Investment-Saving Imbalances with Endogenous Capital Stock

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Ronny Mazzocchi †

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Abstract

The current consensus in macroeconomics, or New Neoclassical Synthesis (NNS), is based on dynamically stochastic general equilibrium (DSGE) modeling with a RBC core to which nominal rigidities are added by way of imperfect competition. The strategy is to minimize the frictions that are required to reproduce both persistent real effects of monetary policy and interaction of interest and prices in a rigorous framework with intertemporal optimization, forward-looking behavior and continuously clearing markets. Unfortunately this “equilibrium” framework do not allow to discuss the effects and the relations between financial markets and real economy, which were the core of the economic crisis of 2008. This paper presents a dynamic model with endogenous capital stock whereby it is possible to assess, and hopefully clarify, some basic issues concerning the macroeconomics of saving-investment imbalances.

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1 Introduction

Modern macroeconomic models can be traced back to a revolution that began in the 1980s in response to the powerful critique authored by Robert Lucas (1976). The key issue was to build models that were specifically based on the aspects of the economy that were beyond the control of the government. All the relationships among the variables need to be ultimately grounded in fundamental features of the economy’s production, such as technology and people’s preferences, i.e., the so-called “deep parameters”\(^1\).

This revolution has led to the so-called Dynamic Stochastic General Equilibrium (DSGE) models. Dynamic refers to the forward-looking behavior of households and firms. Stochastic refers to the explicit inclusion of shocks in the analysis. General refers to the treatment of the whole economy. Finally, equilibrium refers to the constraints and objectives for households and firms that are carefully considered. Another key ingredient of these new models is rational expectations. This term means that households and firms form forecasts about the future as if they were statisticians: it does not mean that they are always right but that they statistically cannot make better forecasts given the available information.

The switch to these new macro-models led to a fierce controversy within the field in the 1980s. Users of the new models not only brought a new methodology, but they also had a surprising substantive finding to offer regarding business cycles. They argued that a large fraction of aggregate fluctuations could be understood as an efficient response to shocks that affected the entire economy (Kydland and Prescott, 1982; Nelson and Plosser, 1982; Long and Plosser, 1983). This became the core of the Real Business Cycle (RBC) theory. Most of these models rely on some form of large quarterly movements in the technological frontier (Prescott, 1986a; 1986b). Some other models consider collective shocks to workers’ willingness to work. Other contributions have large quarterly shocks to the depreciation rate in the capital stock in order to generate high asset price volatilities. The RBC framework uses this notion of shocks only as a convenient shortcut to generate the requisite levels of volatility in endogenous variables. But this assumption leads to the conclusion that most - if not all - government stabilization policies are inefficient.

Despite the formal elegance of these models, the source of disturbances together with the transmission mechanism were patently unrealistic. For a large economy like the United States or the European Union it is implausible for the fluctuations in the efficient level of aggregate output to be as large as

\(^1\)Many economists seem to read the Lucas critique as if it implies we can protect against non-invariance simply by applying microeconomic theory. This is a mistake. As pointed out by Debreu (1974), Sonnenschein (1972; 1973), Felipe and Fisher (2003) and many others, there is no reason to believe that macroeconomic aggregates should behave like microeconomic quantities.
the fluctuations in the observed level of output (Calomiris and Hanes, 1995). The rational expectations hypothesis has been attractive because it provides a simple and unified way to approach the modeling of forward-looking behavior in a wide range of settings, but it is also clearly questionable². Even the assumption of frictionless exchange made solving these models easier but also less compelling. In the real world firms change prices only infrequently (Carlton, 1986; Blinder, 1994; Hall et al., 1997), thus many macro models developed during the 1980s were centered on infrequent price and wage adjustment (Fischer, 1977; Taylor, 1980; Mankiw, 1985). These models were often called sticky prices or New Keynesian models, or NKE (Mankiw and Romer, 1991; Hargreaves-Heap, 1992), and provide a foundation for a coherent normative and positive analysis of monetary policy in face of shocks.

