

Supplementary material for: Likelihood-based Risk Estimation for Variance-Gamma Models

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Abstract

This document contains tables and graphs illustrating some of the outcomes of the simulation experiments and of the real-data analyses described in detail in the paper “Likelihood-based Risk Estimation for Variance-Gamma Models”.

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A Supplementary data

Algo.	Par.	Value	Δ									
			10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}		
MCECM	μ	0.00095	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	σ^2	0.04294	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	γ	-0.00084	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	ν	1.99144	1.155	0.926	0.863	0.518	0.837	0.863	0.863	0.457	1.321	1.321
	iter		264.00	248.37	243.80	210.93	227.28	227.45	222.22	239.10		
	time		3.551	3.114	3.194	3.186	2.943	3.873	3.764	4.309		
ECME	μ	0.00095	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	σ^2	0.04294	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	γ	-0.00084	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	ν	1.99144	1.157	0.937	0.864	0.519	0.839	0.865	0.865	0.458	1.324	1.324
	iter		53.37	47.97	47.31	48.36	48.78	47.80	47.19	50.16		
	time		11.056	9.626	9.733	9.994	9.870	11.489	10.764	13.352		
HECM	μ	0.00095	0.001	0.000	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	σ^2	0.04294	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	γ	-0.00084	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001	0.001
	ν	1.99144	1.16	0.939	0.864	0.519	0.839	0.865	0.865	0.458	1.324	1.324
	iter		270.17	252.88	247.45	214.55	231.01	231.05	225.80	242.87		
	time		4.054	3.523	3.611	3.641	3.389	4.397	4.423	5.786		

Table A.1: RMSE of the estimators of the parameters of the VG distribution, computed using 100 replications and a sample size $n = 500$. Time is the total time to convergence measured in seconds.

Algo.	Par.	Value	Δ								
			10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	
MCECM	μ	0.00095	0.002	0.002	0.002	0.003	0.003	0.002	0.002	0.002	0.002
	σ^2	0.04294	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	γ	-0.3	0.002	0.003	0.006	0.003	0.003	0.002	0.002	0.002	0.002
	ν	1.99144	0.189	0.219	0.165	0.303	0.279	0.243	0.129	0.184	0.184
	iter		56.35	68.25	62.62	72.60	73.00	71.28	62.40	71.94	71.94
	time		3.811	3.644	3.742	3.867	3.847	3.991	3.932	3.708	3.708
ECME	μ	0.00095	0.002	0.003	0.002	0.003	0.003	0.002	0.002	0.002	0.002
	σ^2	0.04294	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	γ	-0.3	0.002	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002
	ν	1.99144	0.189	0.222	0.165	0.344	0.284	0.243	0.129	0.185	0.185
	iter		47.413	59.085	52.866	62.999	62.047	60.449	52.865	61.862	61.862
	time		84.162	82.070	80.692	79.454	78.152	79.873	80.639	91.998	91.998
HECM	μ	0.00095	0.002	0.003	0.002	0.003	0.003	0.002	0.002	0.002	0.002
	σ^2	0.04294	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	γ	-0.3	0.002	0.003	0.003	0.003	0.003	0.002	0.002	0.002	0.002
	ν	1.99144	0.189	0.233	0.166	0.348	0.287	0.244	0.131	0.189	0.189
	iter		59.48	73.12	66.56	77.98	77.95	76.74	67.67	75.86	75.86
	time		4.919	4.548	5.337	4.967	4.803	5.077	4.531	4.559	4.559

Table A.2: RMSE of the estimators of the parameters of the VG distribution, computed using 100 replications and a sample size $n = 500$. Time is the total time to convergence measured in seconds.

A.1 Sensitivity to Δ : $\nu = 0.5$

Δ	μ	σ^2	γ	ν
1e-08	5.402E-08	6.661E-11	1.248E-07	0.004
1e-07	4.043E-08	6.605E-11	1.372E-07	0.004
1e-06	4.869E-08	5.939E-11	1.740E-07	0.003
1e-05	4.636E-08	6.185E-11	1.451E-07	0.003
1e-04	4.747E-08	5.321E-11	1.275E-07	0.003
0.001	3.290E-08	4.208E-11	1.045E-07	0.002
0.01	8.586E-09	5.144E-11	1.223E-07	0.002
0.1	1.767E-08	1.068E-10	1.300E-07	0.003

Table A.3: RMSE ($\sigma^2 = 6.44\text{E-}05$, $\gamma = -8.44\text{E-}04$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	3.966E-03	1.056E-03	4.165E-03	0.013
1e-07	4.570E-03	1.187E-03	4.742E-03	0.017
1e-06	4.416E-03	1.157E-03	4.587E-03	0.015
1e-05	4.160E-03	1.078E-03	4.464E-03	0.013
1e-04	3.798E-03	1.026E-03	4.000E-03	0.012
0.001	9.901E-04	2.929E-04	1.162E-03	0.002
0.01	2.074E-05	5.891E-06	1.136E-04	0.000
0.1	3.460E-08	4.135E-09	7.021E-04	0.001

