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New perspectives from the Euro Zone

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Phillips Curve and output expectations: New perspectives from the Euro Zone

Giuliana Passamani^a   Alessandro Sardone^b   Roberto Tamborini^c

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Abstract

When referring to the inflation trends over the last decade, economists speak of "puzzles": a "missing disinflation" puzzle in the aftermath of the Great Recession, and a "missing inflation" one in the years of recovery to nowadays. To this, a specific "excess deflation" puzzle may be added during the post-crisis depression in the Euro Zone. The standard Phillips Curve model, in this context, has failed as the basic tool to produce reliable forecasts of future price developments, leading many scholars to consider this instrument to be no more adequate. The purpose of this paper is to contribute to this literature through the development of a newly specified Phillips Curve model, in which the inflation-expectation component is rationally related to the business cycle. The model is tested with the Euro Zone data 1999-2019 showing that inflation turns out to be consistently determined by output gaps and experts' survey-based forecast errors, and that the puzzles can be explained by the interplay between these two variables.

Keywords: key1, key2, key3

JEL Codes: key1, key2, key3

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1. Introduction

The Great Recession of 2008-09 represents a watershed for macroeconomics. Kernel relationships between real and monetary variables, as assumed up to that point, broke down, thus showing fragility of macroeconomic models and their inability to make accurate predictions. The price Phillips Curve (PC), which describes the relationship between inflation and the cyclical position of economic activity (we shall refer to this format of the PC unless otherwise stated), is on the forefront. Its failure to produce reliable forecasts of future price developments, has led many scholars to consider this instrument in need of substantial revisions ¹.

A brief account of the recent empirical "puzzles" of the PC may start from the evidence of its "flattening" during the Great Moderation era put forward by Blanchard et al. (2015) who reported two widely shared stylized facts. First, the substantial reduction of the structural slope of the PC plummeting from more than 1 in the mid-1970s to about 0.3. Second, the "anchoring of expectations", meaning that the expectational component of inflation had become more stable and tighter (less weight of lagged inflation rates), implying that both expected and actual inflation would tend to converge faster towards the official inflation target.

These features seemed to persist in the aftermath of the Great Recession, when the "missing deflation puzzle" emerged in the US, where the collapse of about 10% of GDP relative to trend was followed by a modest decline of 1.5% of inflation (Williams 2010). In the Euro Zone (EZ), however, concerns at the European Central Bank (ECB) and among scholars were of opposite sign: a deflationary drift with deanchoring of expectations seemed under way (Draghi 2014; Draghi 2016). Research on the post-crisis EZ challenged the consensus on the worldwide flattening of the PC pointing to its "steepening" (e.g. Riggi and Venditti 2014; Riggi and Venditti 2015; S. Oinonen 2014; Bank of Ireland 2014b). Direct investigation of expectation forecasts showed both their downward drift and a significant underestimation of actual inflation (Riggi and Venditti 2014; Miccoli and Neri 2015). Parallely, direct evidence of the downward deanchoring of expectations was detected in various studies (Buono and Formai 2016; Fracasso and Probo 2017; Nautz et al. 2017; Natoli and Sigalotti 2017).

More recently, since recovery took hold across advanced economies, the new "missing inflation puzzle" has been spurring researchers' efforts. Inflation has remained subdued in spite of more sustained economic activity, falling unemployment and some recovery of wage dynamics. This time the EZ seems to be returned in line with the rest of the advanced world (Ciccarelli and Osbat 2017, Fiedler et al. 2018, Bobeica and Sokol 2019).

Given the intrinsic relationship between accurate economic forecasts and the implementation of appropriate economic policies, it becomes essential to reconsider the equations underlying these models, so as to understand how transmission channels between real and monetary factors operate in the real world. In particular, finding the correct

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¹See for instance the papers collected at the European Central Bank Conference "Inflation in a changing environment", Frankfurt, September 23-24, 2019. A survey of the conference is provided by Ehrmann et al. (2020).

expression of the PC might solve this issue and open up to new hypothesis of public intervention, in order to overcome the persistent economic impasse.

The literature about revisions of the PC has taken two main roads (Ehrmann et al. 2020). One is, as it were, more radical, arguing for a dismissal of the traditional macro-apparatus in favour of a revival of a micro, disaggregated approach to the unemployment-wages-prices transmission. The other remains within the macro-tradition seeking for better specifications of the PC estimation equation. Our contribution presented in this paper may be ascribed to this latter strand. We locate the need for better specification of the PC in its expectational component. Inflation expectations expressed "on the right-hand-side" of the PC are generally treated as independent variables, collected from various available forecast surveys. This practice is not satisfactory for two reasons. First, the PC should be understood as one component of a set of co-evolving macroeconomic relationships (think for instance of the New Keynesian three-equation model). Indeed, to the extent that the central bank controls inflation tightly, it is no surprise that the ex-post inflation-output relationship is seemingly flat. Second, and consequently, the alleged explanatory variables in the PC are co-determined elsewhere in the system, and may not even be independent one of another. This is specifically the case of inflation expectations, which are arguably elaborated, in some way and to some extent, according to the evolution of the macroeconomy.