The divide between NKE and RBC still lives in newspaper columns but has largely vanished in the academia. Both camps have won. On the one hand the RBC won in terms of modeling methodology. On the other hand, the NKE has also won because it is generally agreed that some forms of stabilization policy are useful. Along these lines, a new consensus developed in the late 1990s and it was soon dubbed the New Neoclassical Synthesis, or NNS (Goodfriend and King, 1998; Blanchard, 2000). Like the old Neoclassical Synthesis of Hicks, Modigliani, Samuelson and Patinkin, the NNS tries to link micro- and macroeconomics, using a general equilibrium framework to model some typically Keynesian features³: the RBC part of the model explains the evolution of the potential output, while the transitory deviations from this trend are explained using the sluggish adjustment of prices and wages which were developed in the 1980s by the NKE literature. Like the RBC models, the NNS recognizes that some non-trivial fraction of aggregate fluctuations is actually efficient in nature. Differently from the RBC models, however, the NNS does not consider these fluctuations efficient and desirable and does not think that monetary policy is totally ineffective. In fact, because of the delays in the adjustment of prices and wages, the consequences of real shocks are undesirable. An active economic policy can therefore intervene to reduce these distortions.

There are various versions of the NNS (Rotemberg and Woodford, 1997; ²Among other things, the rational expectation hypothesis implies that economists have no social function and that they are the last to learn (Leijonhufvud, 1992). Moreover longstanding research (Hansen and Sargent, 2001, 2008; Sims, 2003) explores the consequences of relaxing the assumption and has proven challenging both conceptually and computationally.
³Olivier Blanchard summarizes the standard NNS approach using a metaphor: “A macroeconomic article today often follows strict, haiku-like, rules: It starts from a general equilibrium structure, in which individuals maximize the expected present value of utility, firms maximize their value and markets clear. Then, it introduces a twist, be it an imperfection or the closing of a particular set of markets, and works out the general equilibrium implication. It then performs a numerical simulation, based on calibration, showing that the model performs well. It ends with a welfare assessment” (Blanchard, 2008, p.27).
Clarida et al., 1999; Romer, 2000). One authoritative contribution is Michael Woodford’s *Interest and Prices* (Woodford, 2003), which gives a comprehensive representation of the dynamic interaction between interest rates, price level and output\(^4\). Requiring only a small number of equations and variables, the model has proved very helpful in deriving certain important principles for the conduct of monetary policy. As reflected in its title, the book pays respect to Knut Wicksell’s work and his theory of the price level\(^5\). The main aspect of Woodford’s contribution, indeed, is a rediscovery of the Wicksellian nominal interest rate in relation to the “natural” interest rate prevailing at full-employment general equilibrium as the pivot of rule-based monetary policy. As pointed out by McCallum (2010), Woodford’s model has become the bible for a generation of young scholars and it has dominated monetary economics in the last decade.

Some critics (Boianovsky and Trautwein, 2006b; Trautwein and Zouache, 2009; Mazzocchi et al., 2009) have shown that the theoretical structure of the NNS is based on a shaky relationship between the RBC model on one side and a mechanism of price setting which does not depend on excess demand on the other side. At the same time, most of the Wicksellian features that Woodford claims to replicate are notably absent. In particular, the NNS does not consider frictions in the capital market, which generate the first pillar of the Wicksell’s view, i.e. bank intermediation among savers and investors. Moreover, there is no room for information problems and the intertemporal disequilibrium which could produce the well-known dynamics of money creation, prices and nominal income, i.e. the so-called *cumulative process*. These weaknesses are not only a matter of history of thought. For instance, they do not allow to discuss the effects and the relations between financial market and the real economy which were the core of the economic crisis of 2008.

The aim of this paper is to show that there are systematic differences in terms of dealing - or not dealing - with intertemporal disequilibrium, i.e. coordination failures in the market system that have their origin in the capital markets (essentially financial markets) and cannot independently be corrected in the goods or labour markets. Investment-saving imbalances are a logical implication of any theory based on the distinction between market real interest rate and the natural rate. The consequence of a discrepancy between the two rates is that households wish to save more/less whereas

\(^4\)Lucas (2007) argued that NNS models should be reformulated to give a unified account of trends, including those in monetary aggregates. In a similar vein Papademos (2008) and Pesaran and Smith (2011).