Table A.4: RMSE ($\sigma^2 = 6.44\text{E-}05$, $\gamma = -0.2$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	1.009E-02	6.177E-03	9.807E-03	0.013
1e-07	8.382E-03	5.155E-03	8.659E-03	0.012
1e-06	9.506E-03	5.717E-03	9.451E-03	0.014
1e-05	9.340E-03	5.774E-03	8.808E-03	0.014
1e-04	7.958E-03	5.042E-03	8.027E-03	0.010
0.001	2.300E-03	1.605E-03	2.520E-03	0.002
0.01	3.183E-05	1.911E-05	1.351E-04	0.001
0.1	9.679E-09	4.135E-09	1.853E-03	0.001

Table A.5: RMSE ($\sigma^2 = 6.44\text{E-}05$, $\gamma = -0.3$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	5.799E-06	1.526E-05	5.749E-05	0.003
1e-07	7.046E-06	1.129E-05	4.516E-05	0.003
1e-06	4.465E-06	1.316E-05	4.452E-05	0.002
1e-05	7.453E-06	8.842E-06	4.062E-05	0.002
1e-04	5.088E-06	1.383E-05	4.222E-05	0.002
0.001	4.566E-06	1.277E-05	5.495E-05	0.002
0.01	3.815E-06	1.159E-05	6.795E-05	0.002
0.1	1.010E-05	1.709E-05	5.669E-05	0.003

Table A.6: RMSE ($\sigma^2 = 0.03$, $\gamma = -8.44\text{E-}04$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	3.058E-03	6.834E-04	3.253E-03	0.014
1e-07	3.048E-03	6.727E-04	3.138E-03	0.015
1e-06	2.837E-03	6.189E-04	3.044E-03	0.013
1e-05	2.677E-03	5.693E-04	3.045E-03	0.015
1e-04	2.784E-03	6.483E-04	2.730E-03	0.011
0.001	5.651E-04	1.365E-04	9.682E-04	0.002
0.01	1.087E-05	1.215E-05	2.266E-04	0.002
0.1	6.671E-06	1.975E-05	2.513E-04	0.002

Table A.7: RMSE ($\sigma^2 = 0.03$, $\gamma = -0.2$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	6.580E-03	3.575E-03	6.983E-03	0.013
1e-07	6.915E-03	3.776E-03	7.189E-03	0.012
1e-06	6.947E-03	3.702E-03	7.412E-03	0.012
1e-05	7.200E-03	3.853E-03	7.276E-03	0.014
1e-04	6.314E-03	3.619E-03	6.389E-03	0.011
0.001	1.562E-03	9.697E-04	1.511E-03	0.002
0.01	9.529E-06	2.044E-05	5.203E-04	0.001
0.1	4.830E-06	2.408E-05	3.779E-04	0.001

Table A.8: RMSE ($\sigma^2 = 0.03$, $\gamma = -0.3$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	6.536E-06	3.162E-05	6.354E-05	0.003
1e-07	6.203E-06	2.381E-05	5.045E-05	0.002
1e-06	1.026E-05	2.200E-05	7.256E-05	0.003
1e-05	7.136E-06	2.301E-05	6.131E-05	0.002
1e-04	6.284E-06	2.262E-05	5.016E-05	0.003
0.001	6.635E-06	2.346E-05	6.095E-05	0.002
0.01	3.861E-06	2.458E-05	7.591E-05	0.002
0.1	1.128E-05	3.140E-05	8.307E-05	0.004

Table A.9: RMSE ($\sigma^2 = 0.04$, $\gamma = -8.44E-04$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	2.684E-03	5.594E-04	2.699E-03	0.013
1e-07	2.657E-03	6.123E-04	2.644E-03	0.011
1e-06	2.710E-03	5.668E-04	2.922E-03	0.013
1e-05	2.561E-03	4.923E-04	2.744E-03	0.013
1e-04	2.723E-03	5.855E-04	2.712E-03	0.013
0.001	5.708E-04	1.305E-04	7.330E-04	0.002
0.01	7.018E-06	2.899E-05	2.811E-04	0.002
0.1	1.364E-05	3.629E-05	2.637E-04	0.002

Table A.10: RMSE ($\sigma^2 = 0.04$, $\gamma = -0.2$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	6.023E-03	3.359E-03	6.110E-03	0.010
1e-07	6.678E-03	3.405E-03	7.017E-03	0.012
1e-06	6.804E-03	3.748E-03	6.440E-03	0.011
1e-05	6.363E-03	3.512E-03	6.411E-03	0.013
1e-04	6.606E-03	3.555E-03	6.717E-03	0.010
0.001	1.629E-03	9.133E-04	1.977E-03	0.002
0.01	6.638E-06	2.670E-05	4.781E-04	0.001
0.1	8.792E-06	2.907E-05	5.027E-04	0.002

