Monetary Policy, rational confidence, and Neo-Fisherian depressions

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Abstract

We examine the so-called "Neo-Fisherian" claim that, at the zero lower bound (ZLB) of the monetary policy interest rate, and the economy in a depression equilibrium, in order to restore the desired inflation rate the policy rate should be \textit{raised} consistently with the Fisher equation. This claim has been questioned on the ground that the Fisher equation cannot be used mechanically to peg the long-run inflation expectations. It is necessary to examine how inflation expectations are formed in response to, and interact with, policy actions and the evolution of the economy. Hence we study a New Keynesian economy where agents' inflation expectations are based on their correct understanding of the data generations process, and on their probabilistic confidence in the central bank's ability to keep inflation on target, driven by the observed state of the economy. We find that the Neo-Fisherian claim is a theoretical possibility depending on the interplay of a set of parameters and very low levels of agents' confidence. Yet, on the basis of simulations of the model, we may say that this possibility is remote for most commonly found empirical values of the relevant parameters. Moreover, the Neo-Fisherian policy-rate peg is not sustained by the expectations formation process.

Keywords: conventional monetary policy, Neo-Fisherian theory, formation of inflation expectations, monetary policy at the zero lower bound.

JEL codes: D84, E31, E52

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1 Introduction

Several years after the Great Recession, central banks in some advanced countries still seem entangled in difficulties to restore inflation and economic activity to their desired targets even with nominal interest rates close to the zero lower bound (ZLB). As a consequence, a large body of literature is devoted to analysing the optimal conduct of monetary policy at the ZLB (Eggertsson and Woodford, 2004; Jung et al., 2005; Adam and Billi, 2006, 2007; Nakov, 2008; Werning, 2012; Cochrane, 2017). Using forward-looking New Keynesian models, these papers suggest a forward guidance policy that may sustain expected inflation and stimulate the economy by promising a path of future low (real) interest rates such that inflation may overshoot the target. In recent years, forward guidance, coupled with direct liquidity injections in various forms, has led the so-called "unconventional" monetary policy of major central banks (Williams, 2011; ECB, 2014, Cœuré, 2018). However, the effectiveness of this policy strategy has recently been challenged by several prominent economists (Kool and Thornton, 2015; McKay et al., 2016; Gertler, 2017; Hagerdorn et al., 2018).

A more radical critique, labelled “Neo-Fisherianism”, asserts that inflation may remain low precisely because nominal rates themselves are low (e.g. Cochrane, 2014, 2016a; Williamson, 2017). The forward guidance that the policy rate will remain low for an extended period of time keeps inflation expectations below target because it validates beliefs that low inflation in the current circumstances is inevitable. Central banks should instead raise the policy rate in order to lift inflation from its undesirably low rate.

The Neo-Fisherian approach hinges on the so-called Fisher equation (Fisher, 1930), which is present in modern macroeconomic models with Euler equations. It postulates that in the long-run the real interest rate that can be earned on capital is determined by real factors independent of monetary policy; let us call it \( r^{*}_t \). After Wicksell (1898), this is also called "natural rate of interest". On the other hand, by simple accounting, the current real interest rate \( r_t \) equals the difference between the current nominal interest rate \( i_t \) and inflation expected in the following period \( \pi^{e}_{t+1} \). Using the no-arbitrage equilibrium \( r_t = r^{*}_t \), it follows that the inflation (rational) expectation should be \( \pi^{e}_{t+1} = i_t - r^{*}_t \). Hence a permanent rise in the nominal interest rate must be followed by an equivalent increase in expected inflation, and vice versa. Inflation expectations here are assumed to automatically adjust upwards to the new higher nominal interest rate. The main challenge for this view is specifying the market forces that would push inflation up in the wake
of a nominal rate hike. It is as if firms or households look at high nominal interest rates, associate them with high inflation and simply raise their prices accordingly (Cochrane, 2016a).

A more advanced way of presenting the Neo-Fisherian view can be found in a series of papers by Schmitt-Grohé and Uribe (2014, 2017). The authors show that in DSGE models with a ZLB and Taylor-Rule type monetary policy there are two or more equilibria and there is a path from the good equilibrium with positive inflation to a bad equilibrium of very low or even negative inflation. As a matter of fact, in the so-called "liquidity trap" (low inflation and low output equilibrium) the Taylor rule no longer works\(^1\). The Neo-Fisherian view is that the only way to steer the economy to the good equilibrium is to abandon the interest rate rule and peg the policy rate to its good equilibrium value. Although Friedman (1968) argued that a policy-rate peg would make inflation dynamics explosive, Werning (2012) and Cochrane (2016a, 2016b) solve the model by picking backward stable equilibrium whereby the initial state is consistent with the perfect foresight solution\(^2\). These results appear to hold also when the model is opened to possible frictions and modification, including the preference for money, backward-looking Phillips Curve and different Taylor-Rule specifications.

Empirical support for the Neo-Fisherian view is quite limited so far. Using both an empirical VAR model and a theoretical DSGE model with temporary and permanent monetary shocks, Uribe (2017) estimates the Neo-Fisher effect in the United States and Japan. His estimated model produces dynamics consistent with the Neo-Fisherian prediction that a credible and gradual increase of nominal interest rates to normal levels can generate a rapid reflation of the economy with low real interest rates and no output loss. Similarly, Lukmanova and Rabitsch (2018) find strong evidence for a short run positive co-movement of inflation and the nominal interest rate, at no output cost in U.S. data. However, correlation is not causation; the inverse sense of causation of the Fisher equation is also possible, namely that higher inflation should be matched by a higher nominal interest rate, a basic notion in finance. Indeed, Crowder (2018) tested and found that inflation causes nominal interest rates in the long run, but not the other way around.

The Neo-Fisherian view has garnered mainstream press attention (Ip, 2015) and consideration at policy levels (Bullard 2015), but it remains quite controversial

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\(^1\) The existence of a low-inflation steady state is not only a theoretical possibility depicted by Benhabib et al. (2001) and Schmitt-Grohé and Uribe (2014, 2017), but can be retraced in the empirical data (Bullard, 2015).

\(^2\) Similar conclusion can also be found in Uribe (2017, 2018).
since it contradicts established views of how monetary policy affects the economy. First, the real adherence to the thought of Irving Fisher is disputed. In fact, in Fisher’s theory the nominal interest rate is an endogenous variable, determined by the real rate and expected inflation, while in the Neo-Fisherians approach $i_t$ is fixed by the central bank responding to the inflation gap. Several economists contest the arbitrary assumption of backward stable equilibrium to bypass Friedman’s critique. Although the latter is among all potential equilibria, there is no convincing argument that only this equilibrium selection method should be used (Kortelainen, 2017; Gerke and Hauzenberger, 2018; Spahn, 2018).

