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Trust is bound to emerge (In the repeated Trust Game)

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Abstract

This paper addresses the emergence of cooperation in asymmetric prisoners' dilemmas in which one player chooses after having observed the other player's choice (Trust Game). We use the finite automata approach with complexity costs to study the equilibria of the repeated version of this game. We show that there is a small set of automata that form the unique Closed Under Rational Behavior (CURB) set for this game. This set contains two non-strict Nash equilibria, a cooperative and a non-cooperative one. We show that the cooperative equilibrium is the only (cyclically) stable set under the so called Best Response Dynamics.

JEL Classification C70 C72.

1 Introduction

Explaining the emergence of cooperation in the repeated Prisoner's Dilemma (PD) is a major problem in several fields ranging from economics to biology and political science. There are at least two problems that any successful explanation must address. First, it must explain how cooperators can gain a foothold in a population of defectors. Second, it must explain how cooperation can remain stable. For example, what prevents a conditionally cooperative strategy such as Tit for Tat (TfT) from being invaded by an unconditionally cooperative strategy such as All Cooperation (AC)?

Binmore and Samuelson [5] (B&S henceforth) discuss these questions in a model in which strategies are represented by finite automata (or machines), in the spirit of Abreu and Rubinstein [1]. The payoff an automaton receives depends upon its own payoff during the game and its complexity as measured by the number of states. They prove an extremely strong result: the only machines that can be evolutionarily stable are those that cooperate throughout most of the interaction.

This result is based on somewhat restrictive assumptions. The literature subsequent to B&S showed that much less encouraging results would be obtained if they were relaxed. First, B&S assumed that payoffs are evaluated with the limit-of-the-mean criterion. This amounts to assuming that the time-horizon of the interaction is so long (in fact, infinite) that any loss that occurs in a finite number of rounds can be ignored. Second, complexity costs are ranked lexicographically after the game payoffs. A simpler machine will have an advantage over a more complex one only if it gets at least the same game payoffs. Volij [14] proves that if one allows for the possibility of a trade-off between these two parts of a machine's payoff, the opposite result can be obtained: Defect is the only evolutionary stable strategy.[1]

In this paper we shall deal with the case in which the stage-game is the sequential version of the PD, usually referred to as Trust Game (TG). We shall follow Volij [14] in allowing for the possibility of a trade-off between complexity costs and game payoffs. Game payoffs will be evaluated with the discount criterion, with a finite discount factor $\delta \in (0, 1)$. Our main finding is in line with Binmore and Samuelson. We show that the inefficient equilibrium in which players fail to cooperate cannot be stable under the so called Best Response Dynamics. At the same time, there is at least one stable cooperative equilibrium. Moreover, Best Response Dynamics can lead a population from the non-cooperative equilibrium to the cooperative one, but not vice-versa. In short: Trust is bound to emerge.

To gain an idea of the problems involved, consider the matrixes in Table

[1] Samuelson and Swinkels [13] present a neat discussion of this matter. They show that the difference between B&S and Volji [14] lies in the order in which two limits are taken. Volij fixes a (small) cost of complexity and takes the limit for the size of the invading population that goes to zero. B&S fix a (small) invasion barrier and consider the limit for the cost of complexity that goes to zero.

	D	C
D	P	T
C	S	R

	AD	AC	TfT
AD	P^∞	T^∞	$T+P^\infty$
AC	S^∞	R^∞	R^∞
TfT	$S+P^\infty$	R^∞	R^∞

Table 1: A Prisoner's Dilemma (left) and three strategies for the repeated version (right)

1. The players repeatedly play the PD on the left ($T > R > P > S$) and are restricted to using three strategies for the repeated game: All cooperation (AC), All Defection (AD) and Tit for Tat (TfT). The matrix on the right represents this strategy choice and the outcomes associated with each strategy profile. P^∞, T^∞ and R^∞ stand for infinite streams of P, T and R respectively. $T + P^\infty$ ($S + P^\infty$) stands for a round with payoff T (S), followed by an infinite stream of P.

Suppose for the moment that complexity costs can be ignored and repeated game payoffs are evaluated with the limit-of-the-mean criterion. This implies that P^∞ and $S + P^\infty$ are indistinguishable and therefore that TfT obtains against AD the same payoff that AD obtains against itself. At the same time, TfT obtains against itself a payoff that is strictly larger than the payoff AD obtains against TfT. It follows that AD can be invaded by TfT, and it therefore fails to be evolutionary stable. Note that when the game-payoffs are evaluated with the discount criterion, P^∞ always has a larger value than $S + P^\infty$ and hence TfT obtains against AD strictly less than AD itself. In this case, a population of AD cannot be invaded by a tiny fraction of mutants who play TfT. So the evolutionary instability of AD is crucially based on game payoffs being evaluated with the limit-of-the-mean-criterion.

Suppose now that payoffs are evaluated with the limit-of-the-mean criterion, but complexity costs enter a machine's payoff function. If they are ranked lexicographically after the game payoffs, AD can still be invaded by a negligible fraction of mutants who play TfT. The reason is that TfT mutants obtain larger game payoffs than the incumbents, no matter how small the fraction of mutants is. The larger complexity cost of TfT can be ignored, because it is ranked lexicographically after the game payoff. If the costs of complexity were allowed to directly enter into a machine's payoff function, then a fraction of mutants who play TfT would obtain a smaller payoff than AD players, unless the fraction of mutants crosses a minimum, strictly positive, threshold. This is the intuitive reason why in B&S AD cannot be evolutionarily stable, while in Volij [14] it is the only evolutionarily stable strategy.

Suppose now that the game being repeated is the Trust Game in Figure 1. The first player (the *Sender*) chooses whether to Trust (T) the second player (*Receiver*), who in turn decides whether to Reward (R) the Sender's trust or not (NR). With the assumption that $v_S < 0 < 1 < V_R$, (NT, NR) is the game's unique subgame perfect Nash equilibrium.