It may be said that major institutions indeed base their inflation forecasts (also) on full-blown macro-models. Nonetheless, the PC as a stand-alone equation remains a pivotal macroeconomic tool as testified by the debate summarized above. Therefore, in section 2 we present a modified PC where inflation expectations are no longer considered to be independent of the slack variable (output gap in the specific case), as assumed in standard empirical models. The idea is simply that if the PC is used worldwide to foresee inflation, then its expectational component should in some way be the result of agents using the PC itself². As a consequence, the independent explanatory variables of inflation turn out to be the output gaps and the related forecast errors by agents, with notable consequences from an empirical point of view. First, standard empirical models with exogenous inflation expectations yield a distorted estimator of the output structural parameter. Second, the observed inflation-output relationship may appear flatter or steeper not (only) owing to structural factors, but (also) to the interplay between output expectations and forecast errors. Third, our model allows to disentangle the true output component of inflation from the transitional expectational one. These features relate our model also to the issue of the deanchoring of expectations, namely deanchored expectations may make the PC seemingly steeper in recession and flatter in recovery.

Section 3 will provide an empirical application of our model developed on data collected in the EZ from 1999 to 2019. The ECB Survey of Professional Forecasters is employed as proxy of output expectations. Then, an econometric linear regression model is introduced and tested through the ordinary least squares (OLS) method and the gener-

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²This idea is consistent with the notion of rational expectations in the sense of agents' learned understanding of the data generation process (G. Evans and Honkapohja 2011; Kurz 2011).

alized method of moments (GMM). Specific tests are provided in support of this choice. Our results show that in the EZ the output component on inflation is not so flat, and the extent to which the post-crisis deflationary drift and the post-recovery missing inflation can be explained by the transitory expectational component.

The last section concludes with a view to new perspectives in terms of empirical research and possible implications regarding monetary and fiscal policies undertaken by public institutions.

2. The Phillips Curve with endogenous expectations

A baseline specification of the PC for empirical analysis may be the following (see e.g. Hooper et al. 2019):

$$\pi_t = \alpha + \beta_1 \pi_t^e + \beta_2 x_t + \beta_3 \Lambda(\pi, n) + \beta_4 Z_t + u_{\pi t} \quad (1)$$

where π_t is current inflation, π_t^e is an expectational term as of time t to be specified, x_t is a measure of the business cycle or "economic slack", $\Lambda(\pi, n)$ is the lag operator of inflation of order n , Z_t is a vector of other variables, and $u_{\pi t}$ are random shocks. The intercept α may capture an autonomous drift in inflation.

The empirical issues about the PC revolve around the estimation of coefficient β_2 , which is meant to capture the structural slope of the PC, or better the responsiveness of inflation to the business cycle. As to the relevant variable x_t , it is now common to employ the output gap, defined as the difference between actual output (GDP) and the maximum potential output the economy can produce (potential GDP) as a percentage of GDP. Looking back at the origins, some would rather use the unemployment gap, the difference between the unemployment rate and the natural rate or the NAIRU, or the Phillips' wage PC altogether where the dependent variable is wage inflation. Researchers testing these respecifications of the PC argue that the puzzles of the price PC are less compelling or vanish altogether (Hooper et al. 2019; Nickel et al. 2019).

Key to the positive (non-vertical) inflation-output relationship is the presence of sticky wages and/or prices. This has been the main focus of the New Keynesian reformulation of PC. Staggered price setting à la Calvo is usually employed to model this feature. According to Calvo (1983), nominal individual prices are not subject to continuous revisions. Only a fraction of firms can adjust prices in each period, while for the others the price level remains unchanged. Firms maximize their prices, taking into account expected future price developments and the probability (constant) that they may not be able to adjust them in the following periods. As a consequences, a cyclical measure of the marginal costs may replace x_t or may appear in the vector Z_t (Gali and Gertler 1999).

More radical revisions of the estimation model wish to capture nonlinearities, that is different reactions of inflation to upward or downward phases of the business cycle (Nalewaik 2016; Lindé and Trabandt 2019; Hooper et al. 2019). Indeed, as argued by Lindé and Trabandt, Phillips' original curve was convex, implying that wages (inflation) would rise fast in a boom and fall slow in a recession. If this may account for the missing deflation puzzle, it nonetheless leaves the ongoing missing inflation puzzle unexplained.

In the following we shall concentrate on the expectational term π_t^e of equation 1. By way of this term, the model accommodates the Monetarist critique to the older Keynesian PC, and is also akin to the (inverted) Lucas (1973) "surprise" aggregate supply. The debate on the nature of the expectational term is, however, still open. In the "accelerationist" PC, Friedman (1968) used adaptive expectations formed on the basis of the last-period inflation term. In the Lucas supply curve (1973) it was replaced by the expectation of current inflation. In the New Keynesian, Calvo-type PC there appears the current expectation of next period inflation. The "hybrid" PC proposed by Gali and Gertler (1999), and adopted for example by Blanchard et al. (2015), splits expectations in a forward-looking component (usually the anticipation of the current inflation) and in a complementary backward-looking one (usually the previous-period inflation, not dissimilar to Friedman). This amounts to adding the lagged inflation component $\Lambda(\pi, n)$ in order to allow the model to fit inflation persistence.