Garin et al. (2018) address the more specific policy task of re-setting the inflation target, distinguishing between transitory and once-and-for-all changes. In the context of a standard New Keynesian model, they show that achieving a higher inflation target requires a higher policy rate only in the presence of particular conditions, such as highly persistent changes in the target and high price flexibility. In the limit case of a once-and-for-all reset of a higher target, fully anticipated in agents’ expectations, the policy rate should immediately jump to the higher level dictated by the Fisher equation.

Another strand of studies directly addresses the problem of expectations formation, (e.g. Evans and McGough, 2018a, 2018b; Woodford, 2018; García-Schmidt and Woodford, 2019). These belong to the literature that drops substantive rational expectations (coincident with the "true" data generation process) as an a-priori assumption, and introduces various forms of boundedly-rational expectations. They deny the practical relevance of the perfect foresight solutions (or, more generally, substantive rational-expectations solutions) under a permanent interest-rate peg, even when the commitment is fully credible. Indeed, there are many trajectories that output and inflation may follow to go from the bad equilibrium to the good one, and some of them may imply very protracted recessions. Therefore, there is no reason to discard the practical relevance of these alternative outcomes.

These authors argue that predicting what may happen as a result of a particular policy commitment requires two ingredients. First, the identification of the process that generates inflation in the economy, inclusive of the role of inflation

3 It will be recalled that setting a higher inflation target may be a way to bypass the ZLB of the policy rate when the natural interest rate and the current inflation target sum to less than zero.

expectations. Second, how the inflation expectations are generated, that is, the “process of reflection” of agents (García-Schmidt and Woodford, 2019) by which they arrive at particular expectations taking into account their understanding of the data generation process and the relevant available information.

Both Evans and McGough (2018b), and García-Schmidt and Woodford (2019) study a standard New Keynesian economy where agents are inflation forecasters who iteratively adapt their forecasts until a perfect foresight equilibrium has (ideally) been reached. Both studies deliver a negative verdict about the Neo-Fisherian claim that, starting from a depression at the ZLB, the solution is pegging the policy rate at its Fisherian equilibrium level. A common element is that the abandonment of the interest-rate feedback rule impairs convergence and stability of the system in response to the new policy-rate peg.

In this paper we follow the approach put forward by these studies, introducing a different process of expectations formation. Our agents are (consciously) not engaged in making good forecasts of future inflation but in figuring out their subjective probabilistic beliefs about the future state of the economy (“normality”, with inflation reverting to target, or “depression”, with inflation remaining below target). One main reason is that transitions from normal to depression states are possible but very infrequent, so that agents do not (and know they do not) possess sufficient statistical evidence in order to compute robust "objective" probability distributions of the relevant events - a well-known argument put forward by Keynes (1937). Chung et al. (2012) provide evidence that professional forecasters have underestimated the likelihood of the ZLB threat by "focusing too much on the Great Moderation experience and relying on structural models whose dynamics cannot generate sustained ZLB episodes" (p. 47). Beliefs are formed rationally in the sense defined above (consistency with the data generation process, and updating vis-à-vis the evolution of the state of the economy). We will show that key

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5 The two studies differ in the way the expectations formation process is modelled. Evans and McGough assume an adaptive learning process driven by recursive observation of past realizations of (the determinants of) inflation, whereas in García-Schmidt and Woodford agents recursively update their expected inflation upon observing the realizations of the forward-looking component of the determinants of inflation. The motivations and implications of this difference are discussed by García-Schmidt and Woodford (pp. 90 and ff.), though they conclude that the two treatments may be regarded as complements (p. 91, fn. 7).

6 A related advantage of our approach is avoidance of two controversial grounds. First, assumptions about agents' knowledge of the structure (estimation model) of the data generation process (which may quickly go out-of-date in the face of unusual events). Second, conjectures about how good agents are as econometricians.
to the macroeconomic process is the interaction between the agents' state of confidence in the return to normality and monetary policy. Neo-Fisherian depressions, such that output and inflation would react positively to a rise in the policy rate, are a theoretical possibility due to particularly low states of confidence. Yet, these states depend on the value of some key parameters and initial shocks that may be regarded as remote in light of the consensus empirical literature, and the Neo-Fisherian policy-rate peg is not sustained by the expectations formation process.

In section 2 we introduce the standard New-Keynesian economy consisting of the output gap equation (OG), the Phillips Curve (PC) and the Taylor Rule (TR). Following Woodford (2003) and García-Schmidt and Woodford (2019), by forward iteration of the PC the long-run inflation expectations are determined by the expectation of the future path of the output gaps. The TR ensures convergence of output and inflation towards the "zero-gaps" equilibrium, which is therefore the "anchor" of long-run rational expectations, up to the ZLB of the interest rate. At the ZLB a depression equilibrium is possible. Hence we take a step backward and address three questions. First, as Evans and McGough (2018) put it, under what conditions do agents have reason to believe in the central bank's ability to keep the economy on target? Second, if they revise their inflation expectations as the economy deviates from target, how does this revision interact with the macroeconomic process and the effectiveness of monetary policy? Are there conditions under which the Neo-Fisherian claims apply?

Our contribution is developed in three steps in the subsequent sections 3, 4 and 5. Firstly, in section 3 we introduce agents' long-run expectations based on their probabilistic beliefs about the future state of the economy. As in Arifovic et al. (2017) and Gobbi et al. (2019), upon observing a depression state of the economy, i.e. negative output and inflation gaps, agents elaborate a "regime switch hypothesis", that is, they assign probability \( p_t \) to the hypothesis that the current state will revert to the zero-gaps equilibrium ("normal regime") against the probability \( 1 - p_t \) that it will not ("depression regime"). In other words, \( p_t \) is a measure of agents' confidence in the central bank's ability to keep the economy on track, which is not taken for granted a priori. By way of the OG equation, inflation expectations in turn affect the output gap \( y_t \); thus we have a modified function \( y_t = \zeta(\hat{i}_t, p_t) \), depending both on the Fisherian interest-rate gap \( \hat{i}_t \) (the difference between the policy rate \( i_t \) and \( r^* + \pi_{t+1}^e \)), and on \( p_t \). Initially assuming that \( p_t < 1 \) is associated with a depression state, we show two main results. First, the output and inflation gaps are larger for lower \( p_t \), that is low confidence in the normal
regime acts as amplifier of gaps making the depression regime more likely. Second, we identify conditions whereby a Neo-Fisherian policy is feasible, i.e. raising the policy rate reduces the output gap and hence increases expected inflation, namely when $p_t$ falls below a certain threshold level, i.e. confidence in the normal regime is particularly low.