We shall assume that the game is played repeatedly by finite automata, and

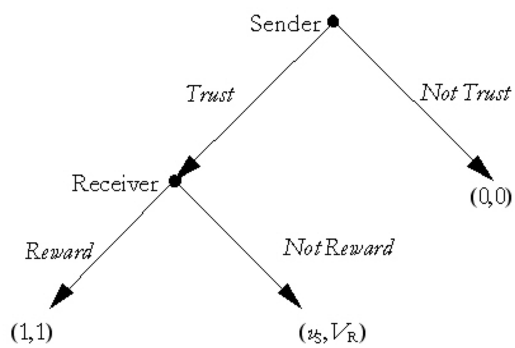

Figure 1: The Trust Game

a stream of payoffs is evaluated with a finite discount factor $\delta \in (0,1)$. The cost of complexity will be measured by the number of states of a machine, and it will enter directly into a machine's payoff function. We are thus assuming conditions that are even more severe for the emergence of cooperation than those assumed by Volij [14].

We shall now sketch what is the main difficulty that this approach must solve. Piccione and Rubinstein [11] proved that in the repeated version of *any* sequential game, the only equilibrium is a constant repetition of one of the equilibria of the stage game. In the TG this would entail that there are no other equilibria than the infinite repetition of (NT, NR).

To see the difference between the TG and the PD in this respect, consider again the matrix in Table 1 (right). No matter how complexity costs enter a machine's payoff function, TfT cannot be in Nash equilibrium with itself. The reason is that AC obtains against TfT the same payoff as TfT itself, but it is simpler. Abreu and Rubinstein's [1] proved that two machines can form a NE only if, in playing one against the other, *all* their states are visited at least once. If one state is never reached, one can always build an alternative machine which produces the same outcome against the other (and hence gets the same payoff during the game) but has a smaller number of states. When applied to the PD, this result implies that, since cooperation is sustained by the threat of some non cooperative behavior, this threat must be carried over at least once during the playing of the game. Intuitively, TfT cannot be a Nash equilibrium with itself because two TfT machines never punish each other.

Both in Abreu and Rubinstein [1] and in B&S's evolutionary model, cooperation is achieved only by machines that put the "punishing phase" first. Each machine starts by "punishing" the other machine by playing Defect for a fixed number of rounds and does not revert to cooperation unless the other machine has played Defect for the same number of rounds. Once the "punishing phase" is over, both machines start cooperating. Switching to defection during the "cooperative phase" is deterred by the threat to start the punishment phase all over again. Abreu and Rubinstein provide a nice interpretation of this initial phase of punishment as a "show of strength": each machine will "test" at the beginning of the play the ability of the other machine to punish an eventual defection. Machines that are unable to "punish" are exploited by unending defection.

Piccione and Rubinstein [11] prove that this argument fails for sequential games like the Trust Game. To see this, consider that in the TG the "punishment phase" would be a finite number of rounds in which the Sender plays NT and the Receiver plays NR. The problem is that Receiver's behavior cannot be observed when the Sender plays NT. It follows that the Receiver can eliminate the states associated with the preliminary phase (in which he plays NR) and reduce the complexity of his strategy. Without the preliminary "punishment phase", however, cooperation cannot be in equilibrium.

Piccione and Rubinstein's result is based on the assumption that players are constrained to choose pure strategies. The exclusion of mixed strategies from the analysis of repeated games played by finite automata, which goes back to Abreu and Rubinstein [1], stemmed "in part from a feeling of unease about the interpretation of mixed strategies in a number of standard economic applications" (p. 1266). A further source of difficulty is that there is no obvious way to model the complexity of a mixed strategy and the associated cost. However, in the evolutionary environment discussed by B&S and Volij [14] there is a natural interpretation of mixed strategies as polymorphisms within populations of agents. A mixed strategy Nash equilibrium can thus be interpreted as a distribution of the population among several machines. All the machines that are represented in the population would earn the same payoff, which is no smaller than the payoff that would be obtained by any machine that is not represented.

We shall show that, once the restriction to pure strategies is removed, the repeated TG admits an equilibrium (in mixed strategies) in which players' average payoffs approach the payoffs of the cooperative outcome of the stage-game as the cost of complexity approaches zero. Besides this equilibrium, there is also a compact set of equilibria in which the Sender never trusts the Receiver and the corresponding payoff is zero for both players.

To assess the stability of each equilibrium we shall use the so called Best Response Dynamics (see Hofbauer and Kuzmics [8]). Our main result is that there is a best-response path that leads from the set of non-cooperative equilibria to the cooperative mixed-strategy NE. At the same time, all paths of the Best Response Dynamics that originate sufficiently close to the cooperative mixed strategy NE will converge to it. It follows that the cooperative mixed strategy NE is socially stable in the sense introduced by Matsui [9], while the set of non-cooperative equilibria is not. This result should be contrasted with Volij [14], who proves that in the repeated PD with complexity costs only Defect is an evolutionary stable strategy, and with Piccione and Rubinstein [11], who prove that in the repeated TG there are only non-cooperative equilibria.

The paper proceeds as follows. Section 2 introduces the necessary technicalities and definitions. Section 3 contains the proof that a cooperative equilibrium exists in the repeated TG. Section 4 introduces the Best Response Dynamics and proves that the cooperative equilibrium is the only stable equilibrium. Section 5 concludes by pointing out some possible lines for further research. Longer proofs are relegated to the appendixes.

2 The model: Definitions

Two players repeatedly play the Trust Game in Figure 1. We shall indicate with $S_S = \{T, NT\}$ and $S_R = \{R, NR\}$ the pure strategy sets for the Sender and the Receiver. The set $E = \{NT, (T, NR), (T, R)\}$ is the set of outcomes of the game. (Note that since the trust game is an extended form game, E does not correspond to $S_S \times S_R$.) $h_i(e)$ is the payoff that player i obtains on reaching the end-node $e \in E$ ($i = Sender, Receiver$).

