public expenditure and growth, with an optimal level of expenditure (taxation) that maximises growth. Public expenditure is fully covered by taxation and taxation depresses growth. Below the optimal level of public expenditure the government does not exploit its growth-enhancing effect, beyond that point the growth-depressing effect of taxation prevails. On the other hand, the bulk of this literature is concerned with the effects of fiscal variables on growth along a balanced budget path, showing a rich variety of results and policy implications (see Zagler and Dürnecker 2003 for an accurate survey), which however are not immediately suitable for analysis of the debt-growth problem.

\(^9\)According to Kneller (1999), distortionary taxes in this context are those which affect the investment decisions of agents (with respect to physical and/or human capital), creating tax wedges and hence distorting the steady-state rate of growth. Non-distortionary taxation, on the other hand, does not affect savings and investment decisions and therefore has no effects on the growth rate.
More to the point, Teles and Cesar-Mussolini (2014) proposed an endogenous growth model in which the effect of fiscal policy on economic growth is negatively affected by the level of the debt-to-GDP ratio. This effect works via the debt interest: a portion of young people’s savings is extracted and paid to elderly people, who do not save, thus implying an allocation exchange between generations. The negative effects of government debt on growth have been shown also by Saint-Paul (1992) and, by studying the impact of fiscal constraints (limited tax and debt capacity) on growth, Aizenman et al. (2007) reached similar conclusions: lower maximal tax rate and higher outstanding debt can lower the growth rate, supporting the fact that differences in growth rates can stem from differences in fiscal policy constraints.

In the subsequent sections we shall characterise and discuss the debt-growth relationship in the four scenarios centred on the efficiency-sustainability criteria recalled above within the intertemporal setup of the fiscal models of endogenous growth, on which we graft public debt as a means to finance productive public expenditure. As will be seen, the model also embeds some specific debt-growth relationships previously presented.

4.2 Endogenous growth with efficient and sustainable public debt

We propose a fiscal model of endogenous growth freely inspired to Barro (1990) and Diamond (1965). Barro (1990), as said above, represents a now standard model of growth sustained by productive public expenditure fully covered by taxation. To introduce debt in this setup, we adopt a sequential economy with two-period generations à la Diamond (1965) where public expenditure is financed by debt in the first period and the debt burden is covered by taxation in the second period.

The key assumption in the Barro model is that the economy consists of competitive firms producing an aggregate output $Y$ according to a neoclassical (Cobb-Douglas) production function augmented by productive public expenditure, where productive means a kind of expenditure in public goods that can raise private factors' productivity (typical examples are infrastructures, education, research, health care, etc.). We reformulate the Barro production function in terms of private capital $K$, and public capital $K_G$ (i.e. the stock of public goods mentioned above):

---

10 We draw on the version by Barro and Sala-i-Martin (1998, ch. 4).
11 The difference here is that the generations do not overlap, but we assume that they comply with the intergenerational pact that each generation leaves the same endowment of capital to the next.
\[ Y_t = A_t K_t^\alpha K_{Gt}^\beta L_t^\beta \quad \alpha + \beta = 1 \]

In order to introduce public debt, we first treat the economy as a sequence of two-period generations. The generation starting in period \( t \) is endowed with labour \( L_t \) and private capital \( K_t \). Labour is supplied inelastically in each period and normalized to 1. Likewise, for the time being, we set the scale factor at constant value \( A_t = 1 \) in each period. The depreciation rate of private and public capital is 100\% and gestation time is one period.

If the private sector realises net investment in addition to depreciation by the amount \( I_t \), the private capital stock \( K_{t+1} = K_t + I_t \) will be operative in \( t+1 \). The government, too, can invest in public capital the amount \( G_t \) financed by debt, \( D_t = G_t \) so that Public capital \( K_{Gt+1} = G_t > 1 \) will also be operative in \( t+1 \). Therefore, feasible production in \( t+1 \) will be:

\[ Y_{t+1} = K_{t+1}^\alpha K_{Gt+1}^\beta \]

The government fulfils its intertemporal budget constraint by taxing all incomes in period \( t+1 \) with the flat rate \( \tau \). Incomes include the public debt service (principal and interests). Therefore, the following government budget equality holds:

\[ \tau Y_{t+1} = (1 - \tau)D_t R_{d_{t+1}} \]

where \( R_{d_{t+1}} \) is the unit debt service.

Under the efficient capital market hypothesis, the interest rate is equalized across the private and public sectors, so that, \( R_{d_{t+1}} = R_{t+1} \) is the gross return to capital, determined as marginal product of private capital, i.e.:

\[ R_{t+1} = \alpha \left( \frac{K_{Gt+1}}{K_t} \right)^\beta \]

Given \( D_t = K_{Gt+1} \), and \( R_{d_{t+1}} = R_{t+1} \), the public budget constraint (7) sets the feasible stock of public capital. Since \( Y_{t+1} \) is determined by (6), the result is

\[ K_{Gt+1} = K_{t+1} \left( \frac{\tau}{(1 - \tau)R_{t+1}} \right)^{1/\alpha} \]

This expression can also be interpreted as the optimal public/private capital ratio \( k_{t+1}^* = K_{Gt+1}/K_{t+1} \). The complementarity between public and private capital is the first key feature of this economy.