Table A.11: RMSE ($\sigma^2 = 0.04$, $\gamma = -0.3$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	1.783E-05	3.232E-04	1.921E-04	0.004
1e-07	3.390E-05	2.647E-04	1.883E-04	0.004
1e-06	2.903E-05	2.329E-04	2.270E-04	0.002
1e-05	3.240E-05	3.399E-04	2.434E-04	0.002
1e-04	2.426E-05	3.042E-04	2.189E-04	0.002
0.001	1.945E-05	2.907E-04	2.418E-04	0.002
0.01	1.463E-05	3.162E-04	3.137E-04	0.002
0.1	3.178E-05	3.586E-04	2.858E-04	0.003

Table A.12: RMSE ($\sigma^2 = 0.14$, $\gamma = -8.44E-04$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	2.162E-03	5.325E-04	2.278E-03	0.008
1e-07	2.215E-03	6.059E-04	2.195E-03	0.009
1e-06	2.039E-03	4.739E-04	2.378E-03	0.009
1e-05	2.087E-03	4.485E-04	2.582E-03	0.009
1e-04	2.105E-03	5.184E-04	2.426E-03	0.009
0.001	9.569E-04	3.468E-04	1.268E-03	0.004
0.01	1.898E-05	2.519E-04	5.678E-04	0.002
0.1	4.399E-05	3.817E-04	4.648E-04	0.003

Table A.13: RMSE ($\sigma^2 = 0.14$, $\gamma = -0.2$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	4.814E-03	2.110E-03	5.279E-03	0.009
1e-07	5.037E-03	2.378E-03	5.373E-03	0.010
1e-06	5.253E-03	2.554E-03	5.474E-03	0.010
1e-05	5.084E-03	2.286E-03	5.809E-03	0.011
1e-04	4.608E-03	2.385E-03	4.555E-03	0.012
0.001	1.673E-03	9.494E-04	1.828E-03	0.003
0.01	2.232E-05	2.637E-04	5.719E-04	0.002
0.1	3.343E-05	3.467E-04	5.953E-04	0.003

Table A.14: RMSE ($\sigma^2 = 0.14$, $\gamma = -0.3$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	2.557E-05	6.790E-04	2.646E-04	0.004
1e-07	4.213E-05	4.004E-04	2.822E-04	0.003
1e-06	2.353E-05	4.098E-04	2.674E-04	0.003
1e-05	3.325E-05	4.225E-04	2.402E-04	0.002
1e-04	3.626E-05	3.476E-04	2.514E-04	0.003
0.001	2.871E-05	5.074E-04	2.693E-04	0.002
0.01	1.913E-05	3.846E-04	3.935E-04	0.002
0.1	3.427E-05	4.918E-04	3.503E-04	0.003

Table A.15: RMSE ($\sigma^2 = 0.17$, $\gamma = -8.44E-04$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	2.004E-03	7.793E-04	2.365E-03	0.008
1e-07	2.067E-03	7.306E-04	2.100E-03	0.008
1e-06	2.307E-03	7.656E-04	2.002E-03	0.009
1e-05	2.003E-03	7.782E-04	2.066E-03	0.008
1e-04	2.051E-03	6.764E-04	2.413E-03	0.007
0.001	7.968E-04	3.984E-04	1.263E-03	0.004
0.01	2.399E-05	3.934E-04	5.758E-04	0.002
0.1	4.935E-05	6.486E-04	5.589E-04	0.003

Table A.16: RMSE ($\sigma^2 = 0.17$, $\gamma = -0.2$, $\nu = 0.5$).

Δ	μ	σ^2	γ	ν
1e-08	4.509E-03	2.441E-03	5.169E-03	0.009
1e-07	4.836E-03	2.469E-03	5.297E-03	0.009
1e-06	4.911E-03	2.447E-03	4.920E-03	0.010
1e-05	4.557E-03	1.878E-03	5.124E-03	0.010
1e-04	4.413E-03	2.049E-03	4.552E-03	0.009
0.001	1.494E-03	9.821E-04	2.294E-03	0.002
0.01	2.732E-05	4.140E-04	6.413E-04	0.001
0.1	4.835E-05	6.667E-04	7.102E-04	0.002

Table A.17: RMSE ($\sigma^2 = 0.17$, $\gamma = -0.3$, $\nu = 0.5$).

