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# Property Rights and Investments: An Evolutionary Approach

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**Abstract**

When contracts are not enforceable, or property rights are not clearly defined, individuals might lack an incentive to carry out costly investments even when they are socially efficient. Some recent contributions such as Ellingsen and Robles (2002) prove that this problem might be less dramatic than standard economic models would suggest. They propose evolutionary models in which only efficient equilibria can be (stocastically) stable. In this paper we show that these results are not robust with respect to the introduction of individual heterogeneity. When individuals have different cost functions, stochastically stable states are inefficient, even when they induce a positive (suboptimal) level of investment.

JEL Classification: L14, C78

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# 1 Introduction

Property rights are crucial for the correct working of virtually any economic activity. When individuals are not sure whether they will be able to enjoy the fruits of their own labor because it might be confiscated by a political authority, investments that are rational from an economic point of view might fail to be undertaken. Similarly, joint ventures in which participants are required to make relation-specific investments will usually fail to work efficiently, unless a way is found to protect the investment made by each participant from the opportunistic behavior of others. Given the ubiquity of this problem, it is not surprising that a large literature emerged on the impact of property rights (or the lack thereof) on economic activities. Part of this literature goes under the heading of "hold-up problem". In its simplest form, the hold-up problem can be described as follows. (See Tirole (1988) for a textbook presentation.) There are two risk-neutral agents $A$ and $B$. $A$ can either invest or not in the production of a pie that has a positive value for both $A$ and $B$. By hypothesis, there is no way for them to sign a binding contract before $A$ makes his investment decision. *After* (and if) the pie has been produced, $A$ and $B$ contract over the division of the pie. $A$ will thus undertake an investment only if it costs less than the value of what she expects to get from the bargaining. Assuming that the pie is divided equally between the two players, $A$ will only invest if the cost of investment is less than half the value of the pie. This implies that investments that are economically efficient will fail to be undertaken because $A$ expects to be (partially) expropriated by $B$.

Ellingsen and Robles (2002) and Troger (2002) build on the evolutionary approach to bargaining due to Young (1993) (1998) and get to a very different conclusion. In Young's approach, players are replaced by large populations of agents and the bargaining game is played repeatedly by pair of agents drawn from these populations. Agents are not rational, but adjust over time their behavior to the way in which the game is played by other agents. The two populations will converge over time to a conventional way to solve the bargaining problem. When such a convention has been selected, it pays to each agent to stick to it.

While Young assumes that bargaining takes place over a fixed and exogenous pie, Ellingsen and Robles (2002) and Troger (2002) endogenize the size of the pie assuming that it is the result of an investment decision made by one of the two agents. The result they get is striking: the two populations will gravitate around efficient equilibria in which the investing agent appropriates a share of the pie which is large enough to cover the costs she paid to bring it about. Investments will thus be undertaken whenever they are efficient. They conclude that the hold-up problem might no be so severe after all, because evolution will promote the emergence of norms and institutions (in the form of bargaining conventions) that mitigate or eliminate it.

There is some reason to be skeptical about these results at least on empirical ground. The the fifty-fifty division of the crop between landlords and tenants (which is a standard example of conventional contract) would be the most obvious counterexample. This custom has traditionally been criticized (for example by Alfred Marshall) for being the source of many inefficiencies in backward agrarian countries such as Italy in the XIX century. (See for example Luporini and Parigi (1996)) In general, there seems to be little empirical ground to the claim that evolution will promote conventional contracts which protect investments and efforts. (See Dawid and MacLeod (2008)) for a more detailed discussion of this point.)

This paper examines this issue from the same perspective as in Ellingsen and Robles (2002), but reaches a different conclusion. We introduce some degree of heterogeneity among agents. In particular, we assume that agents have different cost functions, and that costs are not observable. We show that under this seemingly minor modification, the population will fail to converge to a state in which investors appropriate a sufficiently large share of their efforts, so that investment becomes individually rational. Quite on the contrary, the model shows that the system will tend to gravitate around states in which the investment level is suboptimal.

Besides its contribution to the literature on the hold-up problem, this paper makes a more general point. With the exception of Young (1998), the standard evolutionary approach to social institutions is based on models in which single players are replaced by populations of *identical* agents. (See Young (2008) for a recent review of this literature.) The present paper shows that this hypothesis is in fact extremely strong. We make a first attempt towards a more realistic approach in which a degree of heterogeneity is introduced.[1] We do this in a specific

[1]There are some evolutionary models with heterogeneous agents. For example, Ely and

class of games, namely bargaining games with outside options. Our analysis is further restricted by the hypothesis that there is just one outside option (agents can either invest or not invest) and only two cost functions. Further research is needed to show to what extent these results can be generalized (for example introducing several outside options and cost functions) and extended to other contexts (for example repeated games).

The paper proceeds as follows: Section 2 provides an informal presentation of the model and presents the main result. Section 5 introduces the definitions and the technicalities of the model. Section 6 contains a formal statement of the main result. Section 7 concludes. Proofs are collected in the Appendix.

## 2 Investment and bargaining

There are two risk-neutral individuals $A$ and $B$. $A$ moves first and decides whether to invest or not in the production of a pie. If she decides not to invest, the game is over and $A$ and $B$ get nothing. If $A$ invests, she pays a cost $c \in (0, V)$ and a pie of size $V$ is produced. At this point, $A$ and $B$ bargain over the division of the pie. After production has taken place, the investment cost $c$ is sunk and therefore should not influence $A$'s behavior. The Nash bargaining solution between two risk neutral players would assign half of the pie to each, regardless of who produced the pie. As a consequence, production will take place only if player $A$'s cost is so small that investment yields a positive return

Sandholm (2005) discuss best response dynamics with hetherogeneous preferences. Noldeke and Samuelson (1997) discuss signalling games (with multiple types), in an evolutionary context. None of these papers, however, discusses explicitly the emergence of social institutions.

even when $A$ appropriates only half of the available pie. This requires that $c < \frac{V}{2}$. Inefficiencies will then occur whenever $c \in (\frac{V}{2}, V)$.