Substituting \( k_{t+1}^* \) into the interest rate equation (8), the latter results to be

\[ R_{t+1} = \alpha' \left( \frac{\tau}{1 - \tau} \right)^\beta \quad \alpha' = \alpha^\alpha \]

The second notable result is that the marginal product of private capital, and hence the interest rate, are invariant to the capital stock – a result propaedeutic to
endogenous growth. The interest rate is however a concave function of the tax rate, which leads to the peculiar relationship between debt and growth in this economy.

To this end, we have first to consider the optimal consumption path of the representative household of the \( t \)-th generation, which maximises a time separable logarithmic utility function subject to its two-period budget constraints:

\[
\begin{align*}
\text{max}_{C_t, C_{t+1}} & \quad U(C_t, C_{t+1}) = \log(C_t) + (1 + \rho)^{-1}\log(C_{t+1}) \\
\text{s.t.} & \quad C_t = Y_t - I_t - D_t \\
& \quad C_{t+1} = (1 - \tau)[Y_{Lt+1} + (K_t + I_t + D_t)R_{t+1}] - W_{t+1}
\end{align*}
\]

where \( Y_{Lt+1} \) is labour income, \( \rho > 0 \) is the rate of time preference, and \( W_{t+1} \geq 0 \) is the final stock of wealth (for simplicity we assume \( W_{t+1} = K_t \), i.e. each generation leaves to the next one the same capital endowment received from the previous one).

By combining the first order conditions with respect to \( C_t, C_{t+1} \), we obtain the Euler equation and the optimal consumption path

\[
\gamma \equiv \frac{C_{t+1}}{C_t} = \frac{(1 - \tau)R_{t+1}}{(1 + \rho)}
\]

As usual, an increase in the (after tax) interest rate raises \( \gamma \), while an increase in the rate of time preference decreases it. Substitution of the interest rate equation (10) into (12) yields the growth equation for this economy, namely

\[
\gamma = \frac{\alpha'(1 - \tau)\alpha + \beta}{1 + \rho}
\]

We thus see the crucial result, namely that growth is a concave function of the tax rate necessary to finance public capital. Concavity reflects the double-edge role of taxation: \( \tau^\beta \) is the growth-enhancing effect of financing public capital, \( (1 - \tau)\alpha \) is the growth-depressing effect of taxing capital income. As a consequence, there exists a unique tax rate \( \tau^* \) that maximises \( \gamma \), namely

\[
\tau^* = 1 - \alpha
\]

which is the same result as in the original Barro model.

The public finance implication is that, given \( \tau^* \), each generation has its own optimal public debt which is both sustainable and efficient. In particular, there is neither "crowding out" when debt is created in \( t \) (indeed there is crowding-in)\(^{12}\) nor is there excess fiscal burden in \( t+1 \). As to the debt/GDP ratio, it can first be noted that the relevant ratio (in terms of debt burden) for the \( t \)-th generation is given by the GDP equation (6), i.e.

\[
\text{(8)} \quad \begin{align*}
\text{According to equation (8), as long as } \tau < \tau^*, \text{ raising public capital increases the marginal product of private capital, which allows for a larger private capital stock, and shifts consumption to the future, which generates more saving for capital accumulation.}
\end{align*}
\]
Hence, the key variable is the public/private capital ratio. Equation (9) sets the feasible ratio. Substituting $R_{t+1}$ and $\tau$ with the respective optimal values (10) and (15) the result is

$$d^*_{t+1} = \left(\frac{\beta}{\alpha^2}\right)^\alpha$$

That is to say, the optimal debt/GDP ratio may be whatever is appropriate for each economy and each generation. Different endowments, preferences or technologies, would yield different results, so that no generalisation or comparison is meaningful across time or different economies. In other words, "high" and "low" debt/GDP ratios may equally be efficient and sustainable.

### 4.3 Sustainable but inefficient debt

The previous model provides an immediate instance of cases in which public debt is sustainable but inefficient in terms of growth. Since the relationship between taxation and growth is concave, the cases in consideration occur whenever public debt, i.e. public capital, is either too low or too high with respect to (16). In the former case, the government fails to exploit the full range of growth-enhancing public investment; in the latter excess investment requires excess taxation that depresses growth.