Strategies for the repeated game are represented by means of finite automata. A finite automaton, or a machine, M is a collection of states of which one is the initial one. Each state is associated to a strategy, which is the strategy the automaton plays when in that state. After each round the state of the automaton changes depending upon its current state and the outcome of the previous round.[2]

Formally, a machine for player i is a quadruple $< Q_i, q_i^0, \lambda, \mu >$ with the following characteristics.

- Q_i is a set of states
- q_i^0 is the initial state
- $\lambda_i : Q_i \to S_i$ is the output function
- $\mu_i : Q_i \times E \to Q_i$ is the transition function.

Let $\mathcal{M}_S$ and $\mathcal{M}_R$ be the set of finite automata for player S and R. Two machines (M_S, M_R) playing against each other produce a deterministic history of strategies chosen by the two players (s^t). $E(s^t)$ is the set of end nodes reached as a consequence of the history of play s^t, and $h_i(E(s^t))$ is the set of payoffs obtained by player i in this history. The payoff resulting from (M_S, M_R) for player $i = S, R$ is thus $\pi_i(M_S, M_R) = (1-\delta)\sum_{t=1}^{\infty} \delta^{t-1} h_i(E(s^t))$, where δ is the time discount factor.

Each state after the first one has a cost c, so that a machine M_i will have a cost $c \times |M_i|$, where $|M_i|$ is the number of states of machine M_i minus one. The overall payoff a machine M_i obtains in a match (M_S, M_R) is

$$\begin{aligned} \pi_i^c(M_S, M_R) &= (1-\delta)\textstyle\sum_{t=1}^{\infty} \delta^{t-1} h_i(E(s^t)) - c|M_i| \\ &= \pi_i(M_S, M_R) - c|M_i| \end{aligned}$$

[2] The transition depends upon the *outcome* of the game, not the *strategy profile*. The reason is that at the end of the game only the outcome can be observed.

We shall denote with G^{∞} the quadruple $< \mathcal{M}_S, \mathcal{M}_R, \pi_S^c, \pi_R^c >$. G^{∞} is usually referred to as the *machine game*. The game is assumed to be played simultaneously. Each player chooses a machine at the beginning of the game, and then the repeated game is played by the machines themselves. Players' payoffs are represented by π_S^c and π_R^c.

Let $\bar{\mathcal{M}}_i \subset \mathcal{M}_i$ be a finite set of machines for player $i = S, R$. We shall indicate with $\Delta(\bar{\mathcal{M}}_i)$ the set of probability distributions over $\bar{\mathcal{M}}_i$. The two players are assumed to have *beliefs* about the behavior of the other player. These beliefs will be represented by two probability distributions $p \in \Delta(\bar{\mathcal{M}}_S)$ and $q \in \Delta(\bar{\mathcal{M}}_R)$ for two set of machines $\bar{\mathcal{M}}_S$ and $\bar{\mathcal{M}}_R$. $p(M_S)$ $(q(M_R))$ is the probability with which player R (S) expects S (R) to use her machine $M_S \in \bar{\mathcal{M}}_S$ $(M_R \in \bar{\mathcal{M}}_R)$. Notice that mixed strategies are interpreted as subjective uncertainty and not as conscious randomization. This is the natural interpretation in view of the evolutionary model discussed in the next section.

In choosing his machine $M_S \in \mathcal{M}_S$ when his belief is q, player S's expected payoff is

$$\pi_S^c(M_S, q) = \sum_{M_R \in \bar{\mathcal{M}}_R} q(M_R)\, \pi_S^c(M_S, M_R) = \sum_{M_R \in \bar{\mathcal{M}}_R} q(M_R)\, \pi_S(M_S, M_R) - c|M_S|$$

Note that payoffs are only defined for pure strategies. Since players are only assumed to play pure strategies, we do not need, and do not provide, a definition for the payoff of a mixed strategy.

The best response correspondence for player S is defined as

$$BR_S(q) = \{M_S \in \mathcal{M}_S : \pi_S^c(M_S, q) \geq \pi_S^c(M_S', q) \ for\ all\ M_S' \in \mathcal{M}_S\}$$

The corresponding concepts for player R have similar definitions.

What follows is the definition of equilibrium that we shall use in proving our main result.

Definition 1 *We say that a pair of beliefs (p, q) forms a Nash equilibrium for the machine game G^{∞} if (a) $p(M_S) > 0$ implies that $M_S \in BR_S(q)$ and (b) $q(M_R) > 0$ implies that $M_R \in BR_R(p)$.*

In equilibrium, player i expects player j to use his pure strategy M_j with positive probability only if M_j is a best response to player j's beliefs.

Our second definition extends the notion of a Closed Under Rational Behavior (CURB) set proposed in Weibull and Basu [3] to the game G^{∞}. The need for this definition will become apparent in the proof of Proposition 2.

Definition 2 *Let $(\bar{\mathcal{M}}_S, \bar{\mathcal{M}}_R)$ be two finite sets of automata. We say that $(\bar{\mathcal{M}}_S, \bar{\mathcal{M}}_R)$ is a CURB set for the game G^{∞} if (a) for all $q \in \Delta(\bar{\mathcal{M}}_R)$, $M_S \in BR_S(q)$ implies that $M_S \in \bar{\mathcal{M}}_S$ and (b) for all $p \in \Delta(\bar{\mathcal{M}}_S)$, $M_R \in BR_R(p)$ implies that $M_R \in \bar{\mathcal{M}}_R$. A set is a minimal CURB set if it contains no proper subsets that are CURB sets.*

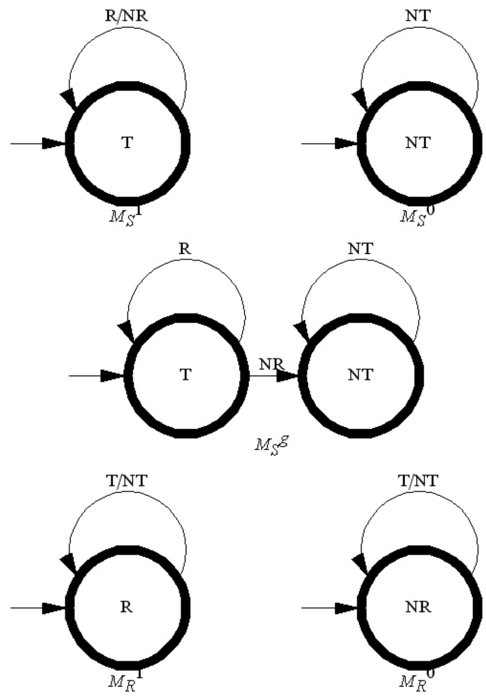

Figure 2: The unique minimum CURB set of the machine game.