Our point is that the expectational term, whatever it is, is generally treated as an independent (exogenous) explanatory variable, proxied by available forecast surveys. This practice is problematic for two reasons. The first is that it isolates the PC from the rest of economy, whereas it should better be understood as co-evolving with the rest of the economy. The consequence can easily be seen, for example, through the New Keynesian three-equation model. If the central bank controls inflation tightly by means of the Taylor Rule, the ex-post PC will seemingly be flat because output fluctuates around its potential due to shocks to the IS curve whereas inflation does not deviate from its target. The second, related, reason is that inflation expectations, too, cannot be taken to be exogenous with respect to the evolution of the rest of the economy, and output in particular. Indeed, if the PC as a stand-alone tool is to survive, a proper treatment of the expectational term may provide the bridge with "the rest of the economy".

Our proposed treatment in the first place considers the case in which π_t^e is the anticipation of the current inflation, with x_t equal to the output gap, the other terms $\Lambda^n(\pi)$ and Z_t being dropped, that is

$$\pi_t = \alpha + \beta_1 \pi_t^e + \beta_2 x_t + u_{\pi t} \quad (2)$$

Further ingredients mentioned above may enrich the picture, but at the cost of some complexification which is not strictly necessary, at least at this first stage. If the central bank has an inflation target π^* , this should be consistent with the stochastic equilibrium of the inflation process (2), i.e. $\pi_t^e = \pi_t = \pi^*$, $x_t = 0$, for $u_{\pi t} = 0$. It follows that $\pi^* = \alpha / (1 - \beta_1)$ ³. Therefore, substituting $\alpha = \pi^*(1 - \beta_1)$, we can rewrite equation (2) as

$$\pi_t = \pi^* + \beta_1 (\pi_t^e - \pi^*) + \beta_2 x_t + u_{\pi t} \quad (3)$$

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³An autonomous inflation drift $\alpha > 0$ may explain why the inflation target is generally positive. Driving the drift to zero would require a permanent negative output gap (see also Woodford 2003, ch. 3).

Where does the expectation of current inflation come from? Treatment of expectation formation is notoriously controversial, but our idea can simply be put this way. If the PC is used worldwide to foresee inflation, its expectational component should in some way come from agents using the PC itself. This idea is consistent with a broad principle of rationality of expectations (of recurrent and stable processes) as the outcome of informed understanding of how the economy works - or the "data generation process" (G. Evans and Honkapohja 2011; Kurz 2011). Applying the limit case of the (unboundedly) rational expectations hypothesis (REH) straightforwardly⁴, π_t^e should be the statistical expected value of inflation, i.e. $\pi_t^e = E(\pi_t)$ conditional on equation (3) (Woodford 2003, ch. 3). Consequently,

$$E(\pi_t) = \pi^* + \frac{\beta_2}{1 - \beta_1} E(x_t) \quad (4)$$

The PC-based rational expectation of inflation at time t can be expressed as a deviation from the target determined by the expected value of the output gap in t . Intuitively, in a price-setters' environment, output expectations may drive economic agents' strategic behaviour, so to justify a variation in the price level.

Substituting equation (4) in (3), and rearranging terms, we obtain

$$\pi_t = \pi^* + \frac{\beta_2}{1 - \beta_1} [\beta_1 E(x_t) + (1 - \beta_1)x_t] + u_{\pi t} \quad (5)$$

The actual inflation path around the target results to be the linear combination of the expected and the actual output gap, where β_1 represents the weight between the two terms, plus random shocks. It may be noted that equation (5) nests two extreme cases. If $\beta_1 = 0$, i.e. no expectational component of inflation, the equation reproduces the old Keynesian PC such that inflation is procyclical, given its structural slope β_2 . If $\beta_1 = 1$, i.e. full pass-through of expected inflation, the PC becomes vertical with respect to *expected nonzero values* of x_t . In other words, any small expected deviation of output from potential is fully transmitted to inflation. This verticalization of the PC, though, is not necessarily due to full wage-price flexibility ($\beta_2 \rightarrow \infty$), but to the expectational component of inflation.

Taking a step forward, we can express actual and expected output gaps in terms of forecast errors

$$v_t = x_t - E(x_t) \quad (6)$$

By way of equation (6), (5) can be rewritten as

$$\pi_t = \pi^* + \frac{\beta_2}{1 - \beta_1} x_t - \frac{\beta_2 \beta_1}{1 - \beta_1} v_t + u_{\pi t} \quad (7)$$

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⁴The standard REH model solution is now regarded as the limit case of the broader notion of rationality of expectations recalled above once a number of conditions hold regarding agents' learning of the data generation process. Recent examples with regard to the PC are G. W. Evans and McGough (2018), García-Schmidt and Woodford (2019).

If the REH holds, forecast errors will be random, with zero mean, and uncorrelated, so that the expected value of (7) coincides with (4).

In the first place, we may employ our inflation equation in connection with the issue of anchoring *vs.* deanchoring of expectations. In the empirical literature, the more common criterion used to test the anchoring of expectations is based on their responsiveness to macroeconomic news. Therefore, we may also say that agents with well-anchored expectations hold $E(x_t) = 0$ at all times irrespective of the observed news about x_t . Consequently, $v_t = x_t$, and

$$\pi_t = \pi^* + \beta_2 x_t + u_{\pi t} \quad (8)$$

By contrast, agents with deanchored expectations will tend to correlate their output-gap expectation to its observed values. In the limit case of fully anticipated output gaps, $v_t = 0$, the result is:

$$\pi_t = \pi^* + \frac{\beta_2}{1 - \beta_1} x_t + u_{\pi t} \quad (9)$$

We can therefore define two macroeconomic regimes depending on the state of expectations in which the PC is different (see also Nalewaik 2016; Gobbi et al. 2018; Hooper et al. 2019). When expectations are anchored (AE regime), equation (8) shows a seemingly old Keynesian PC whose estimation would yield the structural slope β_2 . When expectations are deanchored (DE regime), equation (9) shows that the PC displays a greater slope, which is not due structural factors but to the interplay with the expectational component; hence its estimation would not yield the true structural slope.