Secondly, in section 4 we introduce how the probability $p_t$ may be rationally induced by the actual evolution of the output gap, i.e. by way of a "confidence function" (CF) of the form $p_t = \psi(y_t)$ vis-à-vis $y_t = \zeta(i_t, p_t)$. At any point in time agents hold some probability over the normal vs. the depression regime, which is consistently updated upon observing new realizations of the state of the economy. Consistency is warranted by some necessary properties of the CF, one of which is that the confidence in the normal regime falls as the economy deviates from it. The main result is that depression steady-state equilibria may emerge as fixed points of the two maps $\psi$ and $\zeta$, implying a permanent value of $p < 1$. Depressions may be of Neo-Fisherian type as well as of conventional New-Keynesian type, depending on particular combinations of shocks and parameter values of the OG, PC and CF functions. In short, the occurrence of Neo-Fisherian conditions is an empirical matter.

In section 5 we identify the critical parameters, and by means of simulations we show that for values in line with their consensus range in the empirical literature, the conditions for the Neo-Fisherian policy can be regarded as remote. Moreover, if the economy falls into a Neo-Fisherian depression, the pegging of the policy rate to its Fisherian equilibrium value is not supported by the expectations formation process. Further light is also shed on the conditions underpinning successful conventional policy, which result more stringent – depression states are more likely – than believed in earlier studies as shown by Chung et al. (2012).

Section 6 summarises and concludes pointing out that our findings leave open the question of how the economy can be rescued from a depression state when conventional monetary policy is stuck at the ZLB.

2 The standard New Keynesian model, short-run and long-run expectations

We consider the standard New Keynesian framework for monetary policy (e.g. Gali, 2008). The model is linearized around a zero inflation steady-state. The two equations describing the economy are:

\[
y_t = E_0 y_{t+1} - \alpha(i_t - (r_t^* + E_t \pi_{t+1})) + u_{yt}
\]
\( \pi_t = \beta E_t \pi_{t+1} + \kappa y_t \)

In these expressions, \( y_t \) is the logarithmic difference between the current output and the potential output, \( i_t \) is the nominal interest rate controlled by the central bank, \( \pi_t \) is the inflation rate, \( r_t^* \) is the "natural" (real) interest rate, i.e. the interest rate corresponding to the general equilibrium of the economy at potential output, and \( u_{yt} \) is a white-noise random output disturbance. \( E_t \) is the expectation operator conditional on information available at time \( t \). For simplicity, the natural rate will be kept constant \( r_t^* = r^* \) all \( t \).

Equation (1) is the output-gap equation (OG), derived from the household’s Euler equation. The term in parentheses can be read as the "interest-rate gap" \( \hat{i}_t \), i.e. the deviation of the policy rate from the natural rate and the expected inflation, with \( \alpha \) measuring the (constant) elasticity of substitution of aggregate spending. Equation (2) is the New Keynesian Phillips Curve (PC) and expresses current inflation – the inflation gap for the central bank – as a function of the current output gap and expected future inflation. The parameter \( \beta < 1 \) is the time discount factor, while \( \kappa > 0 \) is a parameter reflecting the degree of price flexibility in the goods market (\( \kappa \) increases with price flexibility).

The expectational terms appearing in the standard New Keynesian model can be dubbed "short-term expectations", i.e. expectations of variables one period ahead. A method to address the formation of (substantively) rational inflation expectations is forward iteration of the equation (2) of current inflation (Woodford, 2003, ch. 2; Garcia-Schmidt and Woodford, 2019), so that, after \( N \) iterations, it proves to be

\[
\pi_t = \beta^N E_t \pi_{t+N} + \kappa (y_t + \sum_{n=1}^{N} \beta^n E_t y_{t+n})
\]

Given that \( \beta < 1 \), as \( N \to \infty \) \( \beta^N E_t \pi_{t+N} \to 0 \), so that current inflation comes to depend on the current and entire future path of output gaps. How is this determined by rational agents?

Consistently with the OG equation (2), the rational expectation of the future path of output gaps is in turn determined by the future path of the policy rate. The key contribution of the New Keynesian theory of monetary policy is that, if the policy rate is pinned down by a feedback rule responding to observed (possibly

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7 In particular, \( \kappa = (1 - \phi)(1 - \phi \beta)\phi^{-1} \), where \( \phi \) is the probability of prices being unchanged (the fraction of firms not changing their price) after a change in aggregate demand (Calvo, 1983). Clearly, \( \phi = 1, \kappa = 0 \), represent the Old Keynesian fixed-price economy where the Phillips Curve is horizontal, and the steady-state inflation is zero, whereas \( \phi = 0, \kappa \to \infty \), represents the New Classical flex-price economy where the Phillips Curve is vertical, and the steady-state inflation is undetermined.
foreseen) inflation gaps (possibly controlling for output gaps), under suitable conditions the economy converges to a steady state with zero gaps. One such feedback rule is the standard Taylor Rule (TR) whereby the central bank adjusts the policy rate in such a way as to be consistent with inflation equalling its target and the Fisher Equation, while smoothing output gaps. With a zero inflation target, the TR equation that closes the model is as follows

\[ i_t = r^* + \gamma_\pi \pi_t + \gamma_y y_t \]

where \( \gamma_\pi > 0, \gamma_y \geq 0 \) are the policy parameters, and \( \tau \) is a time index that can be determined according to various specifications, e.g. "real time" \( \tau = t \), forward looking \( \tau = t+n \) (\( n = 1, \ldots \)), lagged \( \tau = t-n \) (\( n = 1, \ldots \)). A sufficient condition for convergence to the zero-gaps steady state \( (\bar{\pi}, \bar{y}) = 0 \) is \( \gamma_\pi > 1 \).

