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Modelling group processes and effort estimation in Project Management using the Choquet integral: an MCDM approach

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Abstract

We investigate the group processes involved in effort estimation in the context of Project Management. The groups considered are formed by "experts" (people with specific technical competence) and "non-experts" (people with less specific technical competence, usually experts in related fields), because the typically complementary bias of the two classes contribute to a more balanced estimate. In this paper we exploit further the synergies between experts and non-experts in an MCDM framework, aggregating the individual estimates by means of non-additive Choquet integration, and representing the complementary bias by the multiagent interaction structure underlying the capacity. We present some examples and computer simulations whose aggregation results outperform those of the classical weighted mean (additive case), showing lower MMRE (mean magnitude of the relative error between the central estimate and the actual value) and higher HitRate (at which the interval estimate contains the actual value).
Keywords: Group decisions and multi-agent systems, multiple criteria analysis and criteria interaction, aggregation functions, Choquet integration, Project Management, effort estimation.

1 Introduction

Project Managers, today, can choose among many techniques and software to plan and manage their projects. The widespread usage of network approaches,

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like Gantt charts, critical path method (CPM), Critical Chain Method (CCM), etc. have simplified the planning and controlling steps, while project management software have reached a solid maturity level. However, there is one specific area in project management that has no clear and common methodologies or specialized software, i.e., assisting Project managers in formulating estimations regarding the many facets of a Project, like task's duration, resource's allocation, start date, finish date, allocation units and so on.
According to many studies, between 60% and 80% of projects are out of time and/or out of budget , while the average delay is approximately 40% of the estimated duration [56, 60, 49]. Especially when we are talking about complex project with high volatility, like ICT-related projects, project managers' ability to plan and guarantee efficient development mainly depends, especially in these complex environments, on how much accurate effort estimates are. In a survey conducted in 2007 [47] on more than 1,000 IT professionals, two out of three users confirmed that the three most important causes of IT project failure were related to inaccurate effort estimates, mainly deriving from resource under-estimation.
Among the different reasons, it has been shown that these inaccurate estimations were produced by collecting opinions from experts, subjects with relevant experience and strong technical competence. Using experts in estimation formulation is a well-known method especially in social sciences. Respect to other methods, based on mathematical modelling or similarity, this expert-based approach does not require huge amounts of historical series and is able to produce estimations even in settings with high uncertainty and few available information. Nevertheless, competence and experience represent a source of errors in estimate formulation. Psychological research have demonstrated that highly competent experts develop an optimistic bias when formulating estimations, and this in turn can produce an under-estimation of project's risks with, for example, lower values in duration estimation for tasks planning fallacy [37].
When grouping experts together, the individual errors, instead of attenuating, can be amplified by the comparison of analogous views from people with similar perception of the issues and with similar bias. In this case, the interaction within the group reinforces the individual errors, instead of mitigating them [26].
The aim of this paper is to analyze these issues related with experts' opinions, and to formulate a new model regarding how to aggregate experts' view on a certain aspect of a project, in order to reduce errors and obtain more accurate and reliable estimations. For this purpose, aggregation functions play a fundamental role. General reviews of aggregation functions can be found in Calvo, Mayor, and Mesiar [5], Beliakov, Pradera, and Calvo [2], Grabisch, Marichal, Mesiar, and Pap [20].
The classical aggregation scheme of group processes for effort estimation in Project management is the weighted mean. In this paper, however, we consider a more general aggregation framework, with interesting extra degrees of freedom: the Choquet integral, which is defined with respect to a non-additive capacity and corresponds to a large class of aggregation functions, including the classical weighted mean - the additive capacity case - and the ordered weighted mean (OWA) - the symmetric capacity case. Comprehensive reviews of Choquet integration con be found in Grabisch and Labreuche [17, 18, 19], Grabisch, Kojadinovich, and Mayer [16], plus also Wang and Klir [62], Grabisch, Nguyen and Walker [22], Grabisch, Murofushi and Sugeno [21].

In the framework of Choquet integration, in order to control the exponential complexity in the construction of the capacity ($2^n - 2$ real coefficients), Grabisch [14] introduced the so called k-additive capacities, see also Grabisch [15], and Miranda and Grabisch [45]. The 2-additive case in particular (see Miranda, Grabisch, and Gil, [46]; Mayag, Grabisch, and Labreuche, [43, 44]) is a good trade-off between the range of the model and its complexity (only $n(n+1)/2$ real coefficients are required to define a 2-additive capacity). The Choquet integral with respect to a 2-additive capacity is an interesting and effective modelling tool, see for instance Marques Pereira and Bortot [41, 42], Berrah and Clivillé [3], Clivillé, Berrah, and Maurice [9], Berrah, Maurice, and Montmain [4].
In this paper we propose an aggregation model based on Choquet integration with respect to a 2-additive capacity. The groups considered are formed by "experts" (people with specific technical competence) and "non-experts" (people with less specific technical competence, usually experts in related fields), because the typically complementary bias of the two classes contribute to a more balanced estimate. In this paper we exploit further the synergies between experts and non-experts in an MCDM framework, aggregating the individual estimates by means of non-additive Choquet integration, and representing the complementary bias by the multiagent interaction structure underlying the capacity.
The paper is organized as follows. In Section 2 we discuss the group processes in the context of Project Management. Section 3 reviews the basic definitions and results on capacities, particularly in the additive and 2-additive cases, and Choquet integration. In Section 4 we present our model using choquet integration, with some numerical examples. Finally, Section 5 contains computer simulations and some concluding remarks.

2 Group processes in Project management

A project is "... a temporary endeavor undertaken to create a unique product, service, or result" [51]. Unfortunately, the more complex the project is, the more difficult the estimation will be.
This paper deals with the Planning Process Group. Scientific researches have confirmed that the project failure is mainly caused by delays in the delivery, risk under-estimation and inadequate planning phase [63]. On the same line is Van Genuchten [60] that summarizes in too optimistic estimations and under-estimated complexity the main reasons why a project is late or/and fails. Phan et al. [50] agrees sustaining that many estimations are too optimistic, however he adds that other causes may be found in the changes of objectives and requirements of the project. Lastly, according to Standish Group, the main reasons of projects' pitfalls reside in poor users' involvement and requirements incompleteness [56].
Notwithstanding the fact that many complex and elaborate estimation models have been produced along the years, nowadays the most used methods for producing estimates are still those based on experts' opinions [25, 38, 27, 48, 24]. This is due to three main reasons: (a) some formal approaches failed to perform better than experts' opinion [29]; (b) some methods based on mathematical algorithms need many parameters in order to be adequately calibrated, thus discouraging their adoption; (c) historical data are most of the time not avail-