Of course, most of the time the economy will lie in between the two pure regimes, so that inflation will be driven by equation (7). Consequently, in order to identify the structural slope of the PC correctly, it is necessary to take into account both the output gap and its forecast errors. Table 1 displays the effects on the PC of the possible combinations of output gaps and forecast errors.

Table 1: Effects of output gaps and forecast errors on the Phillips Curve, equation (7)

	$v_t > 0$	$v_t < 0$
$x_t > 0$	(underestimated gap) flattening	(overestimated gap) steepening
$x_t < 0$	(overestimated gap) steepening	(underestimated gap) flattening

As can be seen, depending on the sign of the forecast error, the PC may *appear* "steeper" or "flatter". Steepening or flattening mean that the observed change in inflation is greater or smaller than due to the output gap alone. In fact, it is actually due to forecast errors that *shift* the PC as in equation (7). The PC appears steeper with overestimation of output gaps, in which case forecast errors amplify the effect of output gaps. It appears flatter with underestimation of output gaps, in which case forecast errors counterbalance the effect of output gaps. Recall that the effect of the output gap on

inflation is purely determined by the structural factors β_2 only when $v_t = x_t$, i.e. in the AE regime, with underestimation being maximal.

The combinations in Table 1 can accommodate the various "puzzles" detected in the empirical literature. For instance, seemingly missing inflation in a boom as well as missing deflation in a slump may be the consequences of a mis-specified PC that ignores the smoothing effect of agents' underestimation of output gaps or better anchored expectations⁵. By contrast, the excess deflation detected in the course of the EZ recession may be the consequence of the amplifying effect of agents' overestimation of output gaps. This vindicates the concern at the ECB for the deanchoring of deflationary expectations running ahead of the actual downward trend of inflation (Draghi 2014; Draghi 2016). A simple back-of-the-envelope example may give an idea of this effect⁶. Let us consider as the PC structural slope the consensus estimate of about $\beta_2 = 0.2$ during the Great Moderation, a period generally regarded as one of stability and well-anchored expectations. As to the pass-through from expected to actual inflation, the consensus view is that it has increased over time up to about $\beta_1 = 0.7$. According to equation (7), the shift to a DE regime in a recession would rise the output-gap coefficient to 0.67, and each point of overestimation of output gaps would add 0.47 points of deflation. Note that the steepening of the PC found in the EZ during the recession is from about 0.2 to about 0.5 (Riggi and Venditti 2014; Riggi and Venditti 2015; Bank of Ireland 2014a; S. Oinonen 2014).

3. Empirical analysis. The data

Our empirical analysis consists of the econometric estimation of our PC equation (7). The data have been collected from Eurostat AMECO, ECB Statistical Data Warehouse and OECD Main Economic Indicators. All the data are quarterly one-year percentage changes referred to the EZ-19 (fixed composition as of January 1, 2015) from 1999-Q1 to 2019-Q3.

Equation (7) lends itself to straightforward treatment in terms of inflation gap, defined as $\tilde{\pi}_t \equiv \pi_t - \pi^*$. Inflation data, from OECD database, is based on the Consumer Price Index (CPI). The inflation gap is then computed as a simple difference between inflation and 2% inflation target. Quarterly output gaps is obtained by implementing a cubic spline interpolation on AMECO annual data. Data are all calendar and seasonally adjusted.

The expected inflation rate and expected GDP growth are reported as average point forecasts on a 12 months target period (1 year ahead). Both are provided by the ECB Survey of Professional Forecasters (SPF)⁷. The expected inflation gap is computed as the

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⁵"The perceived flattening of the Phillips curve has also generated a good deal of research into the question of why this has happened. The most prevalent explanation is that inflation expectations have both become more important (than lagged inflation) as a determinant of current inflation and have become more firmly anchored as the Fed has more clearly committed to achieving a now stated inflation objective of 2%." (p. 6 Hooper et al. 2019).

⁶See e.g. Blanchard et al. (2015) for the data.

⁷Expected inflation uses the Harmonized Index of Consumer Prices (HICP), while actual inflation is based on CPI. This is not such a major issue, even though the two indicators differ in their calculation.

actual inflation gap is; the expected output gap is calculated as the difference between expected and potential GDP, as a percentage of potential GDP.

A more detailed description of the data is reported in Table 2 (summary statistics of the actual and expected inflation gap) and Table 3 (summary statistics of the actual and expected output gap). First, two interesting elements emerge from Table 2: the difference in the variables' ranges (4.27 for actual inflation, 1.64 for expected) and that in the standard deviations (3 times that of the actual gap with respect to the expected gap). Both hint at the nature of agents' expectations, which are less volatile than observed inflation and fluctuate closer to the equilibrium level. In addition, expected values are on average lower than actual ones (-0.41 against -0.27), which reveals a potential pessimistic or prudential attitude of agents when building their forecasts on the inflation rate. This hypothesis is further confirmed by the fact that 90% of expected values (90th percentile) are negative, against the 50% of actual ones.