\(^5\)In his 1898 treatise (Wicksell, 1898a), Knut Wicksell outlined a theory of price-level determination in which a key role was played by the relationship between the money rate of interest and the natural rate of interest. Likewise in Woodford’s book the gap between the actual interest rate and the natural rate represents the key channel through which central bank actions affect the economy.
firms wish to invest less/more: neither side of the market can achieve the intertemporal equilibrium position. As a result the system exhibits dynamic processes of both output and general price level that deviate from their intertemporal equilibrium values. These gaps persist as long as the one between the market real interest rate and the natural rate persists.

The paper is organized as follows. Section [2] presents some basic theoretical issues underlying the NNS and shows that this framework - both in its neo-Wicksellian and other versions - is precariously based on combinations of continuous intertemporal equilibrium under rational expectations with specific concepts of wage and price stickiness, and other assumptions. Section [3] presents a dynamic model whereby it is possible to assess, and hopefully to clarify, some basic issues concerning the macroeconomics of saving-investment imbalances. The treatment of the capital market is still only remotely connected with the full-fledged financial structure of the economy, therefore the imbalances are dealt with in a way that focuses on deviation of output and inflation from their intertemporal general equilibrium paths, where the possibilities of loan defaults and insolvencies are ignored. The modeling strategy sticks as closely as possible to the NNS model, in order to show that a basic model of capital market failures can be developed from the same underlying framework of RBC theory. This choice requires less modifications of the general equilibrium framework than the NNS setup and it helps, on one hand, to identify the limits of the DSGE modeling and, on the other hand, to explore the gains that can be obtained by moving from the NNS framework towards the realm of intertemporal disequilibrium. Section [4] discusses some dynamic properties of the model with particular attention to the speed of price adjustment, to the variability of the capital stock and to the inflation expectations. Finally section [5] concludes. Proofs are in the Appendix.

2 NNS in a Neo-Wicksellian Framework: strengths and weaknesses

Over the last decade a shift has begun away from the Walrasian price-taker models towards a world where firms may be strategic agents. The NNS approach uses the standard tools of New Classical macroeconomics (NCM): consumers, workers and firms are rational, agents maximize their objective function and markets clear. Yet the output of NNS models follows Keynesian lines: the aggregate economy has multipliers, economic fluctuations are not Pareto optimal, and finally government interventions can be effective. Even if business cycles and growth are analyzed within a single framework that is based on Walrasian principles, “this does not mean that the Keynesian goal of structural modeling of short-run aggregate dynamics has been abandoned. Instead, it is now understood how one can construct and ana-
lyze dynamic general equilibrium models that incorporate a variety of types of adjustment frictions, that allow these models to provide fairly realistic representations of both shorter-run and longer-run responses to economic disturbances (Woodford, 2009, p. 69). Imperfect competition is a key assumption in this approach. It opens new channels of influence of monetary policies but also creates the possibility that an increase in output may be welfare improving (Cooper, 2004). Imperfect competition by itself does not create monetary non-neutrality, but its combination with some other distortions can generate potential real effects (Blanchard, 2000).

Among the various version of the NNS, a prominent position is occupied by Michael Woodford’s book *Interest and Prices* (Woodford, 2003), which represents a good synthesis of New Classical and New Keynesian ideas. This work contains many references to Wicksell’s idea of a pure credit system and his proposal to eliminate inflation by adjusting nominal interest rates to changes in the price level. Since the output and the inflation dynamics are generated by gaps between the natural rate of interest and the market rate, a central bank that controls the latter can close the gap using an appropriate monetary policy. Up to 2008 this contention has become the generally accepted rationale for monetary policy around the world and most monetary authorities use interest rate control to achieve price stability rather than the quantity of money. Such a rules-based approach to policy is discussed at length in the first part of Woodford’s book and taken up again at its end. However, Woodford doubts that the original "Wicksellian theory can provide a basis for the kind of quantitative analysis in which a modern central bank must engage" (2003, p. 5-6) because it does not conform to modern standards of conceptual rigor, i.e. intertemporal general-equilibrium theory. His book seeks to remedy this shortcoming.