Therefore, if agents derive the series \( \pi_{t+n} \) consistently with the full model solution, then \( E_{t} \pi_{t+n} = E_{t} \pi_{t+n} = 0 \) for all \( n \). Consequently, the zero-gaps equilibrium is also the rational, "long-run" expectation of output and inflation. If agents hold these long-run rational expectations, the New Keynesian model boils down to the following two-equation policy control system consisting of the OG (which also drives inflation) and the TR. Provided that shocks are unanticipated, let us consider the basic, "real time" specification (\( \tau = t \)) of the TR. Therefore,

\[\begin{align*}
\dot{y}_t &= -\alpha \hat{i}_t + u_{yt} \\
\dot{i}_t &= (\kappa \gamma_\pi + \gamma_y) y_t
\end{align*}\]

where \( \hat{i}_t = i_t - r^* \) is the Fisherian interest-rate gap. Indeed, the zero-gaps (stochastic) equilibrium \( (\bar{\pi}, \bar{y}) = 0 \) is a solution to the system. By the "Cobweb Theorem", its stability requires that \(|\alpha (\kappa \gamma_\pi + \gamma_y)| < 1 \). It should be noted that the inflation policy parameter now encounters an upper bound: it may be greater than 1, but not unboundedly so. The reason is that policing "unanchored" short-run expectations requires larger changes in the policy rate (i.e. in the market real rate), whereas "anchored" long-run expectations need a softer use of the instrument.

Agents holding the long-run expectation of the zero-gaps equilibrium rationally, presume that the central bank exerts stochastic control on the system, which in turn presumes the rational-expectations hypothesis. In other words, the central bank succeeds in "anchoring" long-run inflation expectations to its target if the agents expect it to succeed, and vice versa. The problem of anchoring expectations seems resolved by assumption.

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8 This is implied by the necessary and sufficient condition for the three-equation system OG-PC-TR to have two eigenvalues within the unit circle, namely \( \kappa(\gamma_\pi - 1) + (1 - \beta)\gamma_y > 0 \) (Galì, 2008, p. 77 and ff.)
To break this circularity, let us take a step backward and address the following question concerning expectation formation: under what conditions do agents have reasons to believe in the central bank’s ability to achieve the zero-gaps equilibrium? In the first place (section 3), we shall introduce agents’ state of confidence as an exogenous probabilistic belief and study the implications of less than full confidence. Then (section 4), we shall rationalise agents’ confidence by relating it to the observed state of the economy.

3 Agents’ confidence, and the existence of Neo-Fisherian depressions

We draw on the idea of "regime switch" put forward by Arifovic et al. (2017) and re-elaborated by Gobbi et al. (2019). Let agents observe, at any point in time $t$, a negative output gap. They consider the possibility of a switch from the "normal regime", where the economy will return to the zero-gaps equilibrium, to a "depression regime" where the output gap will remain constant at the observed value. If agents believe in the normal regime with confidence $p \in [0, 1]$, and in the depression regime with confidence $(1 - p)$, a consistent belief about the future path of output gaps is the $(p; 1-p)$ mean value of the equilibrium gaps in the two regimes, i.e.:

$$E_t y_{t+1} = (1-p)y_t, \ldots, E_t y_{t+n} = (1-p)y_{t+n-1}, \ldots$$

For the time being, we consider $p$ as an exogenous parameter measuring the state of confidence of agents. Given this state of expectations, the iterated PC becomes:

$$\pi_t = \beta^N E_t \pi_{t+N} + \left(1 + \sum_{n=1}^{N} ((1-p)\beta)^n \right)ky_t$$

Taking the limit for $N \to \infty$, we obtain

(6) \quad \pi_t = \omega ky_t

where $\omega \equiv (1 - (1-p)\beta)^{-1}$.

Likewise, we should reformulate also the OG according to the regime switch hypothesis; that is to say, $E_t y_{t+1} = (1-p)y_t$, and $E_t \pi_{t+1} = \omega k E_t y_{t+1}$ (which is indeed the expected value of (6)). Substituting these values into (1) we obtain

(7) \quad y_t = (\alpha \hat{\pi}_t + u_{yt})\theta

where $\theta \equiv \left[\omega(\beta p^2 + (1 - \beta + \alpha \kappa)p - \alpha \kappa)\right]^{-1}$

For $p = 1$, the PC and OG thus obtained coincide with the standard ones when expectations are "anchored" by full confidence in the normal regime, $E_t y_{t+1} = E_t \pi_{t+1} = 0$. The first noteworthy result when agents' full confidence in the normal regime
is not assumed a priori, is that the relationship between inflation and output gaps in the PC equation is amplified ($\omega > 1$) to the extent that $p < 1$. The reason being that the current state of the economy is projected (probabilistically) into the future.

The second result is that $p < 1$ also amplifies output gaps. To see how, let the zero-gaps equilibrium be disturbed by $u_{yt} < 0$. Then let agents assign nonzero probability to switching to the depression regime, i.e. $p < 1$. Since $\partial \theta / \partial p < 0$, lower confidence in the normal regime acts as an amplifier also of the output gap. The reason is that the likelihood of ending up in a depression regime lowers inflation expectations. As a consequence, given the initial $i_t$ and $r^*$, the market real interest rate is increased further, widening the gap. On the other hand, to the extent that the central bank is able to stimulate output by adjusting the policy rate, the gradient of recovery is also amplified. This is good news, since the instrument is more effective when it is most needed, provided that the interest rate "falls faster" than inflation expectations (see Woodford, 2003, p.126 on self-fulfilling inflations and deflations). What if the central bank follows the TR (3)?

In this case, we re-obtain the two-equation system of policy control (4)-(5) with the OG and the TR modified by the degree of confidence $p$:

\[ y_t = (-\alpha \hat{i}_t + u_{yt})\theta \]
\[ \hat{i}_t = (\omega \gamma_{\pi} + \gamma_i ) y_t \]

The zero-gaps equilibrium $(\hat{y}, \hat{i}) = 0$ is still a solution. Its stability requires that $|\alpha \theta (\omega \gamma_{\pi} + \gamma_i )| < 1$, i.e. $\gamma_{\pi}$ has an upper bound, which for $p = 1$ is equal to the one found with "anchored" long-run expectations. Less than full confidence makes a difference in that the upper bound is stricter. Therefore, in order to regain agents' confidence the central bank on the one hand should let the policy rate "fall faster" than expectations, but on the other it should not overshoot too much. In other words, less than full credibility restricts the corridor of stability of the policy parameters.