Table 2: Summary Statistics Table, Real and Expected Inflation Gap

Inflation Gap				
	Percentiles	Smallest ^b		
1% ^d	-2.35899	-2.35899	Obs. ^a	83
10%	-1.64803	-2.10932		
25%	-0.90218	-1.94614		
50%	-0.08601	Largest ^c	Mean	-0.27231
			Std. Dev. ^e	0.91046
75%	0.32367	0.94118	Variance	0.88266
90%	0.69497	1.42165		
99%	1.91123	1.91123		
Expected Inflation Gap				
	Percentiles	Smallest		
1%	-1.24458	-1.24458	Obs.	83
10%	-0.84613	-1.01835		
25%	-0.62022	-0.97811		
50%	-0.36418	Largest	Mean	-0.40716
			Std. Dev.	0.32204
75%	-0.19783	0.70000	Variance	0.10371
90%	-0.02667	0.13147		

As a proof of this, quarterly HICP inflation data have been calculated by interpolating annual HICP, from the ECB dataset, through a cubic spline, and then compared to the CPI inflation data. Since the two do not significantly differ, we opted for using CPI data from the OECD dataset.

99%	0.40000	0.40000

Data Source: Inflation gap data are from OECD Main Economic Indicators (database); Expected inflation gap is calculated on expected inflation from ECB Survey of Professional Forecasters.

^a *Number of valid observations (i.e., not missing) for the variable.*

^b *List of the three smallest values of the variable.*

^c *List of the three largest values of the variable.*

^d *This is the first percentile. Percentiles are calculated by ordering the values of a variable from lowest to highest, and then finding the value that corresponds to the percent, in this case, 1%. Hence, 1% of the values of the variable are equal to or less than -2.35899.*

^e *Standard deviation of the variable.*

On the contrary, the values assumed by actual and expected output gaps are very similar. This is shown again by variables' range (6.63 and 6.89) and standard deviation (1.78 and 1.68), which do not differ significantly. In this case, output forecasts are on average higher than observed GDP (-0.13 against -0.24) and a higher percentage of positive values (the median of expected gap is higher than zero, while that of real gap is below). This could entail an optimistic behaviour of agents when making output forecasts, even though it is rather a weak evidence.

Table 3: Summary Statistics Table, Real and Expected Output Gap

Output Gap				
	Percentiles	Smallest		
1%	-3.79931	-3.79931	Observations	83
10%	-2.87414	-3.49602		
25%	-1.66560	-3.17128		
50%	-0.10090		Mean	-0.23704
		Largest	Std. Dev.	1.78242
75%	0.97338	2.46263	Variance	3.17702
90%	2.10679	2.68344		
99%	2.83215	2.83215		
Expected Output Gap				
	Percentiles	Smallest		
1%	-3.76288	-3.76288	Observations	83
10%	-2.61492	-3.11287		
25%	-1.71506	-2.94907		
50%	0.27325		Mean	-0.13090
		Largest	Std. Dev.	1.67773

75%	1.08466	2.37672		
90%	1.89712	2.37728	Variance	2.52128
99%	3.13635	3.13635		

Data Source: Output gap data are from AMECO (database); Expected Output gap is calculated on expected GDP from ECB Survey of Professional Forecasters.

Let us now turn to the pattern the data exhibit over time. As shown in Figure 1, expectations of the inflation gap have been well anchored (close to zero) from the beginning of the 2000's to 2008 (in correspondence with the outbreak of the financial crisis). From that point on, expectations have fallen about 1 point below the target, showing a higher sensitivity with respect to the trend of observed inflation. This would confirm the hypothesis of de-anchoring of inflation expectations endorsed by a growing number of scholars in last years. Similarly, the expected output gap (Figure 2) has

Figure 1: Inflation Gap and Expected Inflation Gap, 1999 - 2019

Data Source: Inflation gap data are from OECD Main Economic Indicators (database); Expected inflation gap is calculated on expected inflation from ECB Survey of Professional Forecasters.

remained quite stable around the GDP gap until 2009, when agents have dramatically failed to predict the abrupt drop in the output level of the EZ, as highlighted by the fall in the forecast error value (-6.5%). After 2010, instead, the gap between real and expected output has decreased again.

As explained in section 2, key to our model of the PC is the interplay between output gaps and agents' forecast errors. The effects of the four combinations of these variables on the PC are summarized in Table 1. These combinations can easily be mapped onto the data by means of the dispersion graph presented in Figure 3 which plots forecast

Figure 2: Output Gap, Expected Output Gap and Forecast Error, 1999 - 2019

Data Source: Output gap data are from AMECO (database); Expected Output gap is calculated on expected GDP data from ECB Survey of Professional Forecasters.

errors against output gaps. The labels of the four quadrants of the graph reproduce the combinations in Table 1.

The output gaps are divided in four phases: pre-crisis (1999-2007), recession (2008-09), depression (2010-16), recovery (2017-19). In the pre-crisis and recovery phases both output gaps and forecast errors are mostly positive (output gaps are underestimated), hence the predicted result is a flatter PC. In the recession phase, both output gaps and forecast errors are mostly negative (again, output gaps are underestimated), keeping the PC flat. In the depression phase, instead, systematically negative output gaps are mostly associated with positive forecast errors, i.e. they are overestimated by agents, creating a steeper PC. These predictions of changing PCs fit remarkably well the puzzles in the literature, discussed in the previous sections, about missing inflation in the pre-crisis and recovery phases, missing deflation in the aftermath of the recession, and excess deflation in the depression phase, with a clear impact of deanchored expectations.