A.2 Sensitivity to Δ : $\nu = 1.991436822$

Δ	μ	σ^2	γ	ν
1e-08	8.746E-07	2.181E-11	1.094E-06	0.475
1e-07	1.159E-06	2.180E-11	1.385E-06	0.606
1e-06	8.167E-07	2.713E-11	9.202E-07	0.609
1e-05	3.420E-06	2.903E-11	3.470E-06	6.934
1e-04	9.610E-07	2.626E-11	9.725E-07	0.768
0.001	9.371E-07	4.035E-11	1.078E-06	0.634
0.01	1.123E-06	3.318E-11	1.281E-06	0.912
0.1	9.554E-07	2.311E-11	1.076E-06	1.197

Table A.18: RMSE ($\sigma^2 = 6.44\text{E-}05$, $\gamma = -8.44\text{E-}04$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	1.022E-04	1.135E-06	1.304E-04	0.038
1e-07	1.046E-04	1.203E-06	1.599E-04	0.039
1e-06	1.116E-04	1.271E-06	1.504E-04	0.046
1e-05	9.757E-05	1.197E-06	1.182E-04	0.034
1e-04	1.014E-04	1.176E-06	1.494E-04	0.045
0.001	1.668E-04	2.071E-06	2.144E-04	0.049
0.01	9.944E-05	1.083E-06	1.403E-04	0.031
0.1	1.268E-04	1.446E-06	1.807E-04	0.045

Table A.19: RMSE ($\sigma^2 = 6.44\text{E-}05$, $\gamma = -0.2$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	3.040E-04	7.118E-06	4.248E-04	0.054
1e-07	2.879E-04	7.843E-06	3.240E-04	0.043
1e-06	2.674E-04	7.373E-06	3.489E-04	0.037
1e-05	3.324E-04	8.289E-06	3.817E-04	0.049
1e-04	3.474E-04	9.167E-06	4.256E-04	0.050
0.001	3.472E-04	9.166E-06	3.501E-04	0.048
0.01	2.362E-04	6.670E-06	3.002E-04	0.044
0.1	3.516E-04	8.885E-06	4.837E-04	0.047

Table A.20: RMSE ($\sigma^2 = 6.44\text{E-}05$, $\gamma = -0.3$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	4.224E-04	6.163E-06	4.752E-04	2.206
1e-07	4.905E-04	6.338E-06	5.653E-04	2.239
1e-06	4.591E-04	7.571E-06	5.637E-04	1.635
1e-05	4.998E-04	7.709E-06	5.858E-04	1.696
1e-04	2.560E-04	5.841E-06	3.564E-04	0.381
0.001	5.086E-04	5.471E-06	6.541E-04	0.543
0.01	4.691E-04	6.829E-06	5.290E-04	0.449
0.1	4.758E-04	6.109E-06	4.817E-04	1.134

Table A.21: RMSE ($\sigma^2 = 0.03$, $\gamma = -8.44\text{E-}04$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	6.326E-04	1.608E-05	7.289E-04	0.164
1e-07	1.044E-03	1.868E-05	1.056E-03	0.278
1e-06	1.657E-03	2.426E-05	1.788E-03	0.397
1e-05	1.497E-03	2.579E-05	1.568E-03	0.351
1e-04	1.753E-03	2.367E-05	1.867E-03	0.368
0.001	1.499E-03	1.792E-05	1.631E-03	0.413
0.01	1.018E-03	1.976E-05	1.169E-03	0.231
0.1	1.828E-03	2.539E-05	1.993E-03	0.383

Table A.22: RMSE ($\sigma^2 = 0.03$, $\gamma = -0.2$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	1.564E-03	4.770E-05	1.813E-03	0.141
1e-07	1.988E-03	5.971E-05	2.004E-03	0.212
1e-06	2.482E-03	5.529E-05	2.688E-03	0.264
1e-05	1.644E-03	5.695E-05	1.783E-03	0.144
1e-04	1.840E-03	5.848E-05	1.918E-03	0.177
0.001	1.434E-03	4.218E-05	1.655E-03	0.161
0.01	1.342E-03	4.555E-05	1.526E-03	0.137
0.1	1.461E-03	5.219E-05	1.506E-03	0.147

Table A.23: RMSE ($\sigma^2 = 0.03$, $\gamma = -0.3$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	5.655E-04	9.613E-06	6.643E-04	0.472
1e-07	4.977E-04	9.182E-06	6.396E-04	0.557
1e-06	7.471E-04	1.413E-05	7.388E-04	1.262
1e-05	5.065E-04	1.194E-05	5.598E-04	0.593
1e-04	4.982E-04	1.148E-05	6.097E-04	0.420
0.001	4.700E-04	1.189E-05	4.983E-04	0.568
0.01	5.988E-04	1.197E-05	7.071E-04	0.989
0.1	5.350E-04	1.151E-05	6.849E-04	0.925

Table A.24: RMSE ($\sigma^2 = 0.03$, $\gamma = -8.44E-04$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	1.742E-03	2.782E-05	1.818E-03	0.411
1e-07	1.622E-03	3.004E-05	1.701E-03	0.515
1e-06	1.819E-03	3.929E-05	1.996E-03	0.355
1e-05	2.145E-03	3.906E-05	2.425E-03	0.457
1e-04	1.395E-03	2.846E-05	1.428E-03	0.351
0.001	1.569E-03	3.462E-05	1.659E-03	0.531
0.01	2.093E-03	3.486E-05	2.088E-03	0.452
0.1	1.689E-03	2.553E-05	1.716E-03	0.419