This result is crucially based on the hypothesis that in the second stage of the game, when (and if) the pie has been produced, the pie is divided according to the Nash bargaining solution. For instance, this would be the case if in the second stage players bargain according to the alternating offer protocol proposed by Rubinstein (1982). In such an approach, the second stage has a single subgame perfect Nash equilibrium, which approximates the Nash bargaining solution game as players become more patient and therefore implies that no production will take place whenever $c > \frac{V}{2}$.

A different result would follow if, instead of the alternating offer protocol, $A$ and $B$ played a round of the Nash demand game. Suppose that, if production has taken place, each player claims a fraction $x \in (0, V]$ of the available pie. To facilitate the evolutionary analysis that follows, we shall assume that the pie is divisible up to $\delta$, so that players can choose a finite number of claims $x \in \Delta = \{\delta, 2\delta, ..., V - \delta\}$. (Notice that we are assuming that players cannot claim nothing.)

If the two claims are compatible, that is if $x + y \leq V$, each player gets the fraction of the pie she has claimed. If $x + y > V$ they both get nothing. Since both players are risk neutral, their payoffs are $u_A = x - c$ for player $A$ and $u_B = y$ when $A$ produces the pie and $x + y \leq V$. When $A$ does not produce a pie both players get zero. When $A$ produces a pie but the demands in the bargaining stage of the game are incompatible, $A$ gets $-c$ and $B$ gets zero.

A strategy for $A$ is a pair $(i, x)$, with $i \in \{I, NI\}$ where $I$ means invest and $NI$ means not invest. $x \in \Delta$ is the fraction of the pie $A$ claims in the bargaining stage of the game. A strategy for an agent $B$ is just a fraction of the pie $y \in \Delta$ to claim. The *outcome* of the game can either be $NI$, which corresponds to agent $A$ playing $NI$, or $(I, x, y)$, which corresponds to agent $A$ investing and claiming $x$, while agent $B$ claims $y$. (Notice that with a slight abuse of notation we denote with $NI$ both $A$'s strategy and the outcome of the game.)

One can easily see that when production is followed by the Nash demand game a number of subgame perfect equilibria exist, some of which involve production while others do not. To see this, consider that any pair of strategies $((I, x), (V - x))$ form a subgame perfect equilibrium in which $A$ invests in the first stage and claims $x$ in the second stage, provided that $x \geq c$. All these equilibria are socially efficient (because $c < V$), although they differ in the way in which the surplus is divided between $A$ and $B$. Beside these, however, there are other subgame perfect equilibria in which production does not take place because in the second stage $B$ expects $A$ to claim a fraction of the pie $x < c$, and he optimally (given his expectations) claims a fraction of the pie $V - x > V - c$.

Inefficient equilibria in which production does not take place are less compelling than equilibria in which $A$ does invest. Inefficient equilibria are sustained by $A$'s belief that $B$ will claim a share of the pie $y$ larger than $(V - c)$. Such a belief would only be rational if $B$ expected $A$ to produce the pie and claim less than $c$. This strategy yields a negative payoff for sure to $A$ and hence it is strictly dominated (in the normal form version of the game) by the strategy consisting in not producing the pie at all. Once strictly dominated strategies are eliminated, only Nash equilibria in which production does take place survive.

## 3 Evolution and bargaining

There is a connection between the result presented in the previous section and the notion of forward induction. To see this, consider that $A$ has an outside option, not investing, which gives him zero for sure. If $B$ observes that $A$ has not taken her outside option, she should conclude that either he made a , or he hoped to coordinate on one of the equilibria in which he gets a positive payoff. Since the only equilibria which guarantee this are those in which $A$ appropriates at least a fraction $c$ of the total pie, $B$ can make sense of $A$'s decision to invest only assuming that he is going to claim at least $c$. This argument is not conclusive, though, because there are *several* Nash equilibria that yield a payoff larger than zero, so that standard forward induction arguments cannot be applied to this game.

Noldeke and Samuelson (1993) showed that evolutionary models supports forward induction arguments like this to a larger extent than standard models based on rational behavior. Intuitively, the reason is that evolution "solves" the equilibrium selection problem in those cases in which there is more than one Nash equilibrium that yields a player more than her outside option. Ellingsen and Robles (2002) and Troger (2002) exploit this result to prove that evolution provides a solution to the hold-up problem. In this paper I shall follow Ellingsen and Robles's approach, which is closer to Noldeke and Samuelson (1993). My presentation of the technical details of the model will be rapid. The reader is to refer to the original papers for a more user-friendly presentation of the model.

Like in any evolutionary model, single players are replaced by populations of identical agents. The game is played repeatedly by pairs of individuals taken from these populations. Agents are not assumed to be rational. Rather, they have *beliefs* about the way in which the other players will behave at each node of the game, and are characterized by a pure strategy they play at every interaction. Occasionally, their revise their beliefs (observing the way in which the game is played by other agents) and adjust their strategies (by choosing a best response to their beliefs). With a small probability this process fails to work and we say that an agent "mutates". When this happens, the agent picks a belief and a strategy at random.

The instability of Nash equilibria that are incompatible with forward induction depends crucially upon the fact that beliefs at nodes that remain unreached given the state of the population cannot be updated. In our example, beliefs concerning the claims agents in population $B$ would make in the bargaining stage of the game cannot be updated if all agents in the $A$ population fail to invest. In this case, if an agent by mistake changes his beliefs, there would be nothing in what he observes that will induce him to revise them. The same happens for strategies. This process of random changes in beliefs and strategies at unreached nodes is known as *drift*.