For what concerns the econometric modelling, in accordance with recent literature on the empirical analysis of the expectation-augmented Phillips Curve (e.g. Paloviita 2011; Buseti et al. 2017)), and assuming our theoretical equation (7) to take the form of a linear model, a standard estimation procedure for multiple linear regressions, ordinary least squares (OLS), is employed for the estimation. The time series linear model corresponding to equation (7) is as follows:

$$inf_t = \alpha + \gamma_1 out_t + \gamma_2 err_{t-1} + u_t \quad (10)$$

where α is a constant; inf_t represents the gap between actual inflation and inflation tar-

Figure 3: Forecast errors vis-à-vis output gaps, 1999-2019

getting (because of the reasons already explained above); out_t is the output gap measure; $err_{t,t-1}$ is the forecast error. The subscript in forecast error term $(t, t - 1)$ means that the output gap expectations, used to compute the errors, refer to current time t , but are formulated earlier at time $t - 1$. The error component u_t is assumed to be additive.

Some specification tests on the residuals are provided in support of our estimation choices⁸. First issue to consider in time series analysis is the degree of correlation among residuals. Both auto and partial-autocorrelation plots reveal the presence of significant autocorrelation in the residuals up to the first 4 lags; though when employing partial-autocorrelation test, this effect disappears after the first lag⁹. Moreover, tests for multicollinearity (Belsey's diagnostics) and heteroskedasticity (Engle's ARCH Test) are run on the model. For the first test the null hypothesis is rejected; for the second it is not (see Appendix). Heteroskedasticity and autocorrelation robust standard errors (HAC) are then employed in the OLS regression. Another possible issue which is more difficult to detect, is the endogeneity problem. In order to prevent possible sources of endogeneity (unobserved heterogeneity, simultaneous causation, measurement error), which would cause OLS to be biased, we employ additionally a GMM estimation model¹⁰, which provides instead consistent results in the presence of endogeneity.

⁸For details and test results, see the Appendix.

⁹Note that with time series data, it is highly likely that the value of a variable observed at time t is persistent or simply similar to its lagged values; therefore when fitting a regression model to time series it is common to find autocorrelation in the residuals. In this case an OLS estimation could still be unbiased, but it would need a correction of the standard errors.

¹⁰The Generalized Method of Moments is a generic method for estimating parameters. It is widely applied when the full shape of the data's distribution function is not known. It employs instrumental variables to break down autocorrelation or endogeneity problems in the estimation. In estimating our model, the instruments used are the 1 year lagged value of output gap and the 1 year lagged value of forecast error.

We report the results from OLS and GMM estimation of model (10) in table 4. Estimated coefficient are quite reasonable and coherent with our theoretical model. In particular, an increase of 1% in the output gap, leads to an increase of the inflation gap between 0.25% (in OLS) and 0.3% (in GMM); on the contrary, a 1% increase of the forecast error implies a drop of the inflation level of 0.17% with OLS estimation, while the reduction decreases to 0.1% when employing the GMM. Thus the sensitivity of inflation to the output gap would be far from being negligible. In addition, the "regulating effect" of forecast error (which increases or reduces the output gap impact depending on whether the sign of the error is negative or positive, respectively) is also considerable.

Table 4: Phillips Curve Linear Regression Table

	OLS (1) NW	GMM (2) NW
out_t	0.257** (0.1078)	0.292*** (0.0590)
$err_{t,t-1}$	-0.171 (0.1283)	-0.101 (0.0856)
cons	-0.229 (0.1835)	-0.201** (0.0914)
<i>Observations</i>	83	82
R^2	0.265	

Notes: The table reports estimated equation (10). Column (1) contains OLS estimated coefficients, while column (2) contains GMM estimated coefficients. Instruments: lagged output gap out_{t-1} and lagged forecast error $err_{t-1,t-2}$. The sample period is 1999Q1 - 2019Q3, 83 observations. Dependent variable is the quarterly inflation gap (year on year). NW indicates Newey-West (HAC consistence) standard errors. Bartlett GMM weight matrix. SE are displayed in parenthesis.

*** Significant at the 1% level; ** Significant at the 5% level; * Significant at the 10% level.

It might be wise however to consider alternative approaches to our modelling, in order to enrich the analysis. It is indeed true that another crucial question may arise when employing time-series in a regression, that is the stability of parameters over time. In the presence of significant structural breaks in the $\hat{\gamma}_s$, the estimated model might be not completely reliable. A clear representation of this issue comes from the graph in Figure 4. Here it is represented the evolution of output gap and forecast error estimated coefficients of equation (10) over time, obtained by employing a recursive OLS regression with Newey-West robust standard errors. As illustrated, the two parameters are quite

unstable¹¹. In particular, 3 main structural breaks appear around iteration 35, 40 and 55. Further evidence of coefficient instability is provided by the structural change tests "Cumulative-sum-test" (Cusum) and "Chowtest". Both confirm the existence of those

Figure 4: Output Gap and Forecast error Coefficients, Recursive linear regression of model (10)