There is, however, a third result to be discussed. It concerns the sign of the OG equation, and leads to the Neo-Fisherian view. Conventional monetary policy hinges on the negative relationship between the output/inflation gaps and the interest-rate gap: in order to rebalance a negative output/inflation gap, cut the policy rate (create a negative interest-rate gap or reduce a positive one). Yet now this relationship also depends, not only on the magnitude, but also on the sign of $\theta$. The conventional negative relationship holds if $\theta > 0$. It can be seen that this requires $\beta p^2 + (1 - \beta + \alpha \kappa) p - \alpha \kappa > 0$. This expression is equal to zero for two values
of \(p\): both are certainly real, and one is certainly negative. Yet one may be positive,\(^9\) call it \(p^*\). This implies that the conventional sign obtains only if \(p > p^*\). As a consequence, if agents attach *particular low confidence* \((p < p^*)\) to the return to the normal regime, then the relationship among the interest-rate gap, output gap and inflation gap will be inverted. This may be called a "Neo-Fisherian depression", i.e. a state of particular depression such that in order to restore output and inflation it is necessary to *raise the policy rate*, or generate a *positive interest-rate gap*. A straightforward proof is provided by the limit case of confidence falling to zero. In fact, as \(p \to 0\), the output and inflation gap relationships with the interest-rate gap become

\[
y_t = \frac{1 - \beta}{\kappa} \hat{z}_t \\
\pi_t = \hat{i}_t
\]

where the latter confirms the Neo-Fisherian claim that in order to raise inflation the interest rate (gap) should be raised one-to-one.\(^{10}\)

### 4 Rational confidence

So far we have treated the state of agents' confidence in the normal regime of the economy as an exogenous probabilistic belief, showing the consequences of states of less than full confidence. We now want agents' beliefs to be (procedurally) rational, i.e. elaborated in accordance with the actual functioning of the economy and with a viable inference mechanism (see section 1).

We posit that the probability \(p_t\) assigned to the economy being in the normal regime is elicited *vis-à-vis* a (set of) state variable(s) \(z_t\) available to agents in \(t\), such that \(z_t = 0\) when the economy is in the zero-gaps equilibrium and \(z_t \neq 0\) otherwise. The specification of \(z_t\) should reflect agents' understanding of the variables relevant to expectations formation. The natural candidate in our model is the output gap. A general format of the input variable may be \(z_t = \mu(y_t, y_{t-1}, \ldots)\).

A consistent mapping \(\psi\) from \(z_t\) to \(p_t\) should display the following properties,

\[\psi(0) = 1, \quad \psi(z) < 0, \quad \lim_{\hat{z}_t \to \pm\infty} \psi = 0\]

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\(^9\) If \([\beta^2 - 2(1 - \alpha\kappa)\beta + (1 + \alpha\kappa)]^{1/2} + \beta - \alpha\kappa > 1\)

\(^{10}\) Yet, as soon as confidence is re-established, the economy switches back to the normal regime.
That is to say, confidence is maximal as long as the economy is at the zero-gaps equilibrium, it falls and tends to zero as the deviation (either positive or negative) from equilibrium grows larger. We may call this the "confidence function" (CF).

A suitable format is provided by logistic functions, which have wide applications in inference problems in order to transform observed variables into probabilistic assessments of the occurrence of an event. We adopt the following specification, which has in fact the required properties:

\[ p_t = \frac{4e^{\eta z_t}}{(1 + e^{\eta z_t})^2} \]

The choice of the parameter \( \eta \) and different specifications of \( z_t \) enable us to capture different scenarios regarding changes in agents' confidence in the normal regime vis-à-vis changes in the state of the economy, while keeping the model manageable. The parameter \( \eta \) regulates the gradient of the function, i.e. the reactivity of \( p_t \) in response to any observed \( z_t \neq 0 \). Figure 1 depicts the function for increasing values of \( \eta \). Thus, high \( \eta \) may be appropriate when confidence is volatile possibly as a consequence of lack of reputation of the central bank, whereas high reputation of the central bank may be reflected in low \( \eta \).

Since, as a consequence of the CF, inflation expectations deviate from the central bank's target, this mechanism rationalizes the notion of "deanchoring" of expectations in terms of excess sensitivity of long-run inflation expectations to short-run states of the economy (Bernanke, 2007; Gürkaynak et al., 2010; Buono and Formai, 2016; Carvalho et al., 2017; Fracasso and Probo, 2017; Gobbi et al., 2019). Analytically, key to the destiny of the economy is the interaction between the OG and the CF via TR. Let us assume the following sequence:

\[
\begin{align*}
time t: & \quad \text{output/inflation gap} \rightarrow \text{agents: revision of } z \rightarrow \text{revision of } p \\
& \quad \text{central bank: revision of } i \rightarrow \\
\text{time } t+1: & \quad \text{output/inflation gap} \rightarrow \text{.............}
\end{align*}
\]

The system of policy control is now the following:

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11 The most popular application to binary exclusive events as in our case is the so-called logit model, where \( z_t \) is a linear combination of observed variables

12 See Gobbi et al. (2019).

13 It plays a role analogous to the gradient of recursive revisions of estimated parameters in learning models of the data generation process (Evans and Honkapoja, 2001; Evans and McGough, 2018b).
\[ y_t = (-\alpha \check{\gamma}_{t-1} + u_{yt})\theta_{t-1} \]
\[ \check{\hat{\gamma}}_t = (\omega y_\gamma + \gamma)y_t \]
\[ p_t = \frac{4e^{\eta_{z_t}}}{(1 + e^{\eta_{z_t}})^2} \]

Four are the questions to be addressed. Does a steady-state exist with zero gaps and full confidence? Is it stable after a shock? Can the system settle down in a depression steady state (negative output/inflation gaps and less than full confidence)? Can the system fall into a Neo-Fisherian depression?

The first question has a positive answer: \((\bar{y} = 0, \check{\hat{\gamma}} = 0, \check{\bar{p}} = 1)\) is always a solution to the system. Substitution of the TR and the CF into the OG yields a single nonlinear dynamic equation of \(y\). The order of the equation depends on the specification of \(z_t\). Let us first consider the simplest case \(z_t \equiv y_t\), so that we deal with a first-order equation. Hence stability requires that, in the neighbourhood of the steady state, \(| \frac{\partial y_t}{\partial y_{t-1}} |_{y_t < 0} < 1\). This obtains if \(|\alpha(\kappa y_\gamma + \gamma)| < 1\). Indeed, this is exactly the same stability condition that we found for the system with exogenous \(p\), when \(p = 1\).

If \(z_t\) is also a function of lagged values of output gaps, it is not possible to give definite answers. Yet, \(z_t\) can be regarded as an additional independent equation, and to the extent that lagged output gaps enter linearly with weights lower than unity (e.g. a moving average), one may expect the local (if not global) properties of system not to be overturned.