Take now one of the inefficient, subgame perfect equilibria in which all $A$

agents do not invest because they expect agents in population $B$ to claim a share of the pie larger than $(V-c)$. Since production does not take place, what agents in population $A$ and $B$ would claim in case a pie where produced is not observed, beliefs are not updated and they are prone to drift. Because of drift, all $B$ agents might become convinced that, in case a pie where produced, agents $A$ would demand a share $x$ larger than $c$. They would then be willing to claim a share $(V-x)$ of the pie, which is smaller than $c$. When this happens, it takes a single $A$ agent to switch (by mistake, given her expectations) to production, to reveal that, in fact, $B$ players are willing to claim $(V-x)$. Following this event, $A$ agents will switch to a best reply to their new expectations, which consists in producing the pie and claiming $x$. All $B$ players will then start claiming $(V-x)$ and a new equilibrium is selected, in which all A agents invest and the division $(x, V-x)$ is implemented. This argument shows that drift can bring the system from any equilibrium in which production does not take place, to an equilibrium in which it does.

Destabilizing efficient equilibria is not so easy, though. Take one of the equilibria in which all agents in population $A$ claim a fraction $x > c$ and agents in $B$ claim $V-x$. These states are not subject to drift because production does take place and hence agents' claims are revealed at each interaction. These equilibria are thus more stable than those in which $A$ players do not produce. Formally, equilibria with no production are not *locally stable*. Although some of the technical details of the model are fairly involved, this is the basic intuition behind the fact that evolution "solves" the hold-up problem.[2]

## 4 An informal presentation

The obvious weak part of the result presented in the section above is that it relies so heavily on drift. To make this argument work one must assume that agents within each population are exact clones one of another. This assumption is needed because it implies that there is a single threshold level (equal to $c$) such that if agents $A$ expect to get a share of the pie larger than $c$ they will *all* invest, while *none* of them would invest if she expects to get less than $c$. So $B$'s beliefs about $A$'s behavior will drift if the expected size of the pie for $A$ agents is smaller than $c$, and will not if it is larger $c$. In more realistic settings, however, agents in population $A$ will show a certain degree of heterogeneity with respect to costs. Some of them will be more efficient in producing a pie (low $c$), others will be less efficient (high $c$). This difference will be reflected on the fact that $A$ agents will have a different threshold over which production becomes profitable: a share of the pie $x$ might be large enough to induce efficient $A$ agents to produce, while being too small to make production profitable for less efficient $A$ agents.

This paper investigates the effects of introducing this kind of heterogeneity in the $A$ population. To do this, while retaining the assumption that all agents are risk neutral, I stipulate that agents in population $A$ are divided into two

[2]The main reason why the results are involved is that Ellingsen and Robles (2002) and Troger (2002) consider a more sophisticated model in which several levels of investment are considered.

groups: efficient agents whose cost $c_L \in (\frac{V}{2}, V)$ is low and inefficient agents whose cost $c_H \in (c_L, V)$ is high. Notice that the hypothesis that even the less efficient agents $A$ have a cost smaller than $V$ insures that production would be socially efficient both for efficient and for inefficient agents. The only efficient states are thus those in which *all* $A$ agents invest.

This more complex model has two kinds of equilibria in which production takes place. There are equilibria in which players expect with probability one an outcome $(x, V - x)$ in the bargaining stage of the game with $x \in (c_L, c_H)$. In these equilibria only efficient agents invest, and the outcome is therefore inefficient. There are other equilibria in which agents expect an outcome $(x, V - x)$ with $x \in (c_H, V)$, in which both types of $A$ agent invest. There are also equilibria in which production does not take place, because $A$ agents expect to receive from bargaining a value smaller than $c_L$, so that production is not profitable even for the most efficient of them.

The main result, presented in Section 6, partly confirms the results already present in the literature: those states in which production fails to take place cannot be locally stable. The reason is the same as in the existing models: when nobody produces a pie, beliefs and strategies will drift.

However, when agents are heterogeneous there are also equilibria in which some agents invest, and others don't. We prove that these equilibria (which are inefficient) will in fact be locally stable. The intuition is that it takes just a fraction of the $A$ population to invest to stop drift. When some $A$ agents produce a pie, beliefs about what will happen in the bargaining stage of the game are constantly updated. Finally we show that the two populations will tend to gravitate around a state in which the least efficient agents in $A$ will not produce. Formally, states in which *all* $A$ agents produce fail to be stochastically stable, despite being locally stable. This is the paper's main message: evolution will fail to select an efficient equilibrium, when agents have heterogeneous costs.

## 5 The model

There are two large populations $A$ and $B$ of agents. All agents are risk neutral. Time is discrete. Population $B$ numbers $N$ agents. Population $A$ is divided into two subpopulations: there are $N$ efficient agents whose cost of producing a pie is $c_L$ and $N$ inefficient agents whose cost of producing a pie is $c_H$. Hence, in total population $A$ numbers $2N$ agents.

The pie has value $V$ for all $A$ and $B$ agents. We need the following technical assumptions:

**Assumption 1** *i) The pie is divisible up to a fraction* $\delta$*, with* $\delta > \frac{V}{N}$*;*

*ii)* $c_L = n\ \delta$ *and* $c_H = m\ \delta$*, where* $n$ *and* $m$ *are integers;*

*iii)* $c_L > V/2$

*iv) When an A agent is indifferent between producing and not producing, he will not produce.*

Assumption (i) is a standard assumption which guarantees that the population is sufficiently large (with respect to the minimum division of the pie), that a single mutation in one of the two populations will not push the system outside the basin of attraction of a an equilibrium in which all $A$ agents claim $x$ and all $B$ agents claim $V - x$.

Assumptions (ii)-(iv) are only introduced for the sake of simpler proofs and neater results. Removing any of them would leave unaltered the qualitative features of the model, but would result in much more cumbersome propositions. Assumption (ii) stipulates that the costs of investment are multiple of the minimal units that can be claimed in the bargaining stage of the game. Assumption (iii) imposes that efficient agents will not invest unless they expect to get more than half of the pie. Assumption (iv) stipulates that an agent who is able to cover just the cost of his investment will not invest.

At time $t = 1, 2, ...$ all possible combinations of agents from the two populations form and play a round of the investment and bargaining game. Before playing the game, a $B$ agent cannot observe whether the $A$ agent he is playing with belongs to the $AL$ or $AH$ population. In other words, costs are not observable.