Notes: The graph is obtained by employing a recursive (nested-window) regression on equation (10). Number of iterations is reported on the x-axis.

structural breaks detected in figure 4, though the result of the Chowtest suggest the presence of many other possible breaks. In order to address this issue, we introduce then in our analysis an alternative model specification which takes into account the evolution over time of parameters. This kind of models are defined time-varying coefficient linear regressions (Casas and Fernández-Casal 2019). Without prejudice to the importance of linear models, a time-varying regression (tvReg) allows us to verify how the sensitivity of the inflation gap changes over time with respect to the output and agents' forecast error (Fan and Zhang 2008). Results from employing this procedure on our dataset are displayed in figure 5 below. Both parameters are clearly increasing (in absolute value). According to equation (7) this could be determined by an increase of the coefficient of the structural term β_2 .

In view of all this, the hypothesis relative to our endogenous-expectation Phillips Curve can be confirmed. The role played by the forecast error in the years subsequent to the financial crisis, provides a clear explanation to such an apparently unpredictable be-

¹¹Consider that recursive nested-window estimates typically show a high volatility during the initial "burn-in" period, because of the low number of observation in the subsample, which is slightly larger than the number of parameters in the model. After this "trial" period (in our case after iteration 20), any further volatility should be considered as evidence of coefficient instability.

Figure 5: Time-varying Coefficients regression

Notes: *tvReg* model uses the same formula as the linear function (10); "Gaussian" type kernel employed in the coefficient estimation.

haviour of the inflation gap. The two above-mentioned "puzzles" are therefore confirmed by our findings. In particular:

- a the "missing deflation" puzzle in the aftermath of the recession (in particular between 2011 and 2013) coincides with a largely negative value of the forecast component (see figure 2); the negative value of the estimated $\hat{\gamma}_2$ from equation (10) would have then attenuated the impact of the output gap on prices. The result is a flattening PC, which explains why inflation has not fallen together with output;
- b in the second half of the depression phase (between 2014 and 2016), we observe instead a rapid steepening of the curve: the positive forecast error indicates an overestimation of x_t (here between -2% and -1%), strengthening its overall impact on the inflation gap (see table 1);
- c lastly, the "missing inflation" puzzle in the recovery phase (from 2017 onwards) is clearly explained by the increasing negative effect of the expectation term, here positive at least until 2018-Q2, which has caused a strong underestimation of the output gap and hence a natural flattening of the PC.

4. Conclusions and Future Perspectives

That of the Phillips curve is a story of trial and error. It began with A. W. Phillips in 1958 as an empirical observation in need of a theoretical explanation. The notion of Phillips curve was introduced only in 1960 by the economists P. Samuelson and R. Solow, who advanced the first hypothesis of a strong relationship between unemployment and price-inflation. Unfortunately, the PC has demonstrated to be a controversial instrument over time, forcing researcher to adjust it in order to fit always new data. As observed

by Stock and Watson (2010), “the history of the inflation forecasting literature is one of apparently stable relationships falling apart upon publication” (Stock and Watson 2010, pp. 211-212).

The Great Recession of 2008 has further challenged economists and policy makers. The absence of a significant disinflation first (in US and Europe), and of a strong price recovery then (in particular in the Euro Zone), pushed researchers to wonder if the relationship between economic slack and inflation could still be valid. The suggestion coming from Hicks (1946) and Lucas (1973) is that inflation expectations cannot be deemed to be independent of the economic trend (Hicks 1946); therefore the relation inflation-economic slack described by standard expectation-augmented Phillips Curve might be misleading (Gobbi et al. 2018). In this respect, our empirical PC responds precisely to this interrogative: inflation gap expectations are replaced by output gap expectations, using a mathematical expedient. The result is a Phillips Curve in which inflation gap depends no more on its own expectations, but only on real and forecast output gap. In this new expression, output and inflation keep moving in the same direction, while forecast error (calculated as the gap of expectations from observed output) acts in two opposite ways: it amplifies the impact of output on prices when agents overestimate the gap (steepening the curve); or, on the contrary, it reduces the impact when economic agents underestimate the gap (the curve is flatter). The hypothesis underlying the model are confirmed by the data collected on the Euro Zone from 1999 to 2019. The econometric analysis reveals not only that the Phillips Curve is not flat, but that the model is fully able to explain the so-called inflation puzzles following in the wake of the economic crisis.

This empirical work however represents a preliminary analysis of the phenomenon. First, it would be important to extend the research to a country-level analysis, to investigate the impact of structural economic differences between countries in the Euro Area, or to test whether the model is also able to accurately predict the inflation behaviour in the US economy. Furthermore, other variables could serve as a proxy for output forecasts, other than the Survey of Professional Forecasters, to better reproduce households’ and firms’ expectations along the lines of the empirical work by Coibion and Gorodnichenko (2015). In this regard, there exists the Consumer Surveys conducted by the Directorate General for Economic and Financial Affairs of the European Commission, though these are qualitative data that are difficult to integrate into an econometric model¹².