Table A.25: RMSE ($\sigma^2 = 0.04$, $\gamma = -0.2$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	2.099E-03	7.386E-05	2.313E-03	0.216
1e-07	2.213E-03	8.466E-05	2.243E-03	0.233
1e-06	1.941E-03	7.637E-05	2.089E-03	0.180
1e-05	2.204E-03	5.979E-05	2.177E-03	0.215
1e-04	2.351E-03	8.939E-05	2.678E-03	0.170
0.001	2.248E-03	5.796E-05	2.423E-03	0.231
0.01	2.527E-03	7.876E-05	2.691E-03	0.207
0.1	2.263E-03	6.594E-05	2.461E-03	0.263

Table A.26: RMSE ($\sigma^2 = 0.04$, $\gamma = -0.3$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	1.721E-03	1.310E-04	1.771E-03	1.358
1e-07	2.340E-03	1.578E-04	2.882E-03	0.648
1e-06	2.449E-03	1.515E-04	2.565E-03	0.951
1e-05	1.716E-03	1.280E-04	2.140E-03	0.466
1e-04	1.657E-03	1.562E-04	1.845E-03	0.299
0.001	2.510E-03	1.826E-04	2.727E-03	0.734
0.01	2.439E-03	1.325E-04	2.719E-03	0.894
0.1	1.615E-03	1.149E-04	2.061E-03	0.654

Table A.27: RMSE ($\sigma^2 = 0.04$, $\gamma = -8.44E-04$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	3.859E-03	1.781E-04	4.295E-03	0.800
1e-07	2.606E-03	1.940E-04	2.744E-03	0.424
1e-06	4.028E-03	1.972E-04	4.752E-03	0.509
1e-05	4.577E-03	2.056E-04	4.760E-03	0.557
1e-04	2.911E-03	2.672E-04	3.180E-03	0.513
0.001	6.909E-03	2.260E-04	7.498E-03	2.202
0.01	5.379E-03	2.052E-04	5.135E-03	0.831
0.1	3.514E-03	1.541E-04	4.039E-03	0.398

Table A.28: RMSE ($\sigma^2 = 0.14$, $\gamma = -0.2$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	3.703E-03	2.633E-04	4.298E-03	0.377
1e-07	3.149E-03	2.940E-04	3.455E-03	0.409
1e-06	4.061E-03	2.318E-04	4.167E-03	0.425
1e-05	7.472E-03	3.546E-04	7.873E-03	0.631
1e-04	6.393E-03	3.365E-04	7.013E-03	0.709
0.001	5.056E-03	2.230E-04	5.432E-03	0.452
0.01	6.106E-03	2.809E-04	6.137E-03	0.543
0.1	4.831E-03	3.001E-04	4.810E-03	0.553

Table A.29: RMSE ($\sigma^2 = 0.14$, $\gamma = -0.3$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	2.131E-03	1.735E-04	2.643E-03	0.503
1e-07	2.031E-03	2.364E-04	2.167E-03	0.706
1e-06	2.390E-03	1.482E-04	2.757E-03	0.330
1e-05	2.437E-03	1.843E-04	3.058E-03	0.693
1e-04	2.909E-03	2.163E-04	3.042E-03	0.391
0.001	2.360E-03	2.134E-04	2.895E-03	0.630
0.01	2.110E-03	2.056E-04	2.703E-03	0.648
0.1	1.880E-03	2.170E-04	2.142E-03	0.539

Table A.30: RMSE ($\sigma^2 = 0.17$, $\gamma = -8.44\text{E-}04$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	5.231E-03	2.539E-04	6.130E-03	0.580
1e-07	5.117E-03	2.647E-04	5.133E-03	0.829
1e-06	4.837E-03	2.971E-04	4.851E-03	0.714
1e-05	3.417E-03	2.647E-04	4.045E-03	0.706
1e-04	5.211E-03	2.820E-04	5.467E-03	1.114
0.001	3.601E-03	3.311E-04	3.846E-03	0.339
0.01	3.408E-03	2.593E-04	4.222E-03	0.616
0.1	6.771E-03	2.801E-04	7.729E-03	1.141

Table A.31: RMSE ($\sigma^2 = 0.17$, $\gamma = -0.2$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	4.049E-03	2.958E-04	4.723E-03	0.326
1e-07	5.773E-03	4.429E-04	6.547E-03	0.427
1e-06	6.102E-03	3.216E-04	6.997E-03	0.470
1e-05	3.671E-03	3.236E-04	4.298E-03	0.276
1e-04	6.909E-03	4.947E-04	7.540E-03	0.468
0.001	5.085E-03	3.465E-04	5.569E-03	0.342
0.01	7.282E-03	4.012E-04	8.133E-03	0.620
0.1	3.687E-03	3.346E-04	4.100E-03	0.315

Table A.32: RMSE ($\sigma^2 = 0.17$, $\gamma = -0.3$, $\nu = 1.991436822$).