able.
Nowadays there is not a common and widely accepted definition of expert. However, research in the field of Delphi method has highlighted some important aspects that a person must have in order to be considered an expert. According to Cantrill et al. [6] (whose perspective is shared by Sackman [53], Linstone and Turoff [36]), "the definition [of an expert] should include any individual with relevant knowledge and experience on a particular subject...". Moreover, Adler et al. [1] defines four aspects that an expert should embody: (1) a deep knowledge and experience in the field under discussion; (2) capability and willingness to participate to the discussion; (3) enough available time to participate to a discussion; (4) communications skills.
The poor results of the estimation methods based on experts can be identified in two main reasons: planning fallacy and groupthink. Managers make decisions based on delusion optimism rather than on a rational weighting of gains, losses, and probabilities. They overestimate benefits and underestimate costs. They spin scenarios of success while overlooking the potential for mistakes and miscalculations. As a result, managers pursue initiatives that are unlikely to come in on budget or on time - or to ever deliver the expected returns [37].
The planning fallacy is caused by both organization and psychological causes. These reasons can be summarized in the following points: objectives conflict [28], anchoring effect [33], exaggerate own skill, social pressures, inconsistency in estimation. The sum of these effects is that experts are often influenced by a bias that leads to overoptimistic predictions and, therefore, unreliable estimation.
Regarding groupthink, the common sense suggests that a group of people should obtain better results than a single subject, but (especially in Project management) this is not exactly the case. The poor outcomes can be ascribed to many reasons that are part of the theory called groupthink. The term groupthink was widely used by Irving Janis in his studies [26], and it is defined as "quick and easy way to refer to a mode of thinking that people engage in when they are deeply involved in a cohesive in-group, when the members' striving for unanimity override their motivation to realistically appraise alternative courses of action [...]". The principal symptoms of groupthink are:(1) overestimation of group's skill. Groups often are influenced by a strong optimism which lead to an exaggerate evaluation of group's capabilities; (2) mental closure: a group sometimes fails to capture negative signals and therefore to undertake actions to prevent adverse events; (3) pressure towards uniformity: the social pressure within a group can push all the members to align their position to the majority's one.
Starting from the conclusions of Janis, Sunstein [58] illustrated some more causes that may push a group to produce poor results. He identified the following points as important issues to consider when creating a group:

- Amplification of cognitive errors: groups tend to amplify individual biases instead of mitigating them. This effect is known as well as "Garbage in, much garbage out" [34, 33, 11, 58]. As a result, if individuals are biased, the group will be even more biased.

- Hidden profiles and common knowledge: the incapacity of groups in managing information when this is diffused among its members.

- Information cascades: this effect involves the social pressure to uniformity.
- Group polarization: happens when group members conclude a discussion with positions which are more extreme than when the discussion started. This effect prevents to develop a rational discussion and therefore the group is not able to use the information distributed among members anymore.

In conclusion, regarding the delivery of reliable predictions on project management variables, individual experts' estimations are biased by many different factors, specifically their being too optimistic in formulating estimations. This situation is not improved by using groups of experts: the group of homogeneous people presents some internal problem that could produce poor results. The optimism about the estimations, produced by expertise, is a crucial factor in the success of overall project's estimations.

In order to overcome these limitations, we are approaching the issue of project variables' estimation from a different perspective. The approach presented in this paper starts from the idea that, in order to mitigate this innate optimism of experts, we should add to the panel of people that will produce estimations some people that have opposite characteristics, i.e., non-optimistic approach in estimation's formulation. These people are typically non-experts, afraid of delivering their opinion on a topic that they are not fully comfortable about, people that put a lot of attention on releasing crisp numbers and quantities for project estimations.

The idea that non-expert could be useful in projects variables' evaluation has some confirmation in other fields, specifically the web-development projects, where normally input from people in both technical (e.g., programming), and non-technical (e.g., user interaction design) roles are needed. The hypothesis, presented for the first time in a software engineering context by [30] and confirmed by the results of experimentations, is that role and type of competence affect the estimation strategy and performance of people involved. In other words, people with technical competence provided less realistic project effort estimates than those with less technical competence. So more knowledge regarding the implementation phase of a software development project does not always lead to better estimation performance.

In this study, many possible reasons are analyzed, but two of them emerge very clearly. First of all, while technical competence induces a bottom-up strategy, based on what authors call a "construction-based estimation strategy", on the other hand the lack of this competence induces an external view of the project, where a top-down estimation strategy is applied. This outside view of the project may help to be more realistic, or better, not so optimistic in estimations, and force the user to use the history of previous projects and experiences, thus reducing the bias towards over-optimism.

The second (reasonable) element is related with the evaluation of skills of people in technical roles. These people covering these roles perceive a better evaluation of their professionalism when providing low effort estimates. The authors conclude with a corollary to their study: the choice of estimation strategy, estimation evaluation criteria and feedback are important aspects to consider when seeking to improve estimation accuracy, and experts are not necessary the only, but especially, the best people to be involved in an estimation process. Other experiments conducted by the same authors on the estimation of effort required to complete software projects revealed that this amount is often estimated, completely or partially, when using the judgment of experts, whose assessment very often is biased, towards estimates that are overly optimistic. Conscious and unconscious processes seem to be at the basis of this bias, and the degree of bias varies from expert to expert.

The privileged approach to the reduction of this bias is the combination of over-optimism with the judgments of several experts, possibly with different backgrounds combined to provide their estimates through group discussion. The interesting findings for our argumentation are mainly related to the fact that group discussions led to better estimates respect to mechanical averaging of the individual estimates. So group discussion-based estimates were closer to the real effort than the average of the individual expert estimates One of the possible explanations proposed by the original authors is that groups ability to identify a greater number of activities required by the project is among the possible explanations for this reduction of bias.

In conclusion, grouping together people with a great knowledge of the domain (the experts) with people that probably are experts in other domains but not in the one required by the project (non-experts), and that can give a totally different vision and contribution to projects' estimations, is a new approach that can help to mitigate the problems of the approaches studied so far.

3 Capacities and Choquet integrals

In this section we present a brief review of the basic facts on Choquet integration, focusing on the additive and 2-additive cases as described by their Möbius representations. For recent reviews of Choquet integration see [17, 16, 18, 19].

Consider a finite set of interacting agents $N = \{1, 2, \ldots, n\}$, single agents are indexed by $i, j \in N$. The subsets S, $T \subseteq N$ are usually called coalitions.

In order to keep the notation as simple as possible, we denote $\mu(\{i\})$, $\mu(\{i, j\})$, etc by $\mu(i)$, $\mu(ij)$, etc. For the same reason, given a coalition $S \subseteq N \setminus \{i\}$ not involving the element $i \in N$, we write $N \setminus i$ and $\mu(S \cup i)$ instead of $N \setminus \{i\}$ and $\mu(S \cup \{i\})$. Analogously, given a coalition $S \subseteq N \setminus \{i, j\}$ not involving the elements $i, j \in N$, we write $N \setminus ij$ and $\mu(S \cup ij)$ instead of $N \setminus \{i, j\}$ and $\mu(S \cup \{i, j\})$, respectively.