Intuitively, a set of machines $(\bar{\mathcal{M}}_S, \bar{\mathcal{M}}_R)$ is a CURB set if, when player i expects player j to choose with positive probability only strategies in $\bar{M}_j$ he will only choose a strategy within $\bar{M}_i$. As for the notion of Nash equilibrium, the notion of CURB set does not require a definition of the complexity of a mixed strategy, because mixed strategies only appear as player's beliefs.

3 Results

This section introduces the first result of the paper: that is, the existence of a cooperative mixed strategy NE for the repeated TG. We shall prove this result as a corollary of a somewhat stronger proposition. We shall in fact prove first that any machine game G^∞ has a small minimal CURB set, and that there is one cooperative equilibrium within that set.

Figure 2 contains all the automata that form the unique minimal CURB set of game G^∞. These are: all the one-state machines for both players (M_S^0, M_S^1 for the Sender and M_R^0, M_R^1 for the Receiver) plus a two-states machine for the sender M_S^g. The latter machine implements the "grim strategy": it plays T in

	M_R^0	M_R^1
M_S^0	$0,0$	$0,0$
M_S^1	v_S, V_R	$1,1$
M_S^g	$v_S(1-\delta)-c, V_R(1-\delta)$	$1-c,1$

Table 2: A simplified version of the machine game

the first round and keeps playing T as long as the other player has played R.[3] It reverts to a constant play of NT after the first round in which the other player has played NR.

These strategies yield very simple patterns when matched against each other. M_S^0 produces a stream of NT independently of the strategy with which it is matched. (M_S^1, M_R^1) and (M_S^g, M_R^1) produce an uninterrupted stream of (T,R). (M_S^1, M_R^0) produces a continuous stream of (T,NR). (M_S^g, M_R^0) produces (T,NR) in the first round, followed by a stream of NT.

Let $\mathcal{S}_S$= $\{M_S^0, M_S^1, M_S^\infty\}$,$\mathcal{S}_R$= $\{M_R^0, M_R^\infty\}$ and $\mathcal{S} = \{\mathcal{S}_R, \mathcal{S}_S\}$. We shall indicate with $G^{\mathcal{S}}$ the game in which player's choices are restricted to the set $\mathcal{S}$. Table 2 represents $G^{\mathcal{S}}$.

All proofs are based on the following assumption:

Assumption 1 *i)* $\delta \geq \delta_{crit} := \frac{V_R-1}{V_R}$

ii) $c \leq c_{crit} := -\frac{\delta\ v_S}{(1-v_S)}$

δ_{crit} is the threshold value of δ such that when $\delta \geq \delta_{crit}$ (M_S^g, M_R^1) is a Nash equilibrium in the machine game without complexity costs ($c = 0$). This is the familiar condition that players must be sufficiently patient for cooperation to be a Nash equilibrium in a repeated game The second condition imposes that complexity costs are sufficiently small with respect to δ and v_S.

Proposition 1 *Let Assumption 1 hold. Then the set $\mathcal{S}$ is the unique minimal CURB set for G^∞.*

The proof is in the Appendix; here I shall only present a sketch of the proof that $\mathcal{S}$ is in fact a minimal CURB set. First note that against M_S^0 all Receiver's machines obtain the same payoff (zero). Hence, it suffices to consider what any alternative machine can obtain against M_S^1 and M_S^g. The key of the proof is that against M_S^1 no machine can do better than M_R^0, because M_R^0 exploits M_S^1 with the minimum number of states. Against M_S^g no machine that always plays R can do better than M_R^1.

On the other hand, if a machine $\bar{M}_R$ that plays NR for the first time in round $n > 1$ obtains a larger payoff than M_R^1 against the mixed strategy chosen by the Sender, then M_R^0 (which plays NR at the first round) will obtain an even

[3] For the sake of a simple notation, we use the same letters for strategies and outcomes, when this does not create confusion. So in Figure 2 a letter R stands for the outcome (T,R), and NR stands for (T,NR) and so on.

larger payoff, because it would obtain a larger payoff with a smaller number of states.

Similarly, any machine $\bar{M}_S$ with $|M_S| > 0$ that never plays Trust is strictly dominated by M_S^0, because it contains a larger number of states and obtains the same repeated game payoff (zero). Now consider a machine $\bar{M}_S$ that plays T for the first time in round n. The proof consists in showing that any such machine cannot obtain in the course of the repeated game a payoff which is simultaneously larger than the payoff obtained by M_S^g and M_S^1.

The following is a simple corollary of Proposition 1, which is worth stating as a separate result.

Corollary 1 *All the NE for $G^{\mathcal{S}}$ are NE also for G^{∞}.*

The following proposition characterizes the NE for $G^{\mathcal{S}}$, and, because of the previous corollary, of G^{∞}.