Δ	μ	σ^2	γ	ν
1e-08	2.780E-05	2.725E-11	2.799E-05	135.143
1e-07	5.686E-05	3.106E-11	5.663E-05	488.175
1e-06	3.151E-05	2.972E-11	3.140E-05	348.959
1e-05	3.365E-05	2.386E-11	3.404E-05	175.762
1e-04	3.560E-05	3.397E-11	3.577E-05	433.544
0.001	2.233E-05	2.470E-11	2.245E-05	303.611
0.01	2.949E-05	2.734E-11	3.034E-05	271.711
0.1	3.453E-05	1.900E-11	3.422E-05	458.149

Table A.33: RMSE ($\sigma^2 = 6.44E-05$, $\gamma = -8.44E-04$, $\nu = 1.991436822$).

A.3 Sensitivity to Δ : $\nu = 7.5091988$

Δ	μ	σ^2	γ	ν
1e-08	6.847E-03	6.319E-06	6.827E-03	7.334
1e-07	6.783E-03	6.668E-06	6.778E-03	7.432
1e-06	6.693E-03	6.522E-06	6.707E-03	7.695
1e-05	6.889E-03	7.372E-06	6.845E-03	7.560
1e-04	6.643E-03	7.009E-06	6.588E-03	6.923
0.001	7.007E-03	7.317E-06	6.979E-03	6.593
0.01	6.585E-03	7.011E-06	6.605E-03	7.657
0.1	6.875E-03	6.914E-06	6.906E-03	7.212

Table A.34: RMSE ($\sigma^2 = 6.44\text{E-}05$, $\gamma = -0.2$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	1.595E-02	3.728E-05	1.612E-02	7.187
1e-07	1.513E-02	3.444E-05	1.498E-02	7.602
1e-06	1.472E-02	3.362E-05	1.469E-02	7.047
1e-05	1.545E-02	3.560E-05	1.542E-02	6.943
1e-04	1.528E-02	3.506E-05	1.522E-02	7.105
0.001	1.529E-02	3.486E-05	1.532E-02	7.003
0.01	1.483E-02	3.575E-05	1.479E-02	7.310
0.1	1.488E-02	3.422E-05	1.483E-02	6.892

Table A.35: RMSE ($\sigma^2 = 6.44\text{E-}05$, $\gamma = -0.3$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	2.150E-02	6.018E-06	2.170E-02	382.048
1e-07	2.009E-02	7.892E-06	2.060E-02	326.552
1e-06	2.299E-02	7.098E-06	2.283E-02	539.980
1e-05	1.737E-02	6.789E-06	1.738E-02	560.085
1e-04	1.922E-02	7.975E-06	1.969E-02	274.506
0.001	1.745E-02	9.368E-06	1.773E-02	356.283
0.01	1.944E-02	8.157E-06	1.965E-02	270.464
0.1	1.641E-02	8.154E-06	1.655E-02	258.058

Table A.36: RMSE ($\sigma^2 = 0.03$, $\gamma = -8.44\text{E-}04$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	1.057E-02	1.464E-05	1.090E-02	60.099
1e-07	9.546E-03	1.588E-05	9.698E-03	48.853
1e-06	1.015E-02	1.746E-05	9.830E-03	102.658
1e-05	1.202E-02	1.211E-05	1.202E-02	46.384
1e-04	1.197E-02	1.401E-05	1.176E-02	97.060
0.001	1.068E-02	1.230E-05	1.067E-02	83.367
0.01	1.046E-02	1.277E-05	1.025E-02	53.622
0.1	8.998E-03	1.332E-05	8.989E-03	32.382

Table A.37: RMSE ($\sigma^2 = 0.03$, $\gamma = -0.2$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	6.159E-03	2.114E-05	6.244E-03	12.190
1e-07	6.326E-03	2.085E-05	6.515E-03	10.620
1e-06	7.341E-03	1.903E-05	7.233E-03	15.065
1e-05	5.359E-03	1.763E-05	5.535E-03	9.945
1e-04	5.945E-03	1.719E-05	6.079E-03	14.159
0.001	5.275E-03	1.864E-05	5.610E-03	15.400
0.01	5.493E-03	1.942E-05	5.301E-03	8.642
0.1	7.070E-03	2.063E-05	7.157E-03	12.623

Table A.38: RMSE ($\sigma^2 = 0.03$, $\gamma = -0.3$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	2.494E-02	1.302E-05	2.607E-02	237.582
1e-07	1.457E-02	9.691E-06	1.492E-02	188.446
1e-06	2.286E-02	1.207E-05	2.298E-02	363.143
1e-05	2.117E-02	1.255E-05	2.128E-02	207.277
1e-04	1.610E-02	9.860E-06	1.598E-02	241.581
0.001	1.420E-02	1.099E-05	1.430E-02	283.587
0.01	2.457E-02	1.172E-05	2.452E-02	298.689
0.1	2.166E-02	1.429E-05	2.190E-02	279.552