Definition 1. A *capacity* [8] on the set N is a set function $\mu : 2^N \longrightarrow [0, 1]$ satisfying

(i) $\mu(\emptyset) = 0$, $\mu(N) = 1$ (boundary conditions)

(ii) $S \subseteq T \subseteq N \quad \Rightarrow \quad \mu(S) \leq \mu(T)$ (monotonicity).

Capacities are also known as *fuzzy measures* [57] or *non-additive measures* [10]. Given two coalitions S, $T \subseteq N$, with $S \cap T = \emptyset$, the capacity μ is said to be

- additive if $\mu(S \cup T) = \mu(S) + \mu(T)$,
- subadditive if $\mu(S \cup T) < \mu(S) + \mu(T)$,
- superadditive if $\mu(S \cup T) > \mu(S) + \mu(T)$.

If any of these properties holds for all coalitions $S, T \subseteq N$, the capacity μ is said to be additive, subadditive, or superadditive, respectively. In the additive case, in particular, we have $\mu(N) = \mu(\bigcup_{i=1}^{n} i) = \sum_{i=1}^{n} \mu(i) = 1$.

Definition 2. Let μ be a capacity on N. The *Choquet integral* [8, 12, 13] of a vector $\mathbf{x} = (x_1, \ldots, x_n) \in [0,1]^n$ with respect to μ is defined as

$$\mathcal{C}_\mu(\mathbf{x}) = \sum_{i=1}^{n} [\mu(A_{(i)}) - \mu(A_{(i+1)})]\, x_{(i)} \tag{1}$$

where $(\cdot)$ indicates a permutation on N such that $x_{(1)} \leq x_{(2)} \leq \ldots \leq x_{(n)}$. Moreover, $A_{(i)} = \{(i), \ldots, (n)\}$ and $A_{(n+1)} = \emptyset$.

In the additive case, since

$$\mu(A_{(i)}) = \mu((i)) + \mu((i+1)) + \ldots + \mu((n)) = \mu((i)) + \mu(A_{(i+1)}) \tag{2}$$

the Choquet integral reduces to a weighted mean,

$$\mathcal{C}_\mu(\mathbf{x}) = \sum_{i=1}^{n} [\mu(A_{(i)}) - \mu(A_{(i+1)})]\, x_{(i)} = \sum_{i=1}^{n} \mu((i)) x_{(i)} = \sum_{i=1}^{n} \mu(i) x_i \tag{3}$$

where the weights are given by $w_i = \mu(i)$, for $i \in N$.

Definition 3. Let μ be a capacity on N. The *importance index* or *Shapley value* [23, 54] of criterion $i \in N$ with respect to μ is defined as

$$\phi_\mu(i) = \sum_{T \subseteq N \setminus i} \frac{(n-1-t)!\, t!}{n!} [\mu(T \cup i) - \mu(T)] \qquad i \in N\,. \tag{4}$$

where t denotes the cardinality of coalition $T \subseteq N$.
The Shapley value $\phi_\mu(i)$ amounts to a weighted average of the marginal contribution of element i with respect to all coalitions $T \subseteq N \setminus i$ and can be interpreted as an effective importance weight. Moreover, it can be shown [23, 54] that

$$\phi_\mu(i) \in [0,1]\,, \qquad \sum_i \phi_\mu(i) = 1 \qquad i \in N\,. \tag{5}$$

In the additive case, in particular, we have that $\phi_\mu(i) = \mu(i)$, for $i \in N$.
A capacity μ can be equivalently represented by its *Möbius transform* m_μ [52, 15, 45], which is defined as

$$m_\mu(T) = \sum_{S \subseteq T} (-1)^{t-s} \mu(S) \qquad T \subseteq N \tag{6}$$

where s and t denote the cardinality of the coalitions S and T, respectively.
Conversely, given the Möbius transform m_μ, the associated capacity μ is obtained as

$$\mu(T) = \sum_{S \subseteq T} m_\mu(S) \qquad T \subseteq N\,. \tag{7}$$

In the Möbius representation, the boundary conditions take the form

$$m(\emptyset) = 0 \qquad \sum_{T \subseteq N} m(T) = 1 \tag{8}$$

and the monotonicity condition is expressed as follows [45, 7],

$$\sum_{S \subseteq T} m(S \cup i) \geq 0 \qquad i \in N \qquad T \subseteq N \setminus i\,. \tag{9}$$

This form of monotonicity condition derives from the original monotonicity condition in Definition 1, expressed as $\mu(T \cup i) - \mu(T) \geq 0$ for all $i \in N$ and $T \subseteq N \setminus i$. According to the decomposition of the capacity μ in Eq. (7), the Shapley values in Definition 3 can also be expressed in terms of the Möbius transform [14],

$$\phi_\mu(i) = \sum_{T \subseteq N \setminus i} \frac{m_\mu(T \cup i)}{t+1} \qquad i \in N\,. \tag{10}$$

Finally, the Choquet integral in Definition 2 can be expressed in terms of the Möbius transform in the following way [40],

$$\mathcal{C}_\mu(x_1, \ldots, x_n) = \sum_{T \subseteq N} m_\mu(T) \min_{i \in T}(x_i)\,. \tag{11}$$

Defining a capacity μ on a set N of n elements requires $2^n - 2$ real coefficients, corresponding to the capacity values $\mu(T)$ for $T \subseteq N$. In order to control exponential complexity, Grabisch [14] introduced the concept of k-additive capacities.
A capacity μ is said to be *k-additive* [14, 46] if its Möbius transform satisfies $m_\mu(T) = 0$ for all $T \subseteq N$ with $t > k$, and there exists at least one coalition $T \subseteq N$ with $t = k$ such that $m_\mu(T) \neq 0$.
In particular, in the 1-additive (or simply additive) case, the decomposition formula (7) takes the simple form

$$\mu(T) = \sum_{i \in T} m_\mu(i) \qquad T \subseteq N\,, \tag{12}$$

and the boundary and monotonicity conditions (8), (9) reduce to

$$m(\emptyset) = 0 \qquad \sum_{i \in N} m(i) = 1 \tag{13}$$

$$m(i) \geq 0 \qquad i \in N \qquad T \subseteq N \setminus i\,. \tag{14}$$

Moreover, for additive capacities, the Shapley values in (10) are simply

$$\phi_\mu(i) = m_\mu(i) \qquad i \in N \tag{15}$$

and the Choquet integral in (11) reduces to

$$\mathcal{C}_\mu(x_1, \ldots, x_n) = \sum_{i \in N} m_\mu(i)\, x_i. \tag{16}$$

The 2-additive case is the simplest non-linear capacity model and it has been studied and applied in various contexts, see for instance [41, 42, 3, 9, 4, 43, 44]. In the 2-additive case, the decomposition formula (7) takes the form