On the side of policies implication, the inflation puzzles have revealed the ineffectiveness of conventional and unconventional monetary policies in stimulating inflation and economic growth. In addition, the lack of trust that economic agents have vis-à-vis national and supranational institutions have certainly not helped. Hence several scholars suggest a major change of direction in the policies undertaken by the ECB. One of these proposal would be that of a dual-mandate (on the basis of the Federal Reserve’s one) focusing not only on the price stability, but also on the progressive reduction of output and unemployment gaps (International Monetary Fund 2013). This would however re-

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¹²In this respect, Lyziak and Paloviita (2016) quantify consumer expectations using a probability approach

quire higher flexibility from central banks, and a partial relaxation of the constraints on price levels. Therefore, a possible incompatibility between these two objectives is not excluded, given the system in force. Another one would be focusing on a possible range in which inflation could fluctuate, depending on current specific economic circumstances. In this respect, the president of the Federal Reserve Bank of Boston Rosengren (2018), has declared that the FED should define an inflation range between 1.5 and 3%, rather than the actual medium-term target of 2%, in order to avoid hitting the zero lower bound of interest rates to meet the objective. The suggestion coming from Gobbi et al. (2018) instead, is to abandon conventional monetary policies, given their inability to stimulate enough the economy. On the contrary, they recommend to adopt aggressive reflationary policies, aimed at boosting the level of economic activity through a massive direct public spending and a reduction in the taxation level. This last proposal, in particular, might be surprisingly effective in the light of the model developed. Given the twofold impact the output gap has on inflation, it could become fundamental to stimulate directly the real side of the economy, so as to reduce the output gap, drive agents' expectations and, consequently, raise the inflation rate.

Appendix

A. Survey of Professional Forecasters

The ECB Survey of Professional Forecasters is conducted in the first month of every quarter since 1999, with different time horizons, from 3 months to 5 years. These surveys are submitted to a wide number of professionals (about 90) from all the Euro Area countries. Agents are asked to make forecasts about 3 main macroeconomic variable. In this work, we make use only of forecasts made on the following variables:

- Harmonised Index of Consumer Prices (HICP) inflation, published by Eurostat (Annual rates of growth);
- Real gross domestic product (GDP) according to the definition of the European System of National and Regional Accounts 1995 (ESA 95), published by Eurostat; annual rates of growth.

For what concerns inflation surveys, the latest HICP inflation rate available to survey respondents at the time of expectations formation (survey deadline) refers to the last month of the previous quarter. Instead, for GDP surveys output latest projection at forecasters disposal refers to the projection published in the previous quarter. The surveys provide two classes of observation types: a probability distribution class, in which forecasters are asked to provide their personal probability distribution about the forecasted variable; a point forecast, in which forecasters have to indicate a single value for each variable at each time horizon. The latter is the one used in this work. A spreadsheet sample of the survey is available at the following website: https://www.ecb.europa.eu/stats/ecb_surveys/survey_of_professional_forecasters/html/index.en.html#background

B. Statistical Tests

Test for auto and partial autocorrelation are carried on the residuals of OLS regression. The produced test plots show little sign of autocorrelation of order higher than 1; this is due to the fact that time series always have a certain level of persistence (remember for example the stickiness of prices theorized by Calvo (1983)). In the graph 6, autocorrelation test is displayed on the left hand side and partial-autocorrelation on the right one. When removing the influence of lags of order higher than 1 (from auto to partial autocorrelation), the residuals-autocorrelation disappears.

Belsley's diagnostic checks for multicollinearity in linear regression. The default tolerance level for for this test is 30. In this case the test result is:

- $B = 1.21$

hence, there is no evidence of multicollinearity.

After regression, Engle's ARCH test is employed to check for errors heteroskedasticity. The null hypothesis of constant errors' variance is refused. The test result on our linear regression model is:

Figure 6: Auto and Partial Autocorrelation test

· p-value = 6.4901e-07

Robust standard errors (Newey-West) are then employed in both OLS and GMM regressions. Additional proof of the heteroskedasticity is given by figure 7. The high volatility of residuals is self-explanatory.

Figure 7: Linear Regression: Residuals vs Fitted values

The Augmented Dickey-Fueller test the stationarity of variables. The null hypothesis is that the variable contain a unit root, while the alternative is that it follows a stationary process. ADF test up to 4 lags is conducted on the three main variables used; the null hypothesis is not rejected for inflation gap (except for the 4th lag) and and forecast error, but it is rejected for output gap (except for the 1st lag).

The *Cusumtest* statistic is constructed from the cumulative sum of the OLS residuals

to test coefficients' stability and for structural breaks due to changes in the model over time. The cumulative sum of squares test ($CusumSq$) instead is used to detect possible structural breaks in volatility in simulated data. In our case, the null hypothesis of coefficients' instability is rejected, since the blue statistic line never crosses the red confidence bands (figure 8a). On the contrary, the null hypothesis of instability in the volatility (figure 8b) cannot be rejected, as the statistic line crosses the critical lines at least twice. Chow test instead checks the validity of structural breaks in the coefficients when the position of the breaks is known. Assuming they are unknown, we use a loop to test the presence of breaks on the entire sample. The outcome of the test reveals the existence of a multitude of breaks between iterations 30 and 70.

Figure 8: Cumulative Sum Test

(a) Cumulative Sum Test, Coefficients' Stability in New Phillips Curve

(b) Cumulative Sum Test-Squared, Coefficients' Volatility in New Phillips Curve

Notes: Cumulative sum test with 95% confidence bands (dot lines). Total number of iterations produced by the process is 81, equal to the total number of observations (83) minus 2.

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