The attention of readers and economists has focused primarily on the basic version of the model (2003, ch. 4). It combines an IS relation in terms of the first-order condition of intertemporal utility maximization with a New Keynesian Phillips curve, based on imperfect competition and price rigidities, and a Taylor rule as reaction function of monetary policy. Woodford points out that his model "abstracts from the effects of variations in private spending (including those classified as investment expenditure in the national accounts) upon the economy’s productive capacity", therefore the model should be interpreted "as if all forms of private expenditure were like nondurable consumer purchases" (2003, p. 242-243). The preference for a model without endogenous capital stock is often justified on the grounds

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6 The NNS models use mainly monopolistic competition as a form of imperfect competition (Blanchard and Kiyotaki, 1987). This choice derives mainly from the belief that monopolistic competition is pervasive in a modern economy.

7 For a more complete survey see Boianovsky and Trautwein (2006a) and Mazzocchi et al. (2009).

8 The same approach is used by Jeanne (1998).
that capital does not exhibit substantial volatility at business cycle frequencies (McCallum and Nelson, 1997). Moreover, sticky price models with endogenous investment imply unrealistically high volatility in the endogenous variables. In other words, changes in nominal interest rates translate one to one into changes in real rates, therefore leading to excessively high volatility of investment. However, neglecting the endogenous determination of investment eliminates one of the main benefits of the DSGE approach prompted by Kydland and Prescott (1982), namely that it is inherently intertemporal in nature and incorporates the supply side of the economy. Indeed, as King and Rebelo argue, "the process of investment and capital accumulation can be very important for how the economy responds to shock" (King and Rebelo, 2000, p.6). Thus modeling investment demand might help explain some empirical regularities which would be hard to capture if consumption were the only component of aggregate demand. Last but not least, it should be remembered that the intertemporal coordination problem between future consumption (saving) and future production (investment) is the key problem to be solved by the interest rate in general equilibrium theory. Ignoring investment dynamics would leave the sole intratemporal coordination problem between current aggregate demand and supply at each date that is dealt with by the spot price system. For these reasons Woodford (2003, p. 352-378; 2004) - but also other authors (Casaeres and McCallum, 2000; Sveen and Weinke, 2003; 2004) - extend the basic model to include fixed capital and the effects of the related investment dynamic. The purpose of these extensions is not to offer a fully realistic quantitative model of the monetary transmission mechanism, but rather to provide insights regarding several modeling techniques that are used in a number of recent examples of estimated models with optimizing foundations.

The strategy of the NNS to minimize the frictions that are required to reproduce real effects of monetary policy and an interaction between interest and prices comes at the price of some ad-hocery and other shortcomings that have been criticized by way of many papers (Boianovsky and Trautwein, 2006b; Trautwein, 2006; Trautwein and Zouache, 2009; Laidler, 2006; Tamborini, 2006; Tamborini, 2010b). Some of the ad-hocery might be refined and made redundant in future version of the NNS, but some of it are indispensable for intertemporal equilibrium modeling of the current kind (Canzoneri et al., 2004; Blanchard and Gali, 2005). Also the ideal continuity of the NNS with the great founders of modern macroeconomics thought - not only Keynes but also Wicksell - was subject to strong criticism. No doubt, there are many points of apparent coincidence between the NNS and the ideas of Wicksell and Keynes. Probably for many readers the idea that Keynes and Wicksell can coexist in a common framework can be confusing or rather troublesome⁹. However, in light of the contribution made by the NNS, this