$$\mu(T) = \sum_{i \in T} m_\mu(i) \; + \sum_{\{i,j\} \subseteq T} m_\mu(ij) \qquad T \subseteq N\,, \tag{17}$$

and the boundary and monotonicity conditions (8), (9) reduce to

$$m(\emptyset) = 0 \qquad \sum_{i \in N} m(i) \ + \sum_{\{i,\,j\} \subseteq N} m(ij) = 1 \tag{18}$$

$$m(i) \geq 0 \qquad m(i) + \sum_{j \in T} m(ij) \geq 0 \qquad i \in N \qquad T \subseteq N \setminus i\,. \tag{19}$$

Moreover, for 2-additive capacities, the Shapley values in (10) are given by

$$\phi_\mu(i) = m_\mu(i) + \frac{1}{2} \sum_{j \in N \setminus i} m_\mu(ij) \qquad i \in N \tag{20}$$

and the Choquet integral in (11) reduces to

$$\mathcal{C}_\mu(x_1, \ldots, x_n) = \sum_{i \in N} m_\mu(i)\, x_i \ + \sum_{\{i,\,j\} \subseteq N} m_\mu(ij)\, \min(x_i, x_j)\,. \tag{21}$$

In the next section we present our non-additive model based on Choquet integration with respect to a 2-additive capacity μ. The model is constructed at level of the Möbius transform m_μ, and in particular the values $m_\mu(ij)$ represent the interactions between the agents $i, j \in N$.

4 Our Model

Let N be a set of interacting agents. We consider two different classes of agents in N: the class of expert agents N_e and the class of non-expert agents N_a. Clearly $N = N_e \cup N_a$ and $N_a \cap N_e = \emptyset$. Accordingly, for every coalition of agents $T \subseteq N$ we have $T = T_e \cup T_a$ and $T_a \cap T_e = \emptyset$. The cardinality of N_e, N_a, T_e, and T_a is indicated as n_e, n_a, t_e, and t_a respectively, with

$$n_e + n_a = n\,, \qquad t_e + t_a = t\,. \tag{22}$$

In our model we wish to emphasize the inter-class consensus between agents in N_e and N_a with respect to the intra-class consensus between agents in the same class. This effect is obtained by associating positive interaction between agents in different classes, and negative interaction between agents in the same class, in the framework of 2-additive capacities and Möbius representation described in the previous section.
We associate to each agent $i \in N$ a weight $w_i > 0$. These weights are normalized to unit sum $\sum_{i=1}^n w_i = 1$. We suppose that all agents in the same class have the same associated weight

$$w_i = \begin{cases} w_e & \text{if } i \in N_e \\ w_a & \text{if } i \in N_a \end{cases} \qquad i \in N\,. \tag{23}$$

with $w_e \geq w_a$. Therefore, we have

$$\sum_{i \in N} w_i = \sum_{i \in N_e} w_i + \sum_{i \in N_a} w_i = n_e w_e + n_a w_a = 1\,. \tag{24}$$

We define a 2-additive capacity $\mu : 2^N \longrightarrow [0,1]$ in the following way: with reference to Eq. (17), in which the 2-additive capacity μ is expressed in terms of its Möbius transform, we define

$$\mu(T) = \sum_{i \in T} m_\mu(i) \; + \sum_{\{i,j\} \subseteq T} m_\mu(ij) \qquad T \subseteq N \tag{25}$$

where

$$m(i) = w_i / D \qquad i \in N \tag{26}$$

$$m(ij) = -\delta/D \qquad \text{or} \qquad m(ij) = -\Delta/D \qquad i,j \in N \tag{27}$$

depending on whether i,j are in the same class or in different classes, respectively. Our model relies on two interaction parameters $\delta, \Delta \geq 0$ and the normalization factor D is such that $\mu(N) = 1$, see below for the detailed computation of D in terms of n_e, n_a and δ, Δ.
The graph interpretation of this definition, with singletons $\{i\}$ corresponding to nodes and pairs $\{i,j\}$ corresponding to edges between nodes, is the following: the value of the 2-additive capacity μ on a coalition T is given by the sum of the nodes and edges contained in the subgraph associated with the coalition T, as illustrated in Figure 1.

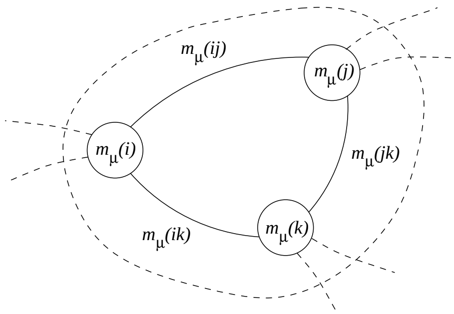

Figure 1: Graph representation of the 2-additive capacity.

Therefore we have

$$\mu(T) = \frac{\sum_{i \in T} w_i + m_1(T)\Delta - m_2(T)\delta}{D} \qquad T \subseteq N \tag{28}$$

where the coefficients $m_1(T)$ and $m_2(T)$ can be computed as follows. The number $m_1(T)$ indicates the number of edges inside the coalition T between agents in different classes, i.e. the number of edges between an expert agent $i \in T_e$ and a non-expert agent in $j \in T_a$. Its value is given by

$$m_1(T) = t_e \cdot t_a \qquad T \subseteq N\,. \tag{29}$$

Similarly, the number $m_2(T)$ indicates the number of edges inside the coalition T between agents in the same class, i.e. the number of edges between two expert agents $i,j \in T_e$ or two non-expert agents $i,j \in T_a$. Its value can be computed as follows

$$m_2(T) = \frac{1}{2} t_e(t_e - 1) + \frac{1}{2} t_a(t_a - 1) \qquad T \subseteq N\,. \tag{30}$$

In particular, for coalitions $T \subseteq N$ of small cardinality, we have

$$\mu(i) = w_i/D \qquad\qquad i, j \in N$$

$$\mu(ij) = (w_i + w_j - \delta)/D \qquad \text{or} \qquad \mu(ij) = (w_i + w_j + \Delta)/D \tag{31}$$

depending on whether i, j are in the same class or in different classes respectively. Note that if the agents i and j belong to different classes, then the capacity μ is superadditive, otherwise if the agents i and j belong to the same class the capacity μ is subadditive.
The normalization factor D is obtained from the boundary condition $\mu(N) = 1$,

$$\mu(N) = \frac{\sum_{i=1}^{n} w_i + m_1(N)\Delta - m_2(N)\delta}{D} = 1 \tag{32}$$

which leads to

$$D = 1 + m_1(N)\Delta - m_2(N)\delta \tag{33}$$

where

$$m_1(N) = n_e \cdot n_a\,, \qquad m_2(N) = \frac{1}{2}n_e(n_e - 1) + \frac{1}{2}n_a(n_a - 1)\,. \tag{34}$$