Table A.39: RMSE ($\sigma^2 = 0.04$, $\gamma = -8.44\text{E-}04$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	1.821E-02	2.847E-05	1.814E-02	64.252
1e-07	1.526E-02	2.926E-05	1.558E-02	96.871
1e-06	1.816E-02	3.289E-05	1.827E-02	91.577
1e-05	1.570E-02	2.351E-05	1.574E-02	88.783
1e-04	1.514E-02	2.769E-05	1.530E-02	120.034
0.001	1.619E-02	2.139E-05	1.612E-02	67.958
0.01	1.700E-02	2.351E-05	1.710E-02	43.539
0.1	1.408E-02	2.339E-05	1.439E-02	78.621

Table A.40: RMSE ($\sigma^2 = 0.04$, $\gamma = -0.2$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	1.114E-02	2.996E-05	1.099E-02	21.961
1e-07	9.016E-03	3.025E-05	8.938E-03	30.541
1e-06	1.075E-02	3.244E-05	1.091E-02	24.981
1e-05	1.043E-02	3.004E-05	1.044E-02	29.989
1e-04	9.375E-03	2.797E-05	9.428E-03	25.186
0.001	9.512E-03	3.190E-05	9.573E-03	16.298
0.01	1.016E-02	3.770E-05	1.005E-02	23.517
0.1	1.012E-02	2.847E-05	1.029E-02	19.746

Table A.41: RMSE ($\sigma^2 = 0.04$, $\gamma = -0.3$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	6.300E-02	1.385E-04	6.491E-02	412.175
1e-07	7.598E-02	1.602E-04	7.760E-02	271.348
1e-06	6.952E-02	1.183E-04	7.221E-02	385.741
1e-05	9.053E-02	1.973E-04	8.969E-02	286.945
1e-04	7.439E-02	1.481E-04	7.563E-02	252.120
0.001	6.775E-02	1.204E-04	6.789E-02	278.723
0.01	4.892E-02	1.280E-04	5.042E-02	175.165
0.1	4.754E-02	9.504E-05	4.747E-02	258.982

Table A.42: RMSE ($\sigma^2 = 0.14$, $\gamma = -8.44E-04$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	6.369E-02	2.121E-04	6.606E-02	208.412
1e-07	7.992E-02	1.906E-04	7.927E-02	243.294
1e-06	1.011E-01	1.834E-04	1.026E-01	239.741
1e-05	8.176E-02	2.356E-04	7.980E-02	162.312
1e-04	6.353E-02	2.211E-04	6.548E-02	220.601
0.001	6.494E-02	1.746E-04	6.561E-02	199.666
0.01	7.538E-02	1.750E-04	7.674E-02	252.802
0.1	6.023E-02	1.883E-04	6.113E-02	162.558

Table A.43: RMSE ($\sigma^2 = 0.14$, $\gamma = -0.2$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	5.845E-02	2.487E-04	5.900E-02	129.307
1e-07	6.566E-02	3.021E-04	6.636E-02	132.985
1e-06	5.999E-02	3.285E-04	6.071E-02	50.635
1e-05	5.999E-02	2.321E-04	5.844E-02	171.731
1e-04	5.697E-02	2.128E-04	5.760E-02	136.386
0.001	5.578E-02	2.337E-04	5.604E-02	188.710
0.01	5.498E-02	1.864E-04	5.462E-02	110.321
0.1	7.254E-02	2.608E-04	7.133E-02	73.033

Table A.44: RMSE ($\sigma^2 = 0.14$, $\gamma = -0.3$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	8.717E-02	2.057E-04	8.724E-02	324.275
1e-07	7.708E-02	1.983E-04	7.905E-02	267.529
1e-06	8.084E-02	2.054E-04	7.996E-02	276.896
1e-05	5.641E-02	1.528E-04	5.635E-02	250.233
1e-04	6.815E-02	1.908E-04	6.796E-02	283.266
0.001	6.419E-02	1.588E-04	6.252E-02	212.855
0.01	8.294E-02	1.776E-04	8.442E-02	205.106
0.1	9.137E-02	2.018E-04	9.168E-02	229.120

Table A.45: RMSE ($\sigma^2 = 0.17$, $\gamma = -8.44E-04$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	1.104E-01	3.382E-04	1.094E-01	300.821
1e-07	1.025E-01	3.082E-04	1.028E-01	165.676
1e-06	8.129E-02	2.654E-04	8.070E-02	350.191
1e-05	1.204E-01	2.757E-04	1.218E-01	254.358
1e-04	7.372E-02	2.045E-04	7.406E-02	246.261
0.001	9.476E-02	3.238E-04	9.718E-02	180.517
0.01	1.067E-01	3.702E-04	1.053E-01	288.772
0.1	9.193E-02	2.264E-04	9.027E-02	175.851

Table A.46: RMSE ($\sigma^2 = 0.17$, $\gamma = -0.2$, $\nu = 7.5091988$).