Each agent is characterized by a strategy and a belief. Let $\nu(.)$ be the beliefs of agents $A$ concerning the behavior of agents in population $B$. Similarly, agents in population $B$ have beliefs $\sigma(.)$ over the behavior of agents in population $A$. Both $\sigma(.)$ and $\nu(.)$ are probability distributions over $\Delta$. Notice that $B$'s beliefs are not conditioned on $A$'s cost.

Let $\theta$ be the *state* of the population. $\theta$ specifies how many agents in both populations have every possible combination of belief and strategy. $z(\theta)$ is the distribution across the terminal nodes of the game, given the state of the population $\theta$.

The state $\theta$ evolves over time on the basis of adaptation and random mutation. At the beginning of each period, each agent in both populations receives with a fixed probability $\lambda$ the opportunity to revise his strategy and his beliefs. We say that an agent receives the learning draw. An agent who receives a learning draw observes $z(\theta)$ and adjusts his beliefs accordingly.[3] Notice that since all agents who receive the learning draw observe the same distribution $z(\theta)$ over outcomes, they will all update in the same manner. Notice also that adjustments in beliefs takes place only in those states $\theta$ in which at least one $A$ agent produces a pie, because only in this event the bargaining stage of the game is reached. In the other cases, the learning draw has no effect on agents' beliefs. After adjusting his beliefs, an agent chooses a best reply to them. If there are multiple best replies, one of them is chosen at random, with the exception of the investment decision, where Assumption 1 (iv) applies.

Beside learning, the state $\theta$ evolves through mutation. Every period each agent mutates with probability $\varepsilon$. An agent who mutates chooses a combination of belief and strategy at random. When mutation produces a change in a strategy or a belief corresponding to a node that is not reached under the current state $\theta$, we say that the state *drifts*.

A state $\theta$ is an *equilibrium* if there is no alternative state $\theta'$ that can be reached from $\theta$ without mutation. The *basin of attraction* of an equilibrium is the set of states $B(\theta)$ from which there is a positive probability that the

[3]We assume here that agents can observe the outcomes of the games, but not the payoffs agents receive. Again, this amounts to assume that costs are not observable.

system can reach $\theta$ without mutations. Let $\theta$ and $\theta'$ be two equilibria such that one can get from one to the other by a single mutation. The set of equilibria with this characteristic is called the *single mutation neighborhood* of $\theta$ and is indicated with $M(\theta)$. A union of equilibria $X$ is a *mutation connected* set if for any $\theta, \theta' \in X$ it is possible to go from $\theta$ to $\theta'$ through a sequence of single mutation transitions. A mutation connected set is said to be *locally stable* if for any $\theta \in X$ a single mutation is not sufficient to enter the basin of attraction of another state $\theta' \notin X$.

The combination of learning and mutation generates a Markov chain over the states $\theta$. When $\varepsilon > 0$, this Markov chain is irreducible because every possible transition among states have positive probability. Therefore, there is a stationary distribution $\mu(\varepsilon)$ which the system will approach regardless of initial conditions. We are interested in the stationary distribution for very small values of the noise $\varepsilon$. Formally, we are interested in the limit distribution $\mu^* = \lim_{\varepsilon \to 0} \mu(\varepsilon)$. Those states that receive a positive probability in $\mu^*$ are called *stochastically stable*.

# 6 Results

Let $x_L = c_L + \delta$, and $x_H = c_H + \delta$. Intuitively, $x_L$ ($x_H$) is the smallest share of the pie that will induce an efficient (inefficient) $A$ agent to invest. (If an $A$ agent with cost $c$ expects to get a fraction of the pie $x = c$ he would be just indifferent between investing and not investing and, because of Assumption 1 (iv), he will not invest.) Let $Q$ be the set of absorbing sets. For a state $\theta$, let $\rho(\theta)$ be the set of outcomes associated with $\theta$.

**Definition 1** *Consider the following classes of equilibria.*

$P_L = \{\theta : \theta \in Q$ *and* $\rho(\theta) = \{(I, x, V - x), NI\}$

$P_H = \{\theta : \theta \in Q$ *and* $\rho(\theta) = \{(I, x, V - x)\}$

$P_{mix} = \{\theta : \theta \in Q$ *and there are at least two demands* $x \neq x'$ *such that* $\{(I, x, V - x), (I, x', V - x')\} \in \rho(\theta)\}$

$NP = \{\theta : \theta \in Q$ *and* $\rho(\theta) = \{NI\}\}$

The intuition behind this definition is: in a state $\theta \in P_L$ only agents in $AL$ invest and only a pair of demands $(x, V - x)$ is observed. Agents in $AH$ do not invest, and therefore two outcomes of the game are observed $(I, x, V - x)$ and $NI$. $\theta$ is an equilibrium provided that all agents expectations put probability one on $(x, V - x)$ being the outcome of the bargaining stage of the game. Of course this requires that $x_H > x \geq x_L$. One can easily check that players receiving a learning draw will not change their strategies and beliefs, and therefore $\theta$ is an absorbing state. In a state $\theta \in P_H$ *all* $A$ agents invest, and the only observed outcome of bargaining is $(x, V - x)$. For $\theta$ being an equilibrium it must be that $x \geq x_H$, because otherwise agents who belong to $AH$ will not play $I$.

$NP$ is the set of states in which none of the $A$ players plays $I$. $\theta \in NP$ is an absorbing set provided that all $A$ agents beliefs are such that $NI$ is a best response to these beliefs. Notice that $\theta$ can be an absorbing set even if these beliefs are false, so that states in $NP$ not necessarily correspond to Nash equilibria of the game.

We shall denote with $\theta_x$ an equilibrium in $P_L \cup P_H$ with $(I, x, V-x) \in \rho(\theta)$. We also say that in $\theta_x$ the convention $(x, V-x)$ has been selected.