Proposition 1 *The capacity μ introduced in (25), (27), (33), satisfies the boundary conditions $\mu(\emptyset) = 0$ and $\mu(N) = 1$. Moreover it is monotonic, that is $\mu(S) \leq \mu(T)$ for $S \subseteq T \subseteq N$, for all $\Delta \geq 0$ and $0 \leq \delta \leq min(\delta_e, \delta_a)$ where $\delta_e = w_e/(n_e - 1)$ and $\delta_a = w_a/(n_a - 1)$.*

Proof: The boundary conditions $\mu(\emptyset) = 0$ and $\mu(N) = 1$ are clearly satisfied, the latter corresponds to the choice of the normalization factor D in (33).
In order to prove monotonicity, it suffices to show that $\mu(T \cup i) \geq \mu(T)$ for all $i \in N$, $T \subseteq N \setminus i$. It is easy to see that the positive factor Δ does not affect the monotonicity, so we can choose $\Delta \geq 0$, instead the monotonicity condition limits the value of δ.
To find this limit, we consider two extreme cases. In the first case, consider a coalition T with $n_e - 1$ experts and we want to add the last expert $e \in N_e \subseteq N$. In this case $T_a = \emptyset$ and $T = T_e = N_e \setminus e$ so the cardinality of the coalition T is $t = t_e = n_e - 1$. We have seen that the value $\mu(T)$ is the sum of the nodes and the edges contained in the subgraph associated with the coalition T, see (25) and (28)

$$\begin{aligned} \mu(T) &= \sum_{i \in T} m_\mu(i) + \sum_{\{i,\, j\} \subseteq T} m_\mu(ij) \\ &= \frac{\sum_{i \in T} w_i + m_1(T)\Delta - m_2(T)\delta}{D} \\ &= \frac{(n_e - 1)w_e - (n_e - 1)(n_e - 2)\delta/2}{D} \qquad\qquad T \subseteq N\,. \end{aligned} \tag{35}$$

If we add the expert e to the coalition of experts T, we have to sum the value associated to the node corresponding to e with all the values associated to the $(n_e - 1)$ arcs connecting that node with the other nodes in the subgraph associated with T. Then

$$\begin{aligned} \mu(T \cup e) &= \mu(T) + m_\mu(e) \ + \sum_{j \in T} m_\mu(ej) \\ &= \mu(T) + \frac{w_e}{D} - (n_e - 1)\frac{\delta}{D} \qquad T \subseteq N\,. \end{aligned} \tag{36}$$

In this extreme case the boundary condition is satisfied iff $w_e - (n_e - 1)\delta \geq 0$, that is

$$0 \leq \delta \leq \frac{w_e}{n_e - 1}\,. \tag{37}$$

We call this value $\delta_e = w_e/(n_e - 1)$.
In the second extreme case, consider a coalition T with $n_a - 1$ non-experts and we want to add the last non-expert $a \in N_a \subseteq N$. In this case $T_e = \emptyset$ and $T = T_a = N_a \setminus a$ so the cardinality of the coalition T is $t = t_a = n_a - 1$. Analogously to the previous case, we have

$$\mu(T) = \frac{(n_a - 1)w_a - (n_a - 1)(n_a - 2)\delta/2}{D} \qquad T \subseteq N\,. \tag{38}$$

Moreover

$$\mu(T \cup a) = \mu(T) + \frac{w_a}{D} - (n_a - 1)\frac{\delta}{D} \qquad T \subseteq N\,. \tag{39}$$

Here the boundary condition is satisfied iff $w_a - (n_a - 1)\delta \geq 0$, that is

$$0 \leq \delta \leq \frac{w_a}{n_a - 1}\,. \tag{40}$$

We call this value $\delta_a = w_a/(n_a - 1)$. Therefore we can conclude that the capacity μ is monotonic for all $\Delta \geq 0$ and $0 \leq \delta \leq min\,(\delta_e, \delta_a)$. $\square$

Example 1. We want to aggregate the estimate given by three agents X_1, X_2, and X_3. Suppose that X_1 and X_2 are expert agents and X_3 is a non-expert agent. Table 1 shows the estimates and the weights associated to each agent.

Agent	Estimate	Weight	Class
X_1	70	0.375	Expert
X_2	80	0.375	Expert
X_3	120	0.250	Non-expert

Table 1: Single agent estimates in Example 1.

Assuming that the final estimate is computed by a weighted mean with weighting vector $\mathbf{w} = (w_1, w_2, w_3) = (0.375, 0.375, 0.250)$, we obtain

$$\mathcal{W}_{\mathbf{w}}(70, 80, 120) = 0.375 \cdot 70 + 0.375 \cdot 80 + 0.250 \cdot 120 = 86.25\,. \tag{41}$$

Now consider the Choquet integral. From Proposition 1 we know that, in this case, monotonicity condition needs $\delta \leq 0.375$. In this example we consider $\delta = 0.3$ and $\Delta = 0.1$. Then the normalization factor D is, see Eq. (33),

$$D = 1 + 2\Delta - \delta = 0.9\,. \tag{42}$$

The consensus between expert agents X_1 and X_2 is standard and therefore its importance in the estimate aggregation through the Choquet integral is downgraded due to the subadditivity of the capacity over agents X_1 and X_2.
Now we can compute the value of our capacity on every coalition of agents

$$\mu(1) = \frac{w_1}{D} \simeq 0,42\,, \quad \mu(2) = \frac{w_2}{D} \simeq 0,42\,, \quad \mu(3) = \frac{w_3}{D} \simeq 0,28$$

$$\mu(1,2) = \frac{-\delta + w_1 + w_2}{D} = 0.5\,, \quad \mu(1,3) = \frac{\Delta + w_2 + w_3}{D} \simeq 0.81 \tag{43}$$

$$\mu(2,3) = \frac{\Delta + w_2 + w_3}{D} \simeq 0.81\,, \quad \mu(1,2,3) = \frac{2\Delta - \delta + w_1 + w_2 + w_3}{D} = 1\,.$$