Proposition 2 *The game $G^{\mathcal{S}}$ has a connected component of NE in which the Sender chooses M_S^0 with probability one and the Receiver chooses M_R^0 with a probability $q^0 \geq \min(\frac{1}{1-v_S}, \frac{1-c}{1-v_S(1-\delta)}) := \tilde{q}_0$. We shall refer to this set of NE as $\mathcal{N}$. If Assumption 1 is met, this game has also a mixed strategy NE (p^*, q^*) where $p^* = (0, \frac{V_R(\delta\ -1)}{\delta\ V_R}, \frac{V_R-1}{\delta\ V_R})$ and $q^* = (-\frac{c}{v_S\ \delta}, \frac{v_S\ \delta+c}{v_S\ \delta})$. The payoffs that the two players receive in the equilibrium (p^*, q^*) converge to $(1, 1)$ as $c \to 0$.*

Proof. When the Sender's conjecture is $(q_0, 1-q_0)$ (where q_0 is the probability with which he expects M_R^0 to be played) the payoff in playing M_S^1 is $v_S q_0 + (1-q_0)$, while the payoff in playing M_S^g is $(v_S(1-\delta)-c)q_0 + (1-c)(1-q_0)$. M_S^0 is thus a best reply provided that both these magnitudes are non positive, which requires $q_0 \geq \frac{1}{1-v_S}$ and $q_0 \geq \frac{1-c}{1-v_S(1-\delta)}$. If $\delta < \delta_{crit}$, M_R^0 is a weakly dominant strategy for the Receiver, and therefore there are no other NE in the game. If $\delta \geq \delta_{crit}$, if the Receiver's conjecture is p^*, he is indifferent between M_R^0 and M_R^1. If the Sender's conjecture is q^*, she is indifferent between M_S^0 and M_S^1. The Sender's payoff in playing these two strategies is equal to $\frac{c-c\ v_S+\delta\ v_S}{\delta\ v_S}$. This magnitude must be non negative, because otherwise M_S^0 would be a best response. This requires that $c \leq c_{crit} = -\frac{\delta\ v_S}{(1-v_S)}$. Finally, the Receiver's payoff in equilibrium is constant and equal to one. As $c \to 0$, $q^* \to (0, 1)$. Since the equilibrium payoff for M_S^1 is $q_0^* v_S + q_1^*$, as $q_1^* \to 1$ this payoff converges to 1 as well. ■

There is a clear intuition behind both (sets of) NE in Proposition 2. First, there is a set of NE in which the Sender chooses the non-trusting machine M_S^0 because she expects, with a sufficiently high probability, the Receiver to choose the non rewarding machine M_R^0. These are not *strict* NE, though, because when the Sender chooses M_S^0 the Receiver is in fact indifferent between M_R^0 and M_R^1.

The second NE is slightly more complex. The Sender expects the Receiver to always play Reward (M_R^1) or always play Not Reward (M_R^0) and the probability he puts on these two strategies are such that he gets the same payoff by choosing the unconditionally trustful machine M_S^1, and the grim machine M_S^g. M_S^g is more complex than M_S^1. However, it obtains a larger payoff against M_R^0 because M_S^g quits trusting after the first time that the Receiver has played NR. So, while M_S^1 yields higher payoffs when the Receiver plays M_R^1 with sufficiently high probability, M_S^g becomes the best reply when the non rewarding machine M_R^1 is expected with a larger probability. The trick of the proof is that when the cost of an extra state is sufficiently low (i.e. when $c < c_{crit}$) it pays to have an extra state to discriminate between M_R^0 and M_R^1 rather that reverting to the simpler (but not discriminating) machine that never trusts M_S^0.

4 Learning

The previous section has shown that, contrary to the result obtained by Piccione and Rubinstein [11], the repeated TG admits a cooperative equilibrium (in mixed strategy). However, there is also a set $\mathcal{N}$ of equilibria in which there is no cooperation. In this section we shall investigate the dynamic stability of these two equilibria.

Consider the following extremely simplified model of learning.[4] There are two large (infinite) populations of agents which, with an abuse of notation, we shall denote as S (Sender) and R (Receiver). The game G^∞ is played by pairs of individuals drawn at random from S and R. Each agent in each population adopts a finite machine. The *state* of the two populations is represented by a pair $(p(t), q(t))$, where $p(t)$ and $q(t)$ are the distributions among the machines within the S and R population respectively. Let $p_{M_S}(t)$ and $q_{M_R}(t)$ be the fraction of the population S and R that use machines M_S and M_R respectively at time t. As above, let $BR_S(q(t))$ and $BR_R(p(t))$ be the set of best replies for the Sender and the Receiver respectively when the state of the two populations is $(p(t), q(t))$.

Agents in each population revise their strategies at a fixed rate. When revising her strategy, an agent will switch to one of the best replies. These hypotheses ensure that the states of the two populations evolve according to the differential inclusions

$$\begin{aligned} \dot{p}(t) &= b_S(t) - p(t) \\ \dot{q}(t) &= b_R(t) - q(t) \end{aligned} \tag{1}$$

where $b_S(t) \in BR_S(q(t))$ and $b_R(t) \in BR_R(q(t))$ for all t.[5] This is the well known Best Response Dynamics (BRD).

Let p_0, p_1 and p_g be the fractions of population S which play M_S^0, M_S^1 and M_S^g respectively and q_0 and q_1 be the fractions of R that play M_R^0 and M_R^1. With an abuse of notation we shall indicate with $\Delta(\mathcal{S})$ the set of population states in which only machines in $\mathcal{S}$ are represented.

[4] This is the continuous and deterministic counterpart of the stochastic learning model proposed by Volij [14].

[5] Note that these are not differential equations, because best replies might not be unique, so that more than one orbit can originate from the same initial condition.

Proposition 3 *The set $\Delta(\mathcal{S})$ is invariant under the BRD. That is, for any initial condition $(p(0), q(0)) \in \Delta(\mathcal{S})$, for any $t > 0$ $(p(t), q(t)) \in \Delta(\mathcal{S})$.*

Proof. This is an immediate consequence of the definition of the BRD and CURB set. ■

This proposition allows us to study the stability properties of the equilibria in the only minimal curb set of the game as if that were an independent game. In fact, if the system starts at any point of $\Delta(\mathcal{S})$, the dynamics will not take it out of $\Delta(\mathcal{S})$.