**Proposition 1** *The sets in definition 1 form a partition of the set of absorbing sets $Q$.*

Proposition 1 implies that all absorbing sets are singletons, so that we can speak of equilibria rather than absorbing sets. Notice however, that even if an absorbing set contains only one state, it might well be that the set of outcomes associated with that state is not a singleton. This is the case of any equilibrium $\theta_x \in P_L$, where the associated outcomes are both $NI$ (because inefficient $A$ agents do not invest) and $(I, x, V-x)$ (because efficient $A$ agents invest and claim $x$).

**Proposition 2** *Only equilibria in $P_L$ and $P_H$ are locally stable.*

This proposition is analogous to Ellingsen and Robles (2002), Proposition 4.1. The intuition behind this result is the same as in their model. Equilibria that are in $NP$ cannot be locally stable because none of the $A$ agents invest, and therefore the bargaining stage of the game is never reached. Agent's expectations and behavior are thus free to drift to a point in which all $B$ agents are willing to claim a fraction of the pie smaller than $V - c_H$, for example $\delta$. When a state like this is reached, it takes a single mutation from an $A$ agent to reach the basin of attraction of the equilibrium $\theta_{V-\delta}$.

Notice, however, that Proposition 2 does not allow us to infer that evolution "solves" the hold-up problem, because all equilibria $\theta_x \in P_L$ are inefficient. To see whether evolution will favor efficient equilibria we have to use the stricter requirement of stochastic stability. This is the content of the main proposition of this paper.

**Proposition 3** *The state $\theta_{x_L}$ is the only stochastically stable state.*

While the proof of this result is fairly involved, the intuition should be straightforward at least for those readers who have a certain familiarity with the original results in Young (1993) (1998). From Proposition 2 we know that only equilibria that are in $P_L$ and $P_H$ are locally stable. All we have to do is to construct a minimum resistance tree connecting these equilibria. Young proved that the easiest way out from an equilibrium $\theta_x$ in which convention $(x, V-x)$ is selected is through mutations involving agents in the $A$ population switching to $x+\delta$ and agents in the $B$ population switching to $V-x+\delta$. When all agents have the same utility function and $x > \frac{V}{2}$, this implies that it is easier to move from $\theta_x$ to $\theta_{x-\delta}$ than vice versa. So the minimum resistance tree among locally stable equilibria would be rooted at $\theta_{x_L}$. In the appendix we prove that taking into consideration heterogeneous agents in the $A$ population does not change this state of affairs. The easiest transitions are still those that go toward the more "egalitarian" convention $(\frac{1}{2}, \frac{1}{2})$. However, the most egalitarian convention compatible with production, that is $x_L$, is not efficient because it does not induce inefficient $A$ agents to invest.

# 7 Conclusions

The results presented in the previous section cast doubt about the ability of evolutionary forces to shape efficient contracts. The main reason for the difference between the results presented here and those of Ellingsen and Robles (2002) and Troger (2002), is that here it is assumed that the same contract (e.g. the fifty-fifty division) applies to a large variety of different cases. When there is large variation among individuals' productivity, the same contract might provide incentives to invest to some agents and not others. For example, it can be easily shown that with $c_L < \frac{1}{2}$ and $c_H > \frac{1}{2}$, the fifty-fifty division will be stochastically stable, even if other divisions (more favorable to the investing agents) would be more efficient. Incidentally, this seems to be in line with the empirical research on conventional contracts in agriculture. For example, Young and Burke (2001) notice that "fifty-fifty is used in almost all farms despite the fact that there is a wide range of soil qualities on which it is used." (11-12) In general, their study shows that there is no evidence that conventional contracts are tailored to the characteristics of the single agents, like the models so far discussed in the literature would suggest. The present paper shows that when this is the case, the emerging conventional contract is likely to be inefficient.

## Appendix. Proofs

**Lemma 1** *Let* $\{z_i\}_{i=1}^k$ *be a set of demands. The set of behavioral best replies to a state of the population in which these demands are made is a subset of* $\{V - z_i\}_{i=1}^k$.

**Proof.** The proof is the same as in Ellingsen and Robles (2002) Lemma A.1. The only difference here is that $A$ agents have different costs, which however do not matter for the computation of the behavioral best response in the bargaining stage of the game. ■

Proof of Proposition 1

**Proof.** We will prove that for every state $\theta$, either $\theta \in P_L \cup P_H \cup P_{mix} \cup NP$, or it is possible to reach via a set of learning steps a state $\theta'$ which is an equilibrium and belongs to one of these classes. Since learning alone cannot change an equilibrium, this implies that the original state $\theta$ cannot be member of an absorbing set. Take any state $\theta$ which is not an equilibrium. This means that either some $A$ agents or some $B$ agents (or both) are not playing a best reply to $\theta$. Let start with the case in which some of the $B$ agents are not playing a best reply. This requires that at least one of the $A$ agents plays $I$, so that $(I, ., .) \in \rho(\theta)$. Otherwise all $B$'s strategies would get $B$ the same payoff, zero. Let $BR_B(\theta) = \{y, y', y''...\}$ be the set of best replies for $B$ agents to the state $\theta$. Let $\sigma_B$ be the distribution among the members of $BR_B(\theta)$ of the $B$ agents who are already playing a best reply. Let $BR_A(\sigma_B) = \{x, x', x'', ...\}$ the set of behavioral best replies for $A$ agents against the distribution $\sigma_B$. This means that all strategies in $BR_A(\sigma_B)$ yield the same payoff against the subset of the $B$ population that is already playing a best reply. Notice that, because of Lemma 1, $BR_A(\sigma_B)$ is a subset of $\{V-y, V-y', V-y'', ...\}$. Suppose now that all $B$ agents who are not playing a best reply receive the learning draw and choose the same claim $y$ within $BR_B(\theta)$, such that $(V-y) \in BR_A(\sigma_B)$. In the new state $\theta'$ one clearly has that a behavioral best reply for $A$ agents is $(V-y)$. To see this, consider that $(V-y)$ gave the same payoff as $(1-y', 1-y''...)$ against the distribution $\sigma_B$ of $B$ players who were already playing a strategy in $BR_B(\theta)$. The new distribution in the state $\theta'$ differs from $\sigma_B$ because one or more $B$ agents are now playing $y$, while the number of $B$ agents making other claims $(y', y'', ...)$ remains the same. Clearly, claiming $(V-y)$ is now a strict behavioral best reply to the state $\theta'$. If now all $A$ agents receive the learning draw, they will update their behavioral strategy playing $(V-y)$, and they will choose an investment level which is a best reply against their new conjecture. If $NI$ is a behavioral best reply for all $A$ agents (those belonging to $AH$ as well as those belonging to $AL$) the next state $\theta''$ is such that $\rho(\theta'') = NI$. $\theta''$ is also an equilibrium, because all $A$ players are now playing a best reply to their conjectures. We have thus reached an equilibrium in $NP$. Suppose instead that playing $I$ is a best reply at least for agents in $AL$. In the following state $\theta''$ we have that all $A$ agents who play $I$ claim $(V-y)$ in the bargaining stage of the game. If now all $B$ agents receive the learning draw, they will expect $A$ players to claim $(V-y)$ with certainty, and therefore will adjust to claiming $y$. We have thus reached a state $\theta''$ which is an equilibrium and such that either