The associated Choquet integral leads to a new final estimate. Since

$$x_1 = 70 \, < \, x_2 = 80 \, < \, x_3 = 120 \tag{44}$$

we obtain

$$\begin{aligned} C_\mu(70, 80, 120) &= \sum_{i=1}^{3} (\mu(A_{(i)}) - \mu(A_{(i+1)})\, x_{(i)} \\ &= [\mu(1,2,3) - \mu(2,3)]\, x_1 + [\mu(2,3) - \mu(3)]\, x_2 + \mu(3) x_3 \\ &\simeq [1 - 0,81]\, 70 + [0,81 - 0,28]\, 80 + [0,28]\, 120 \\ &= 0.19 \cdot 70 + 0.53 \cdot 80 + 0,28 \cdot 120 \\ &= 89.30\,. \end{aligned} \tag{45}$$

Note that using the Choquet integral we obtain a different final estimate. If we compare (41) with (45) we have

$$C_\mu(70, 80, 120) = 89.30 > \mathcal{W}_{\mathbf{w}}(70, 80, 120) = 86.25\,. \tag{46}$$

In this case, with respect to the classical weighted mean aggregation, the Choquet integral has reduced the joint weight of the consensual agents X_1 and X_2 (both experts) and, accordingly, has increased the single weight of agent X_3

$$0.19 + 0.53 = 0.72 < w_1 + w_2 = 0.75\,. \tag{47}$$

Example 2. Consider now the situation illustrated in Table 2.

Agent	Estimate	Weight	Class
X_1	100	0.375	Expert
X_2	60	0.375	Expert
X_3	110	0.250	Non-expert

Table 2: Single agent estimates in Example 2.

The consensus between expert agents X_1 and X_3 is significant and therefore its importance in the estimate aggregation through the Choquet integral is upgraded due to the superadditivity of the capacity over agents X_1 and X_3.

Assuming that the final estimate is computed by a weighted mean with weighting vector $\mathbf{w} = (w_1, w_2, w_3) = (0.375, 0.375, 0.250)$, we obtain

$$\mathcal{W}_{\mathbf{w}}(100, 60, 110) = 0.375 \cdot 100 + 0.375 \cdot 60 + 0.250 \cdot 110 = 87.50\,. \tag{48}$$

The capacity is the same of the previous example. The associated Choquet integral leads to a new final estimate. Since

$$x_2 = 60 \ < \ x_1 = 100 \ < \ x_3 = 110 \tag{49}$$

we obtain

$$\begin{aligned} C_\mu(60, 100, 110) &= \sum_{i=1}^{3} (\mu(A_{(i)}) - \mu(A_{(i+1)})\, x_{(i)} \\ &= [\mu(2,1,3) - \mu(1,3)]\, x_2 + [\mu(1,3) - \mu(3)]\, x_1 + \mu(3) x_3 \\ &\simeq [1 - 0,81]\, 60 + [0,81 - 0,28]\, 100 + [0,28]\, 110 \\ &= 0.19 \cdot 60 + 0.53 \cdot 100 + 0,28 \cdot 110 \\ &= 95.20\,. \end{aligned} \tag{50}$$

Again using the Choquet integral we obtain a different final estimate. If we compare (48) with (50) we have

$$C_\mu(60, 100, 110) = 95.20 > \mathcal{W}_{\mathbf{w}}(100, 60, 110) = 87.50\,. \tag{51}$$

In this case, with respect to the classical weighted mean aggregation, the Choquet integral has increased the joint weight of the consensual agents X_1 and X_3 (one expert and one non-expert) and, accordingly, has reduced the single weight of agent X_2

$$0.53 + 0.28 = 0.81 > w_1 + w_3 = 0.625\,. \tag{52}$$

5 Computer simulation

In order to analyze the behavior of the Choquet integral as aggregation tool, an empirical test was performed. The model constructed to perform the test was inspired by the factors highlighted by the research during the last twenty years. For expert agents we know that their estimates are strongly influenced by an optimistic bias [32, 59, 48, 31], and the magnitude of the relative error of their estimates usually is included within the 25% and the 40% [61, 55, 35, 29]. Moreover we can assume that is possible to simulate an expert's estimation through the extraction of random numbers from a Gaussian distribution [32].
For non-expert agents we know that their estimates tend to be less optimistic than those of the expert agents [48, 32]. In order to emphasize the difference with the expert agents we will assume that they are pessimistic. We assume that is possible to simulate their estimation through the extraction of random numbers from a uniform distribution. Moreover, we expect that the mean magnitude of the relative error of their estimates will be higher than the experts' one.
Finally, we assume that expert agents will be more confident about their estimate than the non-experts.

The simulation proceeds as follows: firstly the actual value of the project to estimate is fixed ex-ante equal to 1.000 units of time. Secondly, according the points highlighted before, we product ex-post the supposed estimate x_i^c, $i \in N$ for experts and non-expert using a random algorithm. In particular, for expert agents we simulate their estimation from extraction of random numbers from a Gaussian distribution, with respect to two different scenarios, as indicated in Table 3, according to their optimistic bias. For non-expert agents we simulate their estimation from extraction of random numbers from a uniform distribution on the interval $[500, 2200]$.

Scenario	Expert		Non-Expert	
	Mean	StD	MIN	MAX
1	600	400	500	2200
2	750	250	500	2200

Table 3: Parameters for Gaussian and uniform distributions.

We associate a length to every estimate x_i^c, $i \in N$. As highlighted before, since the expert agents are more confident about their estimate than the non-experts, to the expert's estimate is associated a narrow interval and to the non-expert's estimate is associated a wide interval. To do this, we consider two different Gaussian distributions: the first one, for the expert agents, is around 5%, and the second one, for the non-expert agents, is around 10%. Then, we extract a random value and we use it as a percentage to calculate the length.
Now each estimate has an interval form

$$[x_i^l, x_i^r] \qquad i \in N \tag{53}$$

centered in x_i^c. Then we aggregate these estimates using the Choquet integral and the weighted mean. First of all we aggregate all the left endpoints x_i^l, and then we aggregate all the right endpoints x_i^r, $i \in N$, and we obtain the aggregated value $[x^l, x^r]$. We indicate with x^c his center. To do this we associate to each agent a positive weight such that the weight associated with an expert agent is twice the weight associated with a non-expert agent. Moreover, to compute the Choquet integral, we consider a capacity such that $\Delta = \delta/4$. Then we compare the results obtained using the two different aggregation methods.
To evaluate the performance of our aggregation method we use the "magnitude of the relative error" (MRE), which can be computed in the following way:

$$MRE = \frac{|x^c - actual|}{actual} \tag{54}$$

But if we want to evaluate the performance of our aggregation method over many simulations, we use a mean of all MRE values, the so-called MMRE (mean magnitude of the relative error). Observing the maximum value of MRE, we can evaluate the reliability of each aggregation method: the larger is the maximum value of MRE, the lower is the reliability of that aggregation method.
Another index used to analyze the results is the HitRate [32]. This index expresses the rate at which the interval estimate $[x^l, x^r]$ contains the actual value. We propose two different analysis. With the first one we want to identify the ideal composition of the panel, in other words we compare all the available composition for a panel of 10 agents mixing experts and non-experts, and find the

panel with minimum MMRE and maximum HitRate. Ten different composition are possible: starting from 10 experts and no non-experts, to 1 expert and 9 non-experts. We simulate each of these combinations 100 times with respect to the first scenario and 100 times with respect to the second one. Table 4 and Table 5 illustrate the results of this analysis.