Therefore, note in the first place that growth may be sub-optimal not only because debt is "too high". In the second place, even when debt is in fact too high, it remains perfectly sustainable. Indeed, lower growth is due to the fact that the government complies with sustainability by levying excess taxation. This, moreover, need not come as an unexpected event but may be fully anticipated. In other words, inefficiency defines a set of effects of public debt on growth that do not depend either on unsustainability nor on default risk but, quite the contrary, on the anticipation of the sustainable path of fiscal policy. Finally, sustainable debt is not synonymous with efficient fiscal policy and optimal growth of the economy. As explained in the previous section, the threshold between efficient and inefficient level of debt is hard to draw in comparisons over time or across different countries.

### 4.4 Efficient but unsustainable debt: again on fiscal consolidation

The third case we examine is one where public debt is ex-ante efficient and sustainable whilst it is not ex-post. We model this situation by means of an unanticipated shock that in period 2 of the $t$-th generation lowers total factor
productivity. This simple device may capture different situations: from true recessions, to ex-ante overvaluation of public investment productivity, or misbehaviour of the government that deviates a share of debt to unproductive uses. In any case, the consequence which we focus on is the necessity of fiscal consolidation in period 2, i.e. a fiscal adjustment that guarantees debt solvency.

The first period of the t-th generation is the same as in the base case, except that the coefficient $A$ in the production function is now a (positive) random variable of unit expected value which in $t+1$ takes the value $A_{t+1} < 1$. Note that, by assumption, the stocks of private and public capital operational in $t+1$ have been installed in $t$ and are irreversible (denoted by a bar) Consequently,

$$Y_{t+1} = A_{t+1} \bar{K}_{t+1}^\alpha \bar{R}_{Gt+1}^\beta = A_{t+1} Y^*_{t+1}$$

where $Y^*_{t+1}$ denotes the ex-ante optimal GDP as in the base case. Therefore, the government budget as given by equations (7) and (8) and the optimal tax rate $\tau^* = 1 - \alpha$ can no longer be satisfied. A fiscal consolidation is necessary, and to this end the government changes the tax rate so that

$$\tau_{t+1} A_{t+1} Y^*_{t+1} = (1 - \tau_{t+1}) D_t \bar{R}_{t+1}^d$$

From the base model, we know the ex-ante optimal values of $Y^*_{t+1}$ and $\bar{R}_{t+1}^d$. Upon substituting these values and $D_t = \bar{K}_{Gt+1}$ into (18), we find that the solvency tax rate is

$$\tau^s_{t+1} = \frac{1-\alpha}{1-\alpha(1-A_{t+1})}$$

which clearly shows that the new tax rate should be higher than $\tau^*$ the worse the productivity shock.

What are the concomitant effects of fiscal consolidation on the economy? The first is that the growth rate is reduced, yet this is the direct and exclusive effect of the productivity shock on the GDP path, not of fiscal consolidation by itself. The second effect, directly due to fiscal consolidation, is on households’ consumption which necessarily deviates from the optimal path given by (12). The increase in the tax rate, in addition to the productivity shock, affects the $t+1$ budget constraint as follows

$$C_{t+1} = (1 - \tau^s_{t+1})[(1 - \alpha)A_{t+1} Y^*_{t+1} + (K_t + I_t)R_{t+1} + D_t \bar{R}_{t+1}^d] - K_t$$

where $(1-\alpha)A_{t+1} Y^*_{t+1}$ is the gross income share of labour. Likewise, we can write

$$(K_t + I_t)R_{t+1} = \alpha A_{t+1} Y^*_{t+1},$$

and therefore

$$C_{t+1} = (1 - \tau^s_{t+1})[A_{t+1} Y^*_{t+1} + D_t \bar{R}_{t+1}^d] - K_t$$

13 Unlike the equity-based private capital, the debt-based public capital is committed to paying the interest rate established ex ante.
As can be seen, households suffer from lower gross income from the private sector, and higher tax rate. Moreover, their consumption is fully constrained by current disposable incomes, so that the economy also displays this "Keynesian" feature.

4.5 Inefficient and unsustainable debt

Debt-financed public expenditure may be ex-ante inefficient for a number reasons, ultimately because the government spends and taxes too much (beyond the optimal level identified in the first scenario) or because the projects are in fact less productive. To simplify our analysis of the fourth scenario, we can note that when the economy is hit by an adverse shock as in the third scenario, public debt observationally results both inefficient and unsustainable ex post. Drawing on the political economy literature on the default choice mentioned in section 3, we now examine the point that, since fiscal consolidation is a costly decision for the government, it may consider the option of default. Yet also default is a costly decision.