⁹Long standing Keynes' hexegetics highlights the paradigmatic divorce that occurred
interminable theoretical dispute disappears by means of a simple and very popular mechanism: *sticky prices*. In NNS treatment, sticky prices are necessary and sufficient to translate Wicksell’s interest-rate theory of the price level into a theory of output and prices fluctuations with apparent Keynesian features. As Woodford outlines “it is only with sticky prices that one is able to introduce the crucial Wicksellian distinction between the actual and the natural rate of interest, as the discrepancy between the two arises only as a consequence of a failure of prices to adjust sufficiently rapidly” (Woodford, 2003, p.232). Whereas the NNS concentrates the non-Walrasian features of the economy in the goods (and/or labour) market, the capital market remains perfectly Walrasian granting continuous intertemporal equilibrium. This is the key point in which the NNS theoretical framework differs from old macroeconomics substantially, so much that one wonders whether the points of coincidence may survive to closer inspection (Mazzocchi et al., 2009). Indeed both Keynes and Wicksell assumed that the problem was neither the price of goods nor the wage, but the price of loans, i.e. the interest rate (Leijonhufvud, 1981).

A first departure from the assumption of perfect capital markets is the existence of intermediaries (the banking system) between savers (households) and investors (firms). If the banking system plays an active role, the idea that the equilibrium on the capital market - defined by the forces of productivity and thrift - is found at the full-employment level is no longer valid. All three actors on the capital market act with incomplete and limited information, which is the reason why deviations of the market interest rate from the natural rate may arise. The interference of the banking system with the natural rate can occur because Wicksell’s cashless economy is not a moneyless economy (Laidler, 2006, p. 3). Therefore each agent can increase his/her nominal purchasing power either selling goods and services or borrowing money from someone else. As long as non-bank agents borrow and lend one with the other, the total amount of nominal purchasing power in the economy is redistributed but cannot increase. By contrast, a bank is in a position to grant additional nominal purchasing power to anyone with no one else in the economy undergoing an equivalent reduction. Moreover, a bank can increase its own nominal purchasing power by borrowing from the central bank. Thus the problem is that the banking system as a whole

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in his thought between the early Wicksellian inspiration of the Treatise on Money and the General Theory (see Chick, 1983; Rogers, 1989). There is ample textual evidence (notably Keynes, 1937a, 1937b, 1937c) that in the search of a consistent explanation of fluctuations in real income, Keynes divorced from Wicksell not on the grounds of imperfect goods market. He realized that a different theory of the interest rate was needed. The idea of the monetary nature of the interest rate related to liquidity preference was conceived as the wedge to be driven in the self-equilibrating mechanism of saving and investment without postulating the role of the banking system. Paving the way of the liquidity preference hypothesis, Keynes ultimately negated the possibility of stability (De Antoni, 2009, p. 11).
might both expand the total nominal purchasing power in the economy and allocate it at terms that differ from those dictated by full-employment saving-investment equilibrium. The capital market is thus imperfect in that the banking system may fail to manage the nominal interest rate consistently with the natural rate. But this is not a question of policy failure, rather a result of the fact that the natural rate is unobservable and neither the financial system nor anybody else has direct information about it.\footnote{Up to date econometric research is by no means encouraging on the possibility that central banks can ever obtain all information necessary to target the natural rate precisely (Amato, 2005; Garnier and Wilhelmsen, 2005; Caresma et al., 2005). A growing literature is now concerned with monetary policy under imperfect information (Orphanides and Williams, 2002b).}

The consequence of the market real interest rate on loans being higher/lower than the natural rate is that households wish to save more/less whereas firms wish to invest less/more: neither side of the market can achieve intertemporal equilibrium of plans. Thus, changes in prices and quantities are the symptom that excess saving or excess investment are being accommodated at the “wrong” market rate and the economy was driven out of the intertemporal equilibrium path. The dynamics of both output and inflation are disequilibrium phenomena and they should necessarily be examined as out-of-equilibrium processes. This interpretation of changes in the price level and production is in sharp contrast with the one put forward in the NNS model, where they are consistent with all markets being cleared and households and firms being in intertemporal equilibrium continuously. Yet ruling saving-investment imbalances out of the theory constitutes a major theoretical weakness (Leijonhufvud, 1981; Van der Ploeg, 2005).