The stability concept we shall use was introduced by Matsui [9]. Intuitively, a set of states X is stable if there is no best response path that leads from any element of X to a state which is not in X. To make this precise, we need some further piece of terminology. A strategy distribution $(p, q) \in \Delta(S)$ is *directly accessible* from (p', q') if there exists a best reply path such that $(p(0), q(0)) = (p, q)$ and $(p(T), q(T)) = (p', q')$ for some $T \geq 0$. Also (p', q') is *accessible* from (p, q) if one of the following holds true: (i) (p', q') is directly accessible from (p, q); (ii) there exists a sequence (p_n, q_n) converging at (p', q') such that (p_n, q_n) is directly accessible from (p, q) for any n; (iii) if (p', q') is accessible from another (p'', q'') which is accessible from (p, q).

We need one final definition: a set of states $\mathcal{F} \subset \Delta(\mathcal{S})$ is a *cyclically stable* set (CSS) if (i) any $(p', q') \notin \mathcal{F}$ (p, q) is not accessible from any $(p, q) \in \mathcal{F}$, and (ii) any $(p, q) \in \mathcal{F}$ is accessible from any $(p', q') \in \mathcal{F}$. The idea of a CSS is that a set of states $\mathcal{F}$ is stable if the best response dynamics (1) cannot leave $\mathcal{F}$.

We are now ready to formulate our main proposition

Proposition 4 *In game G^∞, the mixed strategy NE (p^*, q^*) is cyclically stable. The set of NE $\mathcal{N}$ is not cyclically stable.*

The proof is in the Appendix. Here we shall only provide a graphical illustration. Consider Figure 3. This represents the state space $\Delta(\mathcal{S})$, with the two (sets of) Nash equilibria (p^*, q^*) and $\mathcal{N}$. It also represents two orbits generated by the BRD. The first orbit originates from x, which lies on the face where $p_0 = 0$. and has the familiar appearance of the orbits generated in 2×2 games with a single mixed strategy NE (see for example Berger [4]). p_0 remains constantly equal to zero, while the population converges towards (p^*, q^*). The second orbit starts from $y \in \mathcal{N}$ and converges to (p^*, q^*), just like the first one. The logic of the proof is to show that from any point in $\mathcal{N}$ there is a best response path that approaches (p^*, q^*), while there are no best response paths going from (p^*, q^*) to $\mathcal{N}$. Actually, (p^*, q^*) attracts all best response paths originating in a sufficiently small neighborhood.

5 Conclusions

The results presented here show that the details of the interaction in PD situations do have a crucial impact on the chances that cooperation has of emerging.

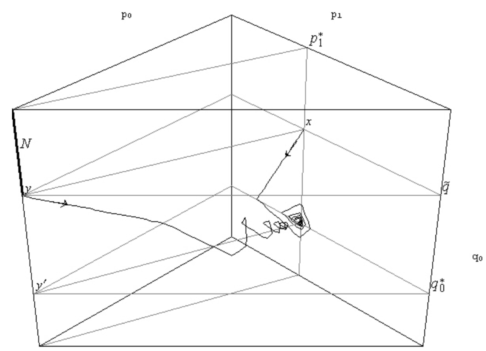

Figure 3: Orbits generated by the BRD on $\Delta(\mathcal{S})$

Volij [14] and Samuelson and Swinkels [13] prove that cooperation can only emerge if the time horizon is so long that the limit of the mean criterion can be employed, and complexity costs are so small that they can be ranked lexicographically after game payoffs. We have proven that these conclusions only hold if the game being repeated is the simultaneous PD. In the repeated TG, the non cooperative equilibrium is dynamically unstable even when payoffs are evaluated with the discount criterion, and complexity costs directly enter into a machine's payoff function. There are several ways in which this result could be extended. For example, dynamics other than the Best Response Dynamics could be used. Also, it would be interesting to analyze the case in which choices are noisy, so that players occasionally play Not Reward even when they intend to play Reward. These and many other extensions are left for future research.

6 Acknowledgments

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A Appendix

A.1 Proof of Proposition 1

We shall prove Proposition 1 with the help of three Lemmata.

Lemma 1 *The set of strategies $\mathcal{S}$ is a minimal CURB set for any G^n.*

Proof. We must show that any machine for the Sender and the Receiver $M_i \notin \mathcal{S}$ $(i = S, R)$ yields, against any element of $\Delta(\mathcal{S})$, a payoff which is strictly smaller than the payoff offered by at least one element of $\mathcal{S}$. First consider any machine for the sender $\bar{M}_S \notin \mathcal{S}$. $\bar{M}_S$ has at least two states, because otherwise it would belong to $\mathcal{S}$. If it never plays T, its payoff is $0 - c|\bar{M}_S| < 0$ $(c \geq 1)$ and hence it is strictly dominated by M_S^0, whose payoff is zero. Suppose thus that $\bar{M}_S$ plays T at the beginning of the game (there is no loss of generality in this assumption, because none of the machines in $\mathcal{S}_R$ behave differently depending on the round in which the first T takes place). Its payoff against M_R^0 is thus bounded above by $v_S(1-\delta) - c|\bar{M}_S|$. Its payoff against M_R^1 is bounded above by $1 - c|\bar{M}_S|$. As a consequence, $\bar{M}_S$ is strictly dominated by M_S^g whenever $|\bar{M}_S| > 1$. If $|\bar{M}_S| = 1$, $\bar{M}_S$ is a two-states machine whose initial state is T and the other is NT. (If both states were T, $\bar{M}_S$ would be dominated by M_S^1, because it would always play T but it would have two states rather than one). It is a tedious exercise to show that all two states machines in which the fist state is T are strictly dominated by M_S^g against any probability distribution involving only M_R^0 and M_R^1.