$\rho(\theta'') = \{(I, y, V - y)\}$ (and in this case $\theta \in P_H$) or $\rho(\theta'') = \{(I, y, V - y), NI\}$ (in which case $\theta \in P_L$.)

Consider now population $A$ and let $BR_{AL}(\theta)$ and $BR_{AH}(\theta)$ be the set of best responses to $\theta$ for agents $A$ with a low and high cost respectively. If $BR_{AL}(\theta) = BR_{AH}(\theta) = \{NI\}$, $\theta \in NP$ and we are done. Suppose thus that $(I, x) \in BR_{AL}(\theta)$ and that some $A$ agents are not playing a best reply. Let all of them receive the learning draw and choose a best reply to $\theta$. Let $\theta'$ be the state thus reached. There are two possibilities. Either also $B$ agents are choosing a best reply to $\theta'$, in which case $\theta'$ is an equilibrium because both $A$ and $B$ agents are playing a best reply; or in $\theta'$ some of the $B$ players are not playing a best reply to $\theta'$. In the latter case, a set of steps analogous to those described in the previous point will take the system to an equilibrium. ■

We prove Proposition 2 with the two following lemmata.

**Lemma 2** *If $\theta \in P_{mix} \cup NP$, then $\theta$ is not locally stable.*

**Proof.** Let $\theta \in P_{mix}$. Since more than one claim is made in equilibrium by $B$ agents, it must be that all these claims yield the same payoff. Suppose now a single $A$ agent who is playing $(I, x)$ switches to $(I, x')$. Since the distribution of claims is changed in the $A$ population, we are now in a new state $\theta'$ in which at least one $B$ agent is not playing a best reply. The same argument developed in the proof of Proposition 1 shows that it is possible from this state to reach an equilibrium in $NP \cup P_H \cup P_L$. As a consequence, the original equilibrium $\theta$ is not locally stable.

Suppose now that $\theta \in NP$. Since $\rho(\theta) = \{NI\}$, agents' conjectures about the bargaining stage are free to drift. With a series of single mutation transitions we can get to a point in which all $B$ agents expect $A$ agents to claim $V - \delta$. Suppose now an agent $A$ makes a mistake (given her expectations, that have not changed), and claims $V - \delta$. If all agents now receive the learning draw they will choose $(I, V - \delta)$ if they are $A$ agents and $\delta$ if they are $B$ agents. So a new equilibrium $\theta'$ is reached in which $\rho(\theta') = \{(I, V - \delta, \delta)\}$, so that $\theta' \in P_L$. ∎

**Lemma 3** *If $\theta \in P_L \cup P_H$, then $\theta$ is locally stable.*

**Proof.** Consider any equilibrium $\theta_x \in P_L \cup P_H$. Only one solution to the bargaining problem $(x, V - x)$ is observed in this equilibrium, with $x \geq x_L$. As a consequence, investing is a strict best reply for $AL$ agents (if $x < x_H$) or for both $AL$ and $AH$ agents (if $x \geq x_H$). Therefore, either $N$ or $2 \times N$ $A$ agents play $Invest$ in these equilibria. We shall only check that a single mutation in the $A$ population cannot push the system into the basin of attraction of a different equilibrium. The case for the $B$ population can be treated symmetrically and is omitted.

Several cases must be distinguished:

i) an $A$ agent who (optimally) plays $(I, x)$ in $\theta_x$ switches to $(NI)$. This will clearly not alter the distribution of claims in the $A$ population (where all the other agents continue to claim $x$), and therefore will not change $B$'s best response, and therefore will not induce $B$ agents to change their strategy. Since by definition the mutating $A$ agent is now playing a non optimal strategy $(NI)$, she will switch back to invest as soon as she receives the learning draw and the original equilibrium is restored.

ii) Consider now an $A$ agent who is (optimally) playing $(I, x)$ in $\theta_x$ and switches to $(I, x')$ with $x' \neq x$. If $x' > x$, the payoff for a $B$ agent who claims $(V - x)$ is $(\bar{N} - 1)(V - x)$, where $\bar{N}$ is either equal to $N$, if $\theta \in P_L$, or to $2N$, if $\theta \in P_H$. If she switches to claiming $(V - x')$ a $B$ agent will get $(V - x')\bar{N}$. $B$ agents will not change their strategies provided that $(\bar{N}-1)(V-x) > (V-x')\bar{N}$. This requires $\bar{N} > \frac{V-x}{x'-x}$, which is satisfied because Assumption 1 ($i$) guarantees that $\bar{N} \geq N > \frac{V}{\delta} > \frac{V-x}{x'-x}$. (The last inequality is justified by noticing that $V > V - x$ and $\delta \leq x' - x$.) If $x' < x$, the payoff of a $B$ agent claiming $(V - x)$ is $(V - x)\bar{N}$, while if she switches to $(V - x')$ she gets just $(V - x')$. Thus not switching is a best response as long as $(V - x)\bar{N} > (V - x')$, which requires $\bar{N} > \frac{V-x'}{V-x}$. Invoking again Assumption 1 ($i$), one has that $\bar{N} > \frac{V}{\delta} > \frac{x-x'}{V-x}$.