As to the third question, a depression steady state is characterized by \(\bar{y} < 0, \check{\hat{\gamma}} = 0, \check{\bar{p}} < 1\). If such a state exists, it is a fixed point between the CF and the OG functions. Hence this state ensures self-consistency of beliefs: if agents observing \(\bar{y}\) assign only probability \(\check{\bar{p}}\) to the normal regime, then \(\bar{y}\) is in fact the output gap at which the economy settles down. Confidence below unity (but above zero) means that agents have no further evidence in favour either of the return to the normal regime or of wider deviation from the current steady state. As previously seen, depression states may be New Keynesian, with conventional policy effects, if \(\check{\bar{p}} > p^*\), or Neo-Fisherian, with inverted policy effects, if \(\check{\bar{p}} < p^*\).

The expression of \(p^*\) is:
\[ p^* = \left\{ \left[ \beta^2 - 2(1 - \alpha \kappa) \beta + (1 + \alpha \kappa)^2 \right]^{1/2} - (1 - \beta + \alpha) \right\} / 2 \beta \]
For \(\beta \in [0, 1]\), \(p^*\) is increasing in \(\alpha\), the elasticity of expenditure to the interest-rate gap, and \(\kappa\), the price flexibility parameter (in the Calvo sense). Higher \(p^*\) means that the economy enters Neo-Fisherian depressions for smaller falls in
agents' confidence, or that conventional monetary policy faces a narrower corridor of stability.\footnote{High price flexibility is a condition for Neo-Fisherian monetary policy also in the model by Garin et al. (2018), and its limiting role on interest-rate policy is an intuition that can be traced back to Knut Wicksell (1898) (see Leijonhufvud, 1981; Boianovsky and Trautwein, 2004; Mazzocchi et al., 2014).}

We have established the existence of Neo-Fisherian depressions as a theoretical possibility arising from the interplay among the factors that govern the dynamic behaviour of the economy: agents' time discount rate $\beta$, the expenditure elasticity to interest-rate gaps $\alpha$, the degree of price flexibility $\kappa$, the output and inflation parameters $\gamma_y$ and $\gamma_\pi$ in the TR, and the CF driving agents' confidence in the normal regime \textit{vis-à-vis} the state of the economy. Therefore, our conclusion is that the likelihood of Neo-Fisherian depressions in reality is an empirical matter. What can we say in light of the existing consensus evidence on the relevant parameters?

5 Empirical simulations

In order to answer the above question we run empirical simulations of the system (S2). Firstly, we chose the following set of baseline values of the parameters:

\[ \alpha = 0.3, \ r^* = 2\%, \ \beta = 0.98, \ \kappa = 0.3, \ \gamma_y = 0.5, \ \gamma_\pi = 1.5, \ \eta = 1 \]

References to the empirical literature are provided in Appendix 1.

As to the specification of $z_t$, the current output gap may arguably be too raw information in an economy where output can actually fluctuate around the zero-gaps equilibrium. The information our agents seek to extract is whether output is \textit{trending away} from the zero-gaps equilibrium. Hence, a more suitable hypothesis is that they collect a longer series of output gaps and process them by means of some smoothing technique. We opted for a four-period moving average of output gaps $y_t, \ldots, y_{t-3}$. With virtual time set in quarters, this formulation, though simple, corresponds to quite common practice in detecting trends, and it smooths the impact of earlier observations after equilibrium. Starting from zero-gaps equilibrium, this working hypothesis adds some slack in the updating of confidence and avoids overreaction at the early stage of the process.

Secondly, we envisaged a range of possible once-and-for-all output shocks hitting the system a time $t=0$, up to $u_{y0} = -5\%$. In quarterly virtual time, the upper tail of this range includes large and extra-large shocks.
We present our results in two parts. The first one shows the simulations of the baseline system in the range of shocks in discrete unit steps. This will allow better understanding of the properties of the system. In the second part, the simulations map the long-run output-gap values in the joint space of the full range of shocks $u_{y0} \in [0, -5]$ and of three selected parameters: $\eta \in [0, 3]$, $\kappa \in [0, 1]$, $\gamma_\pi \in [0, 4]$ (for the selection of these intervals see Appendix 1). These simulations provide a view of the global properties of the system.

5.1 The baseline system.

The baseline system displays different long-run states depending on the magnitude of the shock. Recall that the system enters the Neo-Fisherian depressions as $p_t < p^*$, which here has value 0.256, i.e. a loss of about 75% of confidence – at first glance, quite a dramatic case. Figure 2 presents the simulation results as the phase diagram of the co-evolution of confidence $p_t$ and the output gap $y_t$.

The path generated by the 1% shock is one of stability. The system converges (quickly) to the zero-gaps equilibrium as expected under the guidance of conventional monetary policy. In particular, the initial shock to $y$ triggers a small fall of $p_t (0.94)$. As the policy rate is reduced $y$ improves; the recovery of $p$ lags behind owing to the agents' use of the moving average of $y$, but eventually the shock is fully absorbed and confidence is recovered. This example clarifies the notion that the policy rate should "fall faster" than expectations.

By contrast, the 2% shock exemplifies what may happen if the the policy rate does not fall fast enough. The stimulus to $y$ is not sufficient to restore $p$ at a sufficient rate. The flat left tail of the diagram represents a New Keynesian depression. That is to say, a ZLB steady state characterized by $(\bar{y} = -1.7\%, \bar{\pi} = -1.0\%, \bar{\ell} = 0, \bar{\rho} = 0.592 > p^*)$. Conventional monetary policy is crippled, but the conventional sign of the OG still holds, so that raising the policy rate would be counterproductive.

The 3% and 4% shocks generate cases of Neo-Fisherian depressions, that is steady states where $(\bar{y} < 0, \bar{\pi} < 0, \bar{\ell} = 0, \bar{\rho} < p^*)$. Apart from the larger losses of output, the key difference with respect to the New Keynesian depressions is that the fall of confidence is such that raising the policy rate has now a positive effect on output.

Finally, larger shocks (not reported in the Figure) generate global instability, i.e., the output gap tends to deviate further away from the initial shock.
To the extent that our baseline system replicates an economy with not too large volatility of confidence, we may say that the occurrence of Neo-Fisherian depressions, with the necessity to invert the sign of monetary policy appear rather unlikely, that is for quite large shocks to output and dramatic fall in agents' confidence in the return to the normal regime. Even when Neo-Fisherian conditions occur, though, our model does not support the prescription that the central bank should simply peg the policy rate to its Fisherian equilibrium value. The reason is similar to the one put forward by Evans and McGough (2018b, sec. 3), that is the extent to which expectations are adjusted upon the implementation of the interest-rate peg. To the extent that \( \bar{p} < p^* \), inflation expectations do recover and so does the output gap, but this remains a once-and-for-all effect which may or may not set the economy back to the zero-gaps equilibrium (or it may even overshoot).