Now consider an alternative machine for the Receiver $\bar{M}_R \notin \mathcal{S}_R$. The analysis is simplified by the fact that all Receiver's machines obtain the same payoff (zero) against M_S^0, and hence it suffices to consider what $\bar{M}_R$ obtains against M_S^1 and M_S^g. $\bar{M}_R$ has at least two states. If the initial state is NR, it is dominated by M_R^0. To see this, consider that against M_S^1 any machine obtains at most $V_R - c|\bar{M}_R| < V_R$, while against M_S^g a machine that plays NR in the first round gets $V_R(1-\delta) - c|\bar{M}_R|$. Suppose then that $\bar{M}_R$ has R as initial state. Neither M_S^1 nor M_S^g play NT, unless the Receiver has played NR once. If $\bar{M}_R$ always plays R after any T, $\bar{M}_R$ is strictly dominated by M_R^1. The reason is that it obtains a constant stream of (T, R) against both M_S^1 and M_S^g, and it has at least two states. So $\bar{M}_R$ must have at least one state in which it plays NR, which is reached after a sequence of Ts. After it has played NR the first time, the best that M_R can do is keep playing NR, for M_S^g will not play Trust any longer, while M_S^1 will continue to play T. As a consequence, M_R obtains the same payoff as M_R^0, beginning at round n. Before that, it obtains a stream of 1. Let $\pi^0(p)$ and $\pi^1(p) = 1$ be the payoff obtained by M_R^0 and M_R^1 resp. when the Sender plays the mixed strategy p. If the Receiver uses $\bar{M}_R$ he obtains: $\bar{\pi}(p) = 1 - \delta^n + \delta^n \pi^0(p) - c|\bar{M}_R|$. Clearly, if $\pi^0(p) \geq \pi^1(p) = 1$, then $\pi^0(p) > \bar{\pi}(p)$, so that $\bar{M}_R$ cannot be a best response. If $\pi^0(p) \leq \pi^1(p) = 1$, then $\pi^1(p) = 1 > \bar{\pi}(p)$, and again $\bar{M}_R$ cannot be a best response. ■

Lemma 2 *Let M_S and M_R be two machines for the Sender and the Receiver resp. such that $|M_i| \geq 2$. If $M_R \in BR_R(M_S)$, then $|M_R| < |M_S|$, while if $M_S \in BR_S(M_R)$, then $|M_R| \geq |M_S|$*

Proof. The second part of the lemma is just a consequence of Piccione and Rubinstein [11] Lemma 1. To prove the first part we have to show that the best reply for the Receiver to any machine M_S of the Sender contains a strictly smaller number of states. Consider any machine $M_S =< Q_S, q_S^1, \lambda_S, \mu_S >$ which has at least two states. Let $b_R : Q_S \rightarrow S_R$ be the policy function that maximizes the Receiver's payoff stream against M_S. Now consider a machine for the receiver $\bar{M}_R$ defined as follows $\bar{M}_R =< Q_S, q_S^1, \lambda_R, \mu_R >$, where $\lambda_R(q_S) = b_R(q_S)$ and $\mu_R(q_S, .) = \mu_S(q_S, E(\lambda_S(q_S), \lambda_R(q_S)))$. This machine has the same set of states of M_S and implements the optimal policy, so that it maximizes the Receiver's repeated game payoffs. We now show that it is possible to construct an alternative machine for the Receiver $\hat{M}_R$ that behaves like $\bar{M}_R$ against M_S (and hence obtains the same payoff), but has a smaller number of states. Consider first a match $(M_S, \bar{M}_R)$. If one state $\hat{q}_S \in Q_S$ is not reached in a match $(M_S, \bar{M}_R)$, one can obtain $\hat{M}_R$ by replacing Q_S with $Q_S - \hat{q}_S$. $\hat{M}_R$ obtains the same payoff as $\bar{M}_R$ and contains a smaller number of states. Suppose then that all states in Q_S are reached. There must be at least one succession of states $\hat{Q}_S = \{\hat{q}_S^1, .., \hat{q}_S^k\}$ (with $k \geq 1$), such that $\mu_S(\hat{q}_S^t, NT) = \hat{q}_S^{t+1}$, and $\mu_S(\hat{q}_t) = NT$. In other words, there must be a succession of (at least one) states in which the Sender plays NT. (If all states were T, the Receiver's best reply would be M_R^0). There are two possibilities. First, $\hat{q}_S^1 = q_S^1$ (the succession of NT starts at the beginning of the game) and $\mu_S(\hat{q}_S^k, NT) = \hat{q}_S^{k+1}$ with $\lambda_S(q_S^{k+1}) = T$ (after k rounds M_S enters a state in which plays T). Consider the following machine $\hat{M}_R =< \hat{Q}_R, q_R^1, \lambda_R, \mu_R >$, where $\hat{Q}_R = Q_S - \hat{Q}_S$ and $q_R^1 = \hat{q}_S^{k+1}$. This machine is obtained by $\bar{M}_R$ by selecting q_S^{k+1} ad the initial state and removing the first k states. It behaves exactly as $\bar{M}_R$ against M_S and therefore obtains the same payoff, with less states. A second possibility is that $\hat{q}_S^1 \neq q_S^1$, that is, the succession of NT does not start at the beginning of the game. Let $\bar{q}_S \in Q_S$ be the state such that $\mu_S(\bar{q}_S, E(\lambda_S(\bar{q}_S), \lambda_R(\bar{q}_S))) = \hat{q}_S^1$ and $\lambda_S(\bar{q}_S) = T$. $\bar{q}_S$ is thus the last state in which M_S plays T against $\bar{M}_R$ before entering the succession $\hat{Q}_S$ of states in which it plays NT. Let $\underline{q}_S \in Q_S$ be the state such that $\mu_S(\hat{q}_S^k, E(\lambda_S(\hat{q}_S^k), \lambda_R(\hat{q}_S^k))) = \underline{q}_S$. So $\underline{q}_S$ is the state M_S enters at the end of the succession $\hat{Q}_S$. If $\underline{q}_S \in \hat{Q}_S$, M_S never leaves $\hat{Q}_S$. In this case $\bar{M}_S$ must be modified as follows: $Q_R = Q_S - \hat{Q}_S$ and $\mu_R(\bar{q}_R, E(\lambda_S(\bar{q}_S), \lambda_R(\bar{q}_S))) = \bar{q}_R$. Finally, if $\underline{q}_S \notin \hat{Q}_S$, then $\lambda_S(\bar{q}_S) = T$, so after playing NT for k rounds, M_S plays T again. In this case $\bar{M}_R$ must be modified as follows: $\mu_R(\bar{q}_S, E(\lambda_S(\bar{q}_S), \lambda_R(\bar{q}_S))) = \underline{q}_S$. In all the cases we obtain a machine that obtains the maximum payoffs against M_S with a strictly smaller number of states. ■

The second part of this lemma is a well known result in this kind of literature: the best reply to a machine never contains more states that the machine itself. The first part of the Lemma depends upon the sequential structure of the Trust Game. This result follows the same argument presented in the Introduction to show that the "show of strength" argument do not work for sequential games.