Number of experts	Number of non-experts	Choquet Integral	Weighted Mean
10	0	0,1380	0,2973
9	1	0,1086	0,2427
8	2	0,1038	0,2161
7	3	0,0831	0,1599
6	4	0,0961	0,1352
5	5	0,1054	0,1134
4	6	0,0888	0,0867
3	7	0,1164	0,1165
2	8	0,1993	0,1879
1	9	0,3404	0,2687
Average		0,1380	0,1824

Table 4: MMRE perspective on aggregation tools.

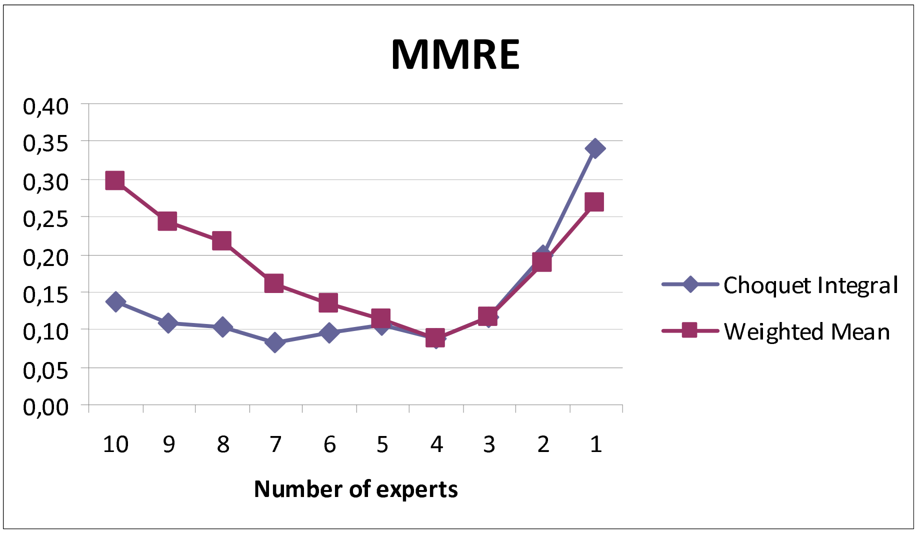

Figure 2: Graphical representation of Table 4.

Table 4 and Figure 2 show that the Choquet Integral produces more accurate prediction in 7 cases out of 10. Moreover it seems that with a panel composed by 7 expert agents and 3 non-expert agents, we obtain the most accurate estimation. This trend is confirmed by the analysis of the HitRate as well. As a result, we can assume that a panel composed by experts for 70% and non-experts for the remaining 30% seems to be the more reliable and accurate one.
The second analysis performed during the simulation is about the size of panel: we want to find the ideal size of the panel. In other words, according to the

Number of experts	Number of non-experts	Choquet Integral	Weighted Mean
10	0	58	0
9	1	100	9
8	2	110	20
7	3	135	51
6	4	115	75
5	5	124	107
4	6	142	148
3	7	132	135
2	8	86	104
1	9	46	83

Table 5: HitRate perspective on aggregation tools.

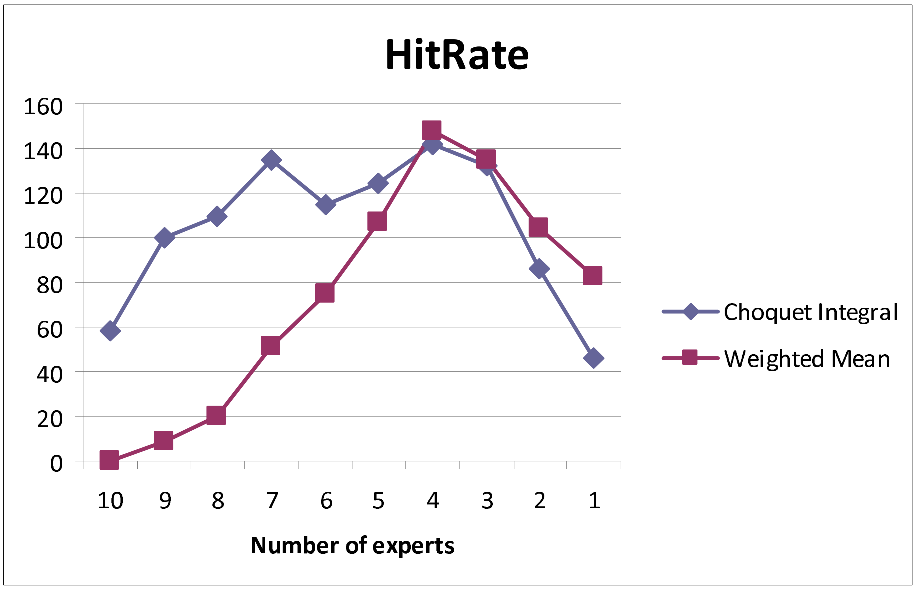

Figure 3: Graphical representation of Table 5.

results of the first analysis, we assume that the panel is composed by expert agents for 70% and non-expert agents for the remaining 30%, and we investigate the influence of the panel's size on the accuracy of the final estimation. We consider eight different sizes, from 3 to 10 agents. As before, we simulate each of these situations 100 times with respect to the first scenario and 100 times with respect to the second one. Table 6 and Table 7 illustrate the result of this second test.

Size of the panel	Choquet Integral	Weighted Mean
3	0,1655	0,1967
4	0,1396	0,2005
5	0,1403	0,2135
6	0,1167	0,1683
7	0,0970	0,1658
8	0,0916	0,1759
9	0,0856	0,1903
10	0,0858	0,1507
Media	0,1153	0,1827

Table 6: MMRE values in reference to panel's size.