Consider the case of 3% shock in Figure 1, ending in a Neo-Fisherian depression where \( \bar{y} = -2.8\% \), \( \pi = -3.8\% \), \( \bar{t} = 0 \), \( \bar{p} = 0.217 < p^* \). Now let the central bank announce that the policy rate is pegged to its Fisherian equilibrium value, namely 2%, with an increase of 0.50% on a quarterly basis. Under the given depressed conditions, the output and inflation gaps improve to \(-1.3\%\) and \(-1.7\%\), respectively. Confidence also improves up to \( p = 0.30 \). Yet the economy remains far from full recovery. As a condition for convergence, a feedback rule of the policy rate is still needed, e.g. a Taylor Rule with inverted signs, with, however, the unpleasant caveat that as soon as \( p \) returns above \( p^* \), the rule should be switched back to the conventional signs. Overall, this promises to be a challenging exercise of stop-and-go policy engineering.

5.2 Sensitivity to selected parameters

In order to check for the sensitivity of the baseline results to different values of shocks and parameters, we present simulations where the system's long-run states are mapped in the space of the full range of shocks \( u_{y0} \in [0, -5] \) vis-à-vis each of the three selected parameters \( \eta \in [0, 3] \), \( \kappa \in [0, 1] \), \( \gamma_{\pi} \in [0, 4] \). The other parameters are kept unchanged at their baseline values. The maps, reproduced in Figures 3 to 5, are organized as follows. The long-run state of the system is gauged by the ratio of residual output gap to the initial shock after 30 iterations according to the following scale\(^{15}\):

- residual output gap < 10% (blue): stability

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\(^{15}\) To control for casual convergence/divergence of the very last observations, the classification is based on the output-gap values after the 25th observation.
• residual output gap > 10%, \( p > p^* \): New-Keynesian depression (green)
• residual output gap > 10%, \( p < p^* \): Neo-Fisherian depression (light brown)
• residual output gap > 100%: divergence: (yellow)

Note that the distinction between New-Keynesian and Neo-Fisherian depressions is only based on whether the policy rate retains its conventional effect or it is reversed. This criterion does not necessarily imply that the former depressions are less severe than the latter, though we may say that is the most common case. As a matter of fact, most frequently both types of depressions are severe in that the residual output gap is more than 50% of the initial shock.

Output shocks and the confidence function

The analytical model suggests that larger shocks and/or higher sensitivity of the CF may reduce the stabilization capacity of conventional monetary policy and increase the likelihood of New-Keynesian or eventually Neo-Fisherian depressions. This property is confirmed by our simulations as reported in Figure 3.

[Figure 3]

The Figure displays a sharp inverse stability frontier between shocks and \( \eta \). Conventional monetary policy grants stability up to shocks of about 2% provided that \( \eta \) does not exceed 1. Historical experience and empirics (see Appendix 1) suggest that this is in fact the region where advanced economies are most likely to be found. Conventional monetary policy fails due to either larger shocks or higher values of \( \eta \), or both. The former tend to push the economy into the region of New-Keynesian depressions. Confirming the results of the baseline system, the region of Neo-Fisherian depressions is relatively limited as it requires a particular set of large shocks combined with high \( \eta \). Moving further north-east in the map, the Figure shows a large region of divergence warning about the existence of unchartered waters where the system may go out of control.

Output shocks and the Phillips Curve

The slope \( \kappa \) of the PC, i.e. the degree of price flexibility, plays a twofold role. On the one hand, increasing \( \kappa \) enhances the reactivity of inflation gaps and reinforces conventional monetary policy; on the other, \( p^* \) is raised enlarging the occurrence of Neo-Fisherian depressions. This double-edged effect clearly emerges in the simulation reproduced in Figure 4.

[Figure 4]

The stability frontier drawn by \( \kappa \) vis-à-vis shocks initially widens and then shrinks. It is worth noting that the former effect, which makes the system more resilient to shocks under conventional policy, operates up to values of \( \kappa \) around 0.4
found by the econometric estimates of "steeper" PC (see Appendix 1). For higher \( \kappa \), its unfavourable effect prevails, so that shocks beyond 1% tend to impair conventional policy. Yet, Neo-Fisherian depressions seem to emerge in a relatively smaller and more remote set of very large shocks and high values of \( \kappa \).

**Output shocks and the Taylor Rule**

We have seen that one of the key factors of stability is that the policy rate "falls faster" than inflation expectations, i.e. confidence in the normal regime. Therefore, it seems reasonable to recommend a reactive inflation parameter \( \gamma_\pi \) in the TR, also in consideration of the fact that the PC may actually be rather flat. On the other hand, we have also shown that, when confidence is endogenous, the inflation parameter should also respect an upper bound for the system to be stable. Thus, working in tandem with \( \kappa \), \( \gamma_\pi \) displays in Figure 5 the same double-edged effect.

Our simulations seem lend support the New Keynesian consensus that recommends setting \( \gamma_\pi \) in the range 1.5-2, where in fact the stability region is maximal, allowing conventional policy to accommodate shocks in the order of 2%. Larger shocks up to 3% shift the system in New-Keynesian depressions, but it takes extra-large shocks to obtain Neo-Fisherian depressions.

**6 Conclusions**

We have examined the Neo-Fisherian claim that, at the ZLB of the monetary policy interest rate, in order to restore the desired inflation rate the policy rate should be raised consistently with the Fisher equation. In our view, the Fisher equation cannot be used mechanically to peg the long-run inflation expectations. It is necessary to examine how inflation expectations are formed in response to, and interact with, policy actions and the evolution of the economy.

Accordingly, we have deployed a model economy where agents' inflation expectations are based on their correct understanding of the data generation process, and on their confidence in the central bank's ability to keep inflation on target in the long run (the normal regime of the economy). Agents' confidence is not taken for granted once and for all, but is expressed as probabilistic beliefs about the economy being in the normal vs a depression regime revised according to the observed state of the economy.

Our main conclusions are that, first, endogenous confidence in the normal regime interacts with the dynamics of output and inflation in such a way that lower
confidence amplifies negative inflation and output gaps making ZLB depressions more likely. Second, conditions for the success of conventional monetary policy require combinations of shocks, reactivity of the agents' beliefs and price flexibility that are not too large. These conditions are more easily flouted than predicted by the standard models. Third, when the economy settles down in a ZLB depression a Neo-Fisherian policy is feasible only when confidence in the normal regime falls below a certain threshold level. Yet, in light of most common empirical values of the relevant factors, and for economies with a history of stability and central bank's credibility, we may say that the latter event is quite unlikely. Finally, we have seen that the expectations formation process may not support the Neo-Fisherian policy prescription of pegging the policy rate.