Lemma 3 *There are no minimal CURB sets other than $\mathcal{S}$ in G^{∞}.*

Proof. By way of contradiction, assume that $\mathcal{S}'$=$\{\mathcal{S}'_R, \mathcal{S}'_S\}$ is a set of machines for the Sender and the Receiver, such that $\mathcal{S}'$ is a minimal CURB set. Of course, $\mathcal{S}' \cap \mathcal{S} = \varnothing$. (If any of the machines in $\mathcal{S}$ were also in $\mathcal{S}'$, $\mathcal{S}'$ would not be a *minimal* CURB set.) Let $M_S \in \mathcal{M}_S$, such that $|M_S| \leq |M'_S|$ for each $M'_S \in \mathcal{S}'_S$. Thus M_S is one of the machines that have the minimal number of states in $\mathcal{S}'_S$. Since $\mathcal{S}$ is a CURB set, it must be that if $M_R \in BR_R(M_S)$, then $M_R \in \mathcal{S}'_R$. Because of Lemma 2, $|M_R| < |M_S|$. Since $\mathcal{S}'$ is a CURB set, it must also be that all best replies to M_R are in $\mathcal{M}_S$. This implies that there exists a strategy $\bar{M}_S$ such that $\bar{M}_S \in BR_S(M_R)$ and $\bar{M}_S \in \mathcal{M}_S$. Because of Lemma 2, this implies that $|\bar{M}_S| \leq |M_R|$. We thus have that $|\bar{M}_S| \leq |M_R| < |M_S|$, which contradicts that $|M_S| \leq |M'_S|$ for each $M'_S \in \mathcal{M}_S$. ■

Clearly, Lemma 2 and Lemma 2 imply Proposition 1.

A.2 Proof of Proposition 4

Proof. The only two candidates for a CSS are the NE (p^*, q^*) and the set of NE $\mathcal{N}$. We prove the proposition by showing that (p^*, q^*) is a CSS and that (p^*, q^*) is accessible from any $(p, q) \in \mathcal{N}$, which of course implies that $\mathcal{N}$ cannot be a CSS. The orbits on the face spanned by p_1 and q_0 can be easily computed, provided that they do not cross the threshold $\tilde{q}_0$. In fact, as long as $q_0(t) < \tilde{q}_0$, M_S^0 is not a best reply, which guarantees that p_0 remains zero. The logic of the proof is to calculate the Poincare section of the two orbits, using the plan $\{x, y, y', (p^*, q^*)\}$, which we shall denote as H. Let $(q_0^n(x))$ and $(q_0^n(y))$ be the values taken by $q_0(t)$ when crossing the plane H starting from x and y respectively where $n = 0, 1, 2...$ denotes the successive crossings. We shall show that for any n, $(q_0^n(y)) = (q_0^n(x))$. This means that the fraction of M_R^0 players when orbits cross the plane H does not depend upon the initial fraction of M_S^0 players within the S population. Since we know that $(q_0^n(x))$ approaches q_0^* monotonically, this implies that the same will happen for $(q_0^n(y))$. Furthermore, $q_0(t)$ remains below $\tilde{q}_0$ for any $t > 0$, and hence along the best response path originating from y, $p_0(t)$ goes monotonically to zero and $p_1(t)$ and $p_g(t)$ approach p_1^* and p_g^*.

Consider any point on H. At any such point $p_1 = p_1^*(1 - p_{00})$, with $p_{00} \in [0, 1]$, and $q_0 \in [q_0^*, \tilde{q}_0]$ One of the best replies for the sender is clearly M_S^g, so that one has that when $p_1(0) = p^*(1-p_{00})$, a best reply path is $p_1(t) = p^*(1-p_{00})e^{-t}$, with $t \in [0, t_1)$. $t_1 = \log(\frac{q_0}{q_0^*})$ is the time it takes for population R to move from q_0 to q_0^*.Let p_{11} be $p_1(t_1)$. When $q_0(t) = q_0^*$, M_S^1 becomes the best reply, so that one has that $p_1(t) = 1 - (1 - p_{11})e^{-(t-t_1)}$. The orbit will cross H when

$\frac{p_1(t)}{p_1(t)+p_g(t)} = \frac{p_1(t)}{1-p_0(t)} = p_1^*$. This requires that

$$\begin{aligned}\frac{1-(1-p_{11})e^{-(t-t_1)}}{1-p_{00}e^{-t}} &= p_1^* \\ \frac{1-(1-(p^*(1-p_{00})e^{-t_1}))e^{-(t-t_1)}}{1-p_{00}e^{-t}} &= p_1^*\end{aligned}$$

Solving the last equation for t, one obtains $t_2 = \log(\frac{p_1^*-e^{t_1}}{1-p_1^*})$, which does not depend on p_{00}. This means that starting from y, p_1 will take the same time t_2 to come back to p_1^*, as when starting from x. At that time, one would have that $q_0(t_2) = \tilde{q}_0 e^{-t_2}$. With an analogous reasoning one can show that it takes the same time t_3 for the system to go back to the plane H, independently of the point (x or y) from which the orbit started. One can iterate this reasoning to show that in fact $(q_0^i(y)) = (q_0^i(y))$ for any i, and therefore that both orbits converge to (p^*, q^*). ■

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