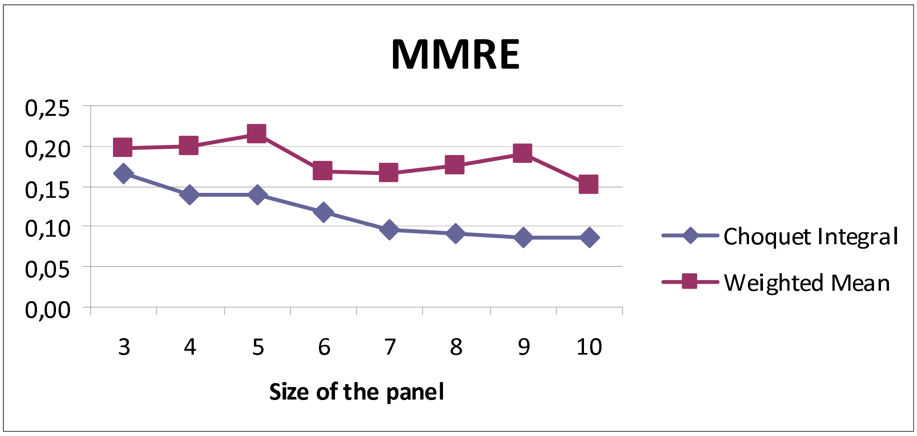

Figure 4: Graphical representation of Table 4.

This second analysis shows that, in accordance with the results highlighted by Makridakis and Winkler [39], it seems that once the panel reaches the size of 7-8 components, adding other agents leads to a poor improvement in the estimation accuracy and on the HitRate as well.
In the last analysis we want to investigate the reliability of each aggregation method, therefore we compare the worst results in terms of MRE obtained employing each aggregation method, see Table 8. Once again, the Choquet Integral is the method that produce estimation with the lower error.
These results lead to some conclusions:

- *Combining estimations*: The simulation shows that to obtain an accurate

Size of the panel	Choquet Integral	Weighted Mean
3	74	40
4	80	31
5	79	28
6	99	44
7	118	45
8	122	28
9	113	15
10	120	42

Table 7: HitRate values in reference to panel's size.

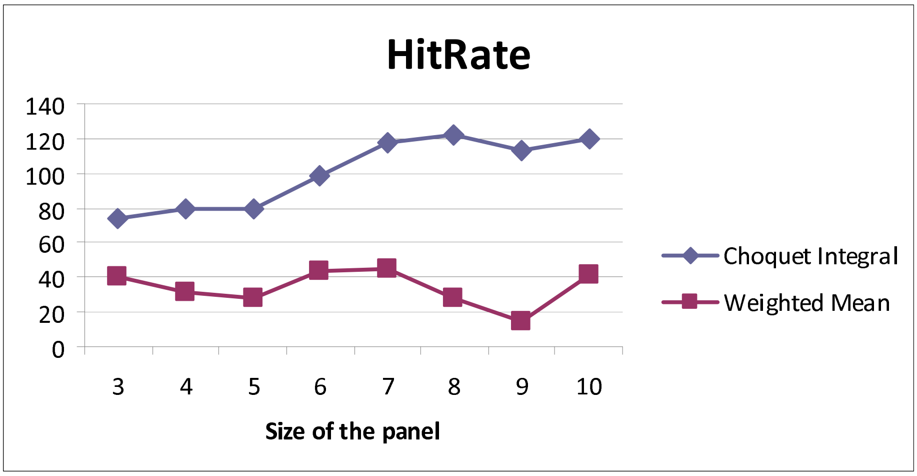

Figure 5: Graphical representation of Table 5

Size of the panel	Choquet Integral	Weighted Mean
3	0,5366	0,6054
4	0,4851	0,5456
5	0,4279	0,5324
6	0,4007	0,4970
7	0,3249	0,4645
8	0,3751	0,5082
9	0,3701	0,4722
10	0,3748	0,4762
Media	0,4119	0,5127

Table 8: Worst case (MRE) obtained during simulation.

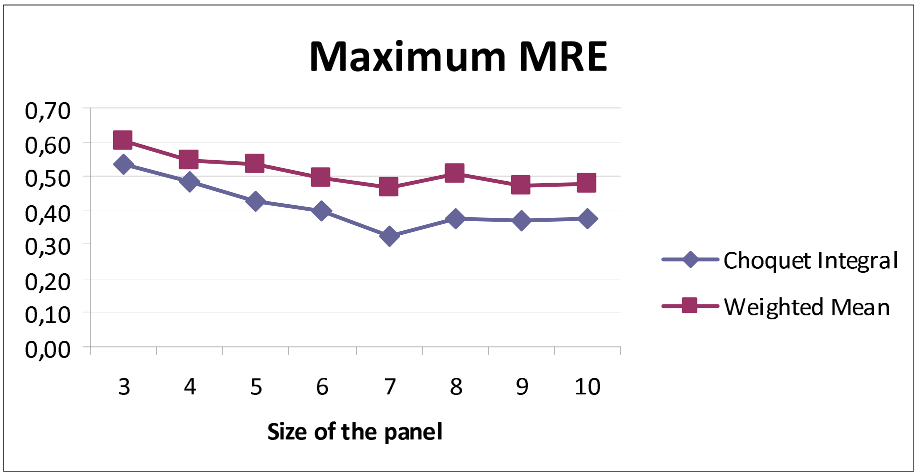

Figure 6: Graphical representation of Table 8

and reliable prediction it is necessary to combine estimates produced by expert agents and non-expert agents. As shows Table 4 and Table 5, if we consider only expert agents or only non- expert agents, the final prediction has a higher MMRE and a lower HitRate.

- *Panel composition*: As mentioned before, both the composition and the size of the panel impact on the accuracy of the aggregated estimation. Regarding the composition, the simulation suggests that an ideal panel should be composed by 70% by expert agents and by the 30% by non-expert agents, with different backgrounds and knowledge. With reference to the size of the panel, the simulation shows that the accuracy of an estimation improves rapidly when adding a new component to the panel obtaining lower MMRE and higher HitRate. But once the panel reaches the size of 7-8 components, adding others estimators leads to a poor improvement in the estimation accuracy.

- *Aggregation methods*: Amongst the aggregation methods analyzed in the simulation, the Choquet Integral seems to be the best one in terms of both accuracy and HitRate. Moreover, the Choquet Integral seems also to be the methods that produce the lower error when analyzing the worst case. Another intersecting aspect of the Choquet integral is that it move the focus of the project manager from choosing the correct weight for each agent to the choice of the most suitable parameters. In fact, when using the Choquet Integral, the project manager has only to chose the value of 2 parameters: the δ and Δ. These two values influence the process of mixing prediction coming from agents having different knowledge and capabilities. In our case, it seems that choosing a Δ much lower than δ allows to obtain very accurate estimations.

Summarizing, in this paper we consider group processes for effort estimation in Project Management and we exploit the synergies between experts and non-experts in an MCDM framework, aggregating the individual estimates by means of non-additive Choquet integration, and representing the complementary bias by the multiagent interaction structure underlying the capacity. The computer simulations presented are encouraging and the aggregation results obtained outperform those of the classical weighted mean (additive case), showing lower MMRE and higher HitRate.

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