Our findings leave the question of how the economy can be rescued from a depression when conventional monetary policy is stuck at the ZLB open to further investigation. Whether additional monetary stimuli are needed or Neo-Fisherian conditions occur, an important point, which often seems disregarded, is that the right policy action depends on the state of agents' confidence. For this is part of the structure of the economy, and it would be a mistake to assume that in a depression agents would react to a policy action as if they had full confidence in the central bank's ability to steer the economy back on track. Deeds not words drive agents' confidence.

References


Figures

Figure 1. The confidence function with increasing values of $\eta$

Figure 2. Phase diagram of confidence in the normal regime ($p$) and output gap ($y$) for increasing output shocks
Figure 3. Map of long-run states for increasing values of output shocks and sensitivity of the confidence function ($\eta$).

Figure 4. Map of long-run states for increasing values of output shocks and slope of the Phillips Curve ($\kappa$).

Figure 5. Map of long-run states for increasing values of output shocks and inflation parameter in the Taylor Rule ($\gamma_\pi$).
Appendix 1

- $\alpha$. The elasticity of expenditure to the interest-rate gap $\alpha$ can be found in calibrations of consumers' intertemporal elasticity of substitution or in econometric estimates of the New Keynesian IS function. The former procedure is common in the Real-Business-Cycle literature, which typically converges on values between 0.5 and 1. Direct econometric estimates yield lower values between 0.2 and 0.3 (e.g. Smets and Wouters, 2003; Laubach and Williams, 2003; Garnier and Wilhelmsen, 2005). Hence we set the baseline value at $\alpha = 0.3$.

- $r^*, \beta$. According to the New Keynesian standard model, the equilibrium value of the natural rate is $r^* = 1/\beta - 1$. The consensus value $r^* = 2\%$, dating to the original specification of the TR (Taylor, 1993), yields the commonly used value of $\beta = 0.98$.

- $\kappa$. Calibration of the slope of the PC $\kappa$ in New Keynesian models yields very low values. For instance, a common order of magnitude of firms not adjusting prices in the face of shocks is around 75% (e.g. Smets and Wouters, 2003; Luk and Vines, 2015); the Calvo equation with $\beta = 0.98$ yields $\kappa = 0.09$. Direct econometric estimates of the slope of the PC equation over the last decades typically provide higher values, in the range of 0.5. However, after Blanchard et al. (2015), various works have produced evidence of "flatter" PC, with $\kappa$ falling between 0.2 and 0.3. More recent works, mostly based on European data, find a "steepening" of the PC in the aftermath of the Great Recession (e.g. Riggi and Venditti, 2014; Bank of Ireland, 2014; Oinonen and Paloviita, 2014), with the estimated slope around 0.4. Note that this finding is consistent with our hypothesis of lower confidence in the normal regime, according to which these estimates may actually be measuring $\omega \kappa$. For the baseline model we chose a mid value among these estimates, i.e. $\kappa = 0.3$, whereas for simulations in section 5.2 we set a range up to twice the largest available estimates, $\kappa \in [0, 1]$, in order to capture the effects of increasing price flexibility (according to the Calvo equation, $\kappa = 1$ results from about 60% of flexible prices).

- $\gamma_y, \gamma_\pi$. We adopted the usual benchmark of Taylor's (1993) original empirical model, $\gamma_y = 0.5, \gamma_\pi = 1.5$, and we considered the range $\gamma_\pi \in [0, 4]$ for the simulations in section 5.2.

- $\eta$. We do not have direct evidence for the reactivity $\eta$ of the CF. As already said above, the suitable empirical counterpart of our model can be seen in works seeking to detect the "deanchoring" of inflation expectations from the central bank's target by gauging their correlation with changes in output. Among them,
the closest to our model is Gürkaynak et al. (2010), who find significant reactivity of long-term inflation expectations to various macroeconomic news in US, UK and Sweden. Following news about real GDP, the estimated reactivity varies between 0.3 in Sweden and 1.8 in the US. In our model, the corresponding relationship is $E_t\pi_{t+1} = \omega(1-p)y_t$. For one point of output gap, the range of values of $\eta$ consistent with the above estimates is (approximately) between 1.8 and 3.4. As can be seen from Figure 1, such an order of magnitude can be regarded as quite reactive, possibly too reactive if the system initial state is in equilibrium (and the central bank enjoys a good reputation). Adopting a similar technique applied to the Euro Zone, Corsello et al. (2019) find a reactivity of 0.142 before 2013 increased to 0.256 afterwards; the implied values of $\eta$ are, respectively, 1.29 and 1.66.

Another indirect empirical insight into the dimension of $\eta$ can be drawn from Chung et al. (2012), who show that a wide selection of major forecast models of the US economy largely underestimated the probability of the economy hitting the ZLB in the course of 2008-12. They also show that the probability can be increased substantially, together with the goodness of forecasts of the main variables, by including parameter and latent variable uncertainty, and extending the sample up to 2010. After shocking the models at 2008:1, the highest probability they obtain for the ZLB lasting at least 1 quarter or 8 quarters is 29% and 6% respectively. But 6% is rather optimistic since the ZLB actually persisted for more than 8 quarters.

Figure A1 plots the path of $p$ (the complement to Chun et al. estimated probability) generated by our CF updated on observing the four-quarter moving average of US output gaps around the major shock of the second half of 2008, i.e. from 2007:1 to 2009:1. Three values of $\eta$ are considered 0.7, 1, 1.5.

Figure A1. Probability of return to the normal regime updated with the moving average of US quarterly output gaps, 2007:1-2008:4

Source: Our elaborations on the FRED database of US quarterly GPD
The highest probability estimated by Chung et al. is matched by the CF with $\eta = 0.7$ at the 7th quarter (2008:3), i.e. $p = 72.8\%$, with further deterioration to 41.6% in the 8th quarter at the climax of the slump. At the same time points, the CF with $\eta = 1$ yields $p = 53.9\%$ and 20.2% respectively, a better-fitting guess of the subsequent inability of the Federal Reserve to return to the normal regime quickly.

Overall, we chose $\eta = 1$ as baseline value of the CF, which seems to fit the phenomena discussed above reasonably well. For the simulations in section 5.2 we chose a range up to the highest value compatible with the estimations by Gürgüynak et al. (2010), i.e. $\eta \in [0, 3]$. 