iii) Finally, consider an $A$ agent who is optimally playing $NI$ in $\theta$ (which implies that $\theta \in P_L$) and switches to $(I, x')$. If $x' > x$, a $B$ agent claiming $(V - x)$ gets $N(V - x)$, while if she switches to claiming $(V - x')$ she gets $(N+1)(V-x')$. $B$ agents will not switch to $(V-x')$ provided that $N(V-x) > (N + 1)(V - x')$, which requires $N > \frac{V-x'}{x'-x}$. If $x' < x$, $B$ agents will not switch to $(V - x)$ provided that $(V - x)(N + 1) > (V - x')$, which requires $N > \frac{x-x'}{V-x}$. Both these conditions are again fulfilled because of Assumption 1 ($i$). ■

Clearly, Lemma 2 plus Lemma 3 imply Proposition 2.

The proof of Proposition 3 requires the following lemmata. We first calculate the minimum number of mutations in the $B$ population required to exit one of the locally stable state equilibria in $P_H$ or $P_L$ to enter any other locally stable equilibrium.

**Lemma 4** *Let $\theta_x$ be an equilibrium in $P_H$ or $P_L$. The number of mutations in the $B$ population to enter the basin of attraction of another equilibrium $\theta_{x'} \in P_L \cup P_H$ is $\frac{x}{x'}N$ if $x' > x$ and $\frac{(x-x')}{x}N$ if $x' < x$.*

**Proof.** The proof is very close to the original Young (1993) model and will not be reproduced here. The only thing to notice is that both types of $A$ agents will switch from $x$ to $x'$ for the same number of mutants in population $B$. When $x' < x_H$, inefficient agents will switch to $NI$, rather than $(I, x')$. ■

**Corollary 1** *Let $\theta_x$ be an equilibrium in $P_H$ or $P_L$. The equilibrium whose basin of attraction can be reached from $\theta$ with the smallest number of mutations in the $B$ population is $\theta_{x-\delta}$, and the number of mutation required is $N\frac{\delta}{x}$*

**Proof.** This is again very close to Young (1993). Clearly $N\frac{x}{x'}$ (with $x' > x$) is minimized when $x' = V - \delta$, that is by $N\frac{x}{V-\delta}$, while $N\frac{x-x'}{x}$ (with $x' < x$) is minimized when $x' = x - \delta$, that is by $N\frac{\delta}{x}$. Since by Assumption 1 (*iii*) $x \geq x_L > \frac{V}{2}$, $\frac{\delta}{x} < \frac{2\,\delta}{V} < \frac{x}{V-\delta}$. (To see this consider that $x > \frac{V}{2}$ implies $\frac{2}{V} > \frac{1}{x}$, and hence $\frac{2\,\delta}{V} > \frac{\delta}{x}$) ■

We now calculate the number of mutations in the $A$ population required to displace a locally stable equilibrium. This procedure requires some care, because the number of agents who play $I$ depends upon the selected convention, and can either be $N$ or $2\,N$. Also, one must take into consideration that beside switching to alternative claims in the bargaining stage of the game, agents $A$ could switch from $I$ to $NI$ and *vice versa*. We shall start with equilibria when only efficient $A$ agents invest.

**Lemma 5** *Let $\theta_x$ be an equilibrium in $P_L$. It takes $\frac{x'-x}{V-x}N$ mutations in the $A$ population to reach the basin of attraction of an equilibrium $\theta_{x'}$, with $x' > x$. It takes $\frac{V-x}{V-x'}N$ mutations in the $A$ population to reach the basin of attraction of an equilibrium $\theta_{x'}$ with $x' < x$.*

**Proof.** Since $x < x_H$, in $\theta_x$ only efficient agents invest. Mutations can be of two kinds: efficient agents who produce a pie and claim $x'$ rather than $x$, and inefficient agents who mutate into producing a pie *and* claim $x'$. Each of these mutations can be of two kinds, depending on whether $x' > x$ or $x' < x$. There are four possible combinations, which we shall consider in turn.

The first two cases concern equilibria $\theta_{x'}$ with $x' > x$.

$i$) Suppose $q$ efficient individuals in $A$ switch to claiming $x' > x$. A $B$ player claiming $(V-x)$ gets a payoff equal to $(V-x)(N-q)$. If she switches to claiming $(V-x')$ she gets a payoff equal to $(V-x')N$. Claiming $(V-x')$ is thus a best reply provided that $(V-x')N > (V-x)(N-q)$, which requires $q > \frac{N(x'-x)}{V-x}$.

$ii$) Consider now $q$ inefficient $A$ agents who switch to invest and claim $x' > x$. $B$ agents claiming $(V-x)$ get a payoff equal to $(V-x)N$, while claiming $(V-x')$ will give them a payoff equal to $(V-x')(N+q)$. Hence, claiming $(V-x')$ is a best reply provided that $(V-x')(N+q) > (V-x)N$, which implies $q > N\frac{x'-x}{V-x'}$.

Since $x' > x$, $N\frac{x'-x}{V-x} < N\frac{x'-x}{V-x'}$, and therefore the easiest transition involves only efficient $A$ players to switch to $x' > x$, and the minimum number of mutations is $N\frac{x'-x}{V-x}$.

The other two cases concern equilibria $\theta_{x'}$ with $x' < x$.

*iii*) Suppose $q$ efficient $A$ players switch to $x' < x$. A $B$ player claiming $(V-x)$ would get $(V-x)N$. If he switches to claiming $(V-x')$, he gets a payoff $(V-x')q$. Therefore, claiming $(V-x')$ is a best reply provided that $(V-x')q > (V-x)N$, which requires $q > N\frac{V-x}{V-x'}$.