Δ	μ	σ^2	γ	ν
1e-08	6.575E-02	3.062E-04	6.538E-02	149.699
1e-07	6.105E-02	3.439E-04	6.237E-02	92.821
1e-06	6.481E-02	2.508E-04	6.596E-02	142.821
1e-05	8.155E-02	4.374E-04	8.330E-02	87.420
1e-04	7.517E-02	4.340E-04	7.401E-02	163.611
0.001	7.942E-02	3.310E-04	7.985E-02	125.001
0.01	8.448E-02	3.322E-04	8.371E-02	273.640
0.1	7.230E-02	3.898E-04	7.078E-02	111.251

Table A.47: RMSE ($\sigma^2 = 0.17$, $\gamma = -0.3$, $\nu = 7.5091988$).

	GHYP	VG	MCECM
$S_0 = 1330$	2.07	2.72	1.21
$S_0 = 1350$	2.07	2.70	1.23
$S_0 = 1390$	2.05	2.67	1.28
$S_0 = 1410$	2.00	2.74	1.26

Table A.48: Average RMSE-ranking of the estimated prices of a European call option with VG-distributed underlying and different starting prices, computed using 100 replications and a sample size $n = 500$. The total number of VG distributions is equal to 30, obtained as all possible combinations of $\mu = 0.000952372$, $\sigma^2 \in \{0.17, 0.14, 0.04, 0.03, 6.44 \cdot 10^{-5}\}$, $\gamma \in \{-0.000843568, -.2, -.3\}$, $\nu \in \{0.5, 1.991437, 7.509199\}$. The remaining parameters are $K = 1370$, $r = 0.02$, $t = 0.56164$, $q = 0.012$.

Figure A.1: Standard & Poor's correlograms: returns (panel a), squared returns (panel b), standardized returns (panel c) and squared standardized returns (panel d).

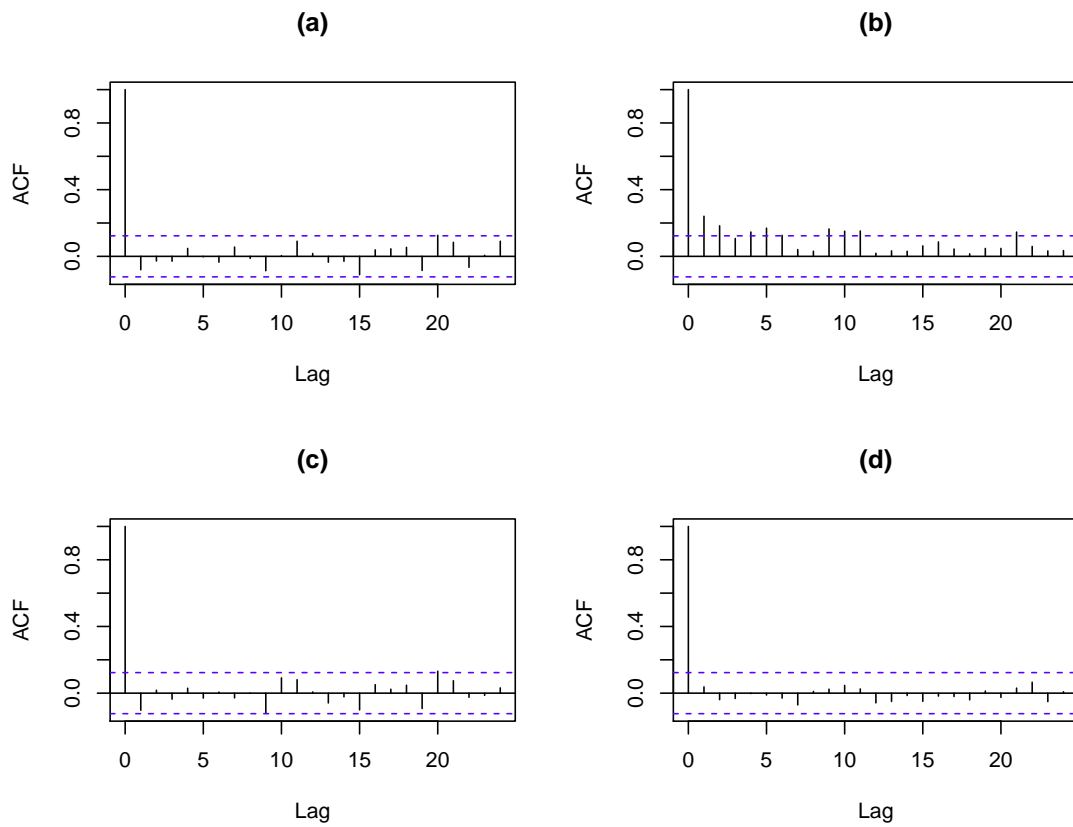


Figure A.2: Monte dei Paschi di Siena correlograms: returns (panel a), squared returns (panel b), standardized returns (panel c) and squared standardized returns (panel d).