To address this problem various specifications of the government's decision are available. In our context, it is natural to assume the representative consumer's utility as the welfare function of the government. Consequently, let us consider the post-shock consumption level in $t+1$:

\[ C_{t+1} = (1 - \tau_{t+1})[A_{t+1}Y^*_{t+1} + (1-\phi)D_t\bar{R}^d_{t+1}] - K_t \]

where the government has two policy variables, the tax rate $\tau_{t+1}$ and the rate of "haircut" of the debt repayment $\phi$. Note that the former variable affects consumption via after-tax income, whereas the latter affects consumption via pre-tax income. The point is that the two variables are inversely related: to the extent that the government lowers $\tau_{t+1}$ below the solvency level $\tau^s_{t+1}$ given by (19), it should raise $\phi$. In fact, on the basis of the government’s budget, we obtain that

\[ \phi = (\tau^s_{t+1} - \tau_{t+1}) \frac{(A_{t+1}Y^*_{t+1} + D_t\bar{R}^d_{t+1})}{D_t\bar{R}^d_{t+1}} \]

where $\tau^s_{t+1} = D_t\bar{R}^d_{t+1} / (A_{t+1}Y^*_{t+1} + D_t\bar{R}^d_{t+1})$. Hence, $\phi = 0$ if $\tau_{t+1} = \tau^s_{t+1}$, and $\phi = 1$ if $\tau_{t+1} = 0$.

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14 This treatment simplifies the scenario in that taxation and haircut hit the same subject. This is not always (often it is not) the case. For instance, part of debt may be held by foreign subjects whose well-being is not relevant to the government. On the other hand, defaulting on foreign obligations may generate other kinds of costs (reputation, market access, etc.) that may be relevant.
Upon substituting $\phi$ into (21), $C_{t+1}$ results in a concave quadratic function of $\tau_{t+1}$. The optimal debt policy is the $(\tau^d_{t+1}, \phi)$ combination that maximises the post-shock consumer utility, i.e.

$$\tau^d_{t+1} = \tau - s_{t+1}/2, \phi = 0.5$$

where $(d)$ denotes that the tax rate implies default.

Interestingly, the optimal debt policy is independent of any other variable except the solvency tax rate $\tau_{t+1}$, but of course this is due to the utility function that we have assumed. It is however generally true that post-shock consumption is concave in $\tau_{t+1}$, i.e. it reaches a maximum for a specific combination $(\tau^d_{t+1}, \phi)$. This result prompts two remarks. First, (partial) default is always a policy option for a government facing (unexpectedly) unsustainable debt. Second, the effect of debt on the economy cannot be gauged independently of whether debt is inefficient/unsustainable and the government chooses the default option.

5 Final remarks

Research on the relationship between public debt and growth has a long history. Interest has been revived by the fiscal consequences of the Great Recession of 2008-09. This new wave of research has been mostly empirical, and largely dominated by the pursuit of "the" debt-to-GDP ratio beyond which debt depresses growth; yet no univocal conclusion has been reached either about the quantification of the critical ratio or even about its existence.

Foundational work is however lacking: why should we expect a negative public debt-growth relationship in the first place? If such a relationship exists, why should it take the specific form of a threshold of the debt-GDP ratio, and why should we expect this threshold to be equally valid across time and space?

In an attempt to address these questions, we have examined a wide range of different literatures concerning public debt and its impact on economic activity. One main problem is that different analytical approaches are intertwined: static, single-period vs. dynamic, intertemporal setup; flow (budget deficits) vs. stock (outstanding debt) analysis; expected solvency vs. expected default. In each of them, or combination of them, debt has distinct effects on the economy. Some agreement has emerged in recent years around the idea that high debt may eventually impair growth through the default risk channel or through the consequences of large, unanticipated consolidation under market pressure after a debt shock or other unforeseen contingencies. The possibility of "expansionary consolidations" has lost force over time both theoretically and empirically. This
consensus view, somewhat paradoxically, hinges more on the existence of Keynesian "large" fiscal multipliers, than on the neoclassical opposite presumption.

Our methodological conclusion is that no meaningful assessment of debt and its effect on growth at any point in time is possible without reference to the entire debt trajectory and the specific state of the economy along the trajectory. In this perspective, we have shown that an orderly and consistent analysis may be developed along two coordinates of debt assessment: sustainability/unsustainability, and efficiency/inefficiency. If, for instance, debt is on a sustainable and efficient trajectory, the debt level, the debt-to-GDP ratio and the growth rate at any point in time may be whatever is consistent with the fundamentals of the economy; the mere comparison between different countries has no informative value. Specific analyses, leading to different predictions, are necessary when public debt is either inefficient or unsustainable, and whether the government wishes to consolidate or not.

If a comprehensive conclusion may be drawn is that each country's debt history and specific characteristics, circumstances, and events have an overwhelming importance that cannot be encapsulated in a single general law. Research should concentrate on the former and abandon the pursuit of the latter.

References


Economy Research Institute, University of Massachusetts Amherst Working Paper No. 322.


Reihart, C. M., Rogoff K. S. (2009), This time is different. Eight centuries of financial folly, Princeton: Princeton University Press.