*iv*) Suppose instead that $q$ inefficient $A$ agents mutate to produce a pie and claim $x' < x$. In this case, claiming $(V-x)$ a $B$ player would get $(V-x)(N+q)$, while claiming $(V-x')$ she would get $(V-x')q$. Claiming $(V-x')$ is a best reply provided that $(V-x')q > (V-x)(N+q)$, which requires $q > \frac{V-x}{x-x'}N$.

Since $V-x' > x-x'$, it follows that $\frac{V-x}{x-x'}N > N\frac{V-x}{V-x'}$, so that the number of mutations to reach $\theta_{x'}$ with $x' < x$ is smaller when efficient agents switch to a lower claim. The minimum number of mutations required to enter the basin of attraction of $\theta_{x'}$ is thus $N\frac{V-x}{V-x'}$. ■

**Lemma 6** *Let $\theta_x$ be an equilibrium in $P_H$. It takes 2 $N\frac{x'-x}{V-x}$ mutations in the A population to reach the basin of attraction of an equilibrium $\theta_{x'}$, when $x' > x$. It takes 2 $N\frac{V-x}{V-x'}$ mutations in the A population to reach the basin of attraction of an equilibrium $\theta_{x'}$, when $x' < x$.*

**Proof.** Since $\theta_x \in P_H$, in $\theta_x$ all $A$ agents play $(I, x)$. Suppose $q$ of them switch to a larger claim $x'$. A $B$ agent who continues to claim $(V-x)$ gets $(2N-q)(V-x)$. If she switches to $(V-x')$ she gets $2N(V-x')$. Claiming $(V-x')$ is thus a best response provided that $2N(V-x') > (2N-q)(V-x)$, which requires $q > 2N\frac{x'-x}{V-x}$. If $q$ agents in $A$ switch to a smaller claim $x' < x$, a $B$ agent who continues to claim $(V-x)$ gets $2N(V-x)$, while a $B$ agent claiming $(V-x')$ gets $q(V-x')$. $(V-x')$ is thus a best response provided that $q(V-x') > 2N(V-x)$, which requires $q > 2N\frac{V-x}{V-x'}$. ■

**Lemma 7** *Let $\theta_x$ be an equilibrium in $P_H \cup P_L$. The minimum number of mutations in population A required to enter the basin of attraction of any other equilibrium is at least min$(N\frac{\delta}{V-x}, N\frac{V-x}{V-\delta})$.*

**Proof.** Reaching from $\theta_x$ the basin of attraction of an equilibrium $\theta_{x'}$ with $x' > x$ requires $N\frac{x'-x}{V-x}$ mutations in the $A$ population if $\theta \in P_L$ (Lemma 5) or $2N\frac{x'-x}{V-x}$ if $\theta \in P_H$ (Lemma 6). These numbers are clearly minimized for $x' = x+\delta$, so that the number of mutations it at least $N\frac{\delta}{V-x}$. To move the population in the basin of attraction and equilibrium $\theta_{x'}$ with $x' < x$ it takes a number of mutations equal to $N\frac{V-x}{V-x'}$ if $\theta_x \in P_L$ (Lemma 5) and $2N\frac{V-x}{V-x'}$ if $\theta_x \in P_H$ (Lemma 6). These numbers are minimized for $x' = \delta$, so that the minimum number of mutations is at least equal to $N\frac{V-x}{V-\delta}$.

The minimum number of mutations in population $A$ required to exit the basin of attraction of an equilibrium $\theta_x$ is thus the minimum between $N\frac{\delta}{V-x}$, when the new equilibrium selected is $\theta_{x+\delta}$, and $N\frac{V-x}{V-\delta}$, when the new equilibrium selected is $\theta \in NP$. ■

We are now ready for the main result of the paper.

*Proof of Proposition* 3.

**Proof.** The construction of the minimum resistance tree is based on an algorithm named *naive minimization test.* (See Binmore e.a. (2003) for an application to bargaining games) It works as follows: join with an arrow any locally stable state $\theta$ with the locally stable state whose basin of attraction can be reached from $\theta$ with the smallest number of mutations. If one obtains a chain with a single loop, remove the arrow associated with the highest resistance. The remaining arrows form a rooted tree, which is the minimum resistance tree. The root of the tree corresponds to the unique stochastically stable state. To see how this process works in the present context, take any state $\theta \in P_L \cup P_H$. Because of Corollary 1 and Lemma 7, the minimum number of mutations required to enter the basin of attraction of another equilibrium is either $N\frac{\delta}{x}$ (when mutations occur in the $B$ population and the system enters the basin of attraction of convention $(x-\delta, V-x+\delta)$) or a number no smaller than $min(N\frac{\delta}{V-x}, N\frac{V-x}{V-\delta})$ (when mutations occur in the $A$ population). Consider first that $N\frac{\delta}{x} \leq N\frac{V-x}{V-\delta}$. Also, by Assumption 1 $(iii)$ $x > \frac{V}{2}$, and therefore $N\frac{\delta}{x} < N\frac{\delta}{V-x}$. This implies that the easiest transition out of $\theta_x$ is towards convention $(x-\delta, V-x+\delta)$. Using the naive minimization test one obtains a chain of arrows that point from any $\theta_x$ to $\theta_{x-\delta}$, with $x \geq x_L$. Finally, one can join $\theta_{x_L}$ with any other $\theta_x$ with $x > x_L$. In fact, leaving $\theta_{x_L}$ requires at least $N\frac{\delta}{x_L}$ $B$ agents who mutate to claiming $V - x_L + \delta$. When this happens, playing $I$ fails to be rational even for efficient agents. An equilibrium in $NP$ is thus reached, which can be linked (with a single mutation) to any equilibrium in $P_H \cup P_L$. The arrow departing from $\theta_{x_L}$ is clearly associated to the highest resistance. Removing that arrow would produce a tree rooted at $\theta_{x_L}$, which completes the proof. ■

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