On the policy front, whereas the distortionary effects of sticky prices are the raison d’être of monetary policy in the NNS, older mainstreams argued that interest rates should be brought under policy control not because prices do not move enough, but because unfettered interest rates may force prices and quantities to move out-of-equilibrium. On the other hand, changes in the price and output levels are a means to re-equilibrate the economy only if they induce the nominal interest rate to close the gap with the natural rate. For reasons that we cannot consider here, NNS macroeconomics took the easier, perhaps realistic, shortcut of sticky prices at the cost of obscuring one of the most important keys to understanding business cycles, that is their dimension of intertemporal coordination failures. Nevertheless other economists maintained the focus on the role of saving-investment imbalances and the underlying capital-market imperfections (Leijonhufvud, 1981; Greenwald and Stiglitz, 1987, 1993; Hahn and Solow, 1995). Despite the methodological differences, common to these views is the idea that the older macroeconomics of saving-investment imbalances does offer guidance for consistent foundations of the interest-rate theory and practice of monetary policy precisely because it focuses on the interest rate as “the wrong
price” in the system and lead us to investigate how the monetary authority can manage to “get it right”.

3 The Model

3.1 Basic Setup

In order to assess and hopefully clarify some basic issues concerning the macroeconomic of saving-investment imbalances, let me introduce the same general equilibrium framework underlying the NNS models, where perfect capital market, monopolistic competition and the sticky price assumptions are replaced by imperfect capital market, perfect competition and flexible prices (Tamborini, 2010a; Tamborini et al., 2014). Adding goods market imperfections, sticky prices and wage rigidities may enrich the model for empirical purposes, but, contrary to what is claimed by the NNS, it is not theoretically indispensable.

The economy consists of three competitive markets - for labour, capital and output - and rational forward-looking agents. Households own the inputs and assets of the economy, including ownership rights in firms, and choose the fractions of their income to consume and save. Firms hire inputs and use them to produce goods that they sell to households or to other firms. All exchanges take place in terms of a general unit of account of value $P_t$, where $P_t$ is the general price level. For sake of concreteness and comparison with the standard NNS model, I have posited specific functional forms for the production function and the utility function.

I assume that the supply side of the economy is characterized by the following technology:

$$Y_t = K_t^a L_t^{1-a}$$  \(3.1\)

Where $Y_t$ is the flow of output, $K_t$ is the available capital stock and $L_t$ is the labour input. The chosen production function satisfies the traditional neoclassical properties. Moreover, we assume a capital accumulation technology such that the share of output transformed into capital at time $t$ takes one period to become operative (Christiano and Todd, 1996; Kydland and Prescott, 1982)$^{11}$. These two conditions permit us to write the transition law of capital stock as:

$$K_{t+1} = I_t + (1 - \delta)K_t$$  \(3.2\)

$^{11}$The accumulation function may take different forms, with (slight) differences in result. Here we have followed the “time-to-build” (Kydland and Prescott, 1982) or “time-to-plan” hypothesis (Christiano and Todd, 1996): the underlying idea reflects the fact that firms often decide to carry out new projects and there is a lag between the decision and its implementation. As argued by Blanchard and Fischer (1989), many capital expenditure decisions take long periods to achieve fruition justifying the assumption of a planning before actual investment expenditure is implemented.
where $I_t$ is net investment and $I'_t = I_t + (1 - \delta)K_t$ is the gross investment (inclusive of capital replacement). For the sake of simplicity we assume that capital depreciates at a constant rate $\delta = 1$. At each point in time all the capital stock wears out and, hence, can no longer be used for production\textsuperscript{12}.

As far as capital is concerned, firms can finance gross investment out of households’ gross saving by issuing one-period bonds $B_t$ bearing a nominal interest rate $i_t$. By analogy with physical capital, bonds are indexed by their maturity, i.e. $t$ denotes bonds issued at time $t - 1$ with maturity $t$. Note, therefore, that the market real interest rate relevant to the saving/investment decisions in period $t$ is given by $R_t = \frac{1+i_{t-1}}{1+\pi_t}$, where $\pi_t$ is the rate of inflation of the output price $P_t$, whereas the actual real interest rate that households earn in each $t$ is given by $R_{t+1} = \frac{1+i_t}{1+\pi_{t+1}}$, where $\pi_t$ denotes an expected value.

In the labour market\textsuperscript{13}, firms bargain with workers over a real wage $\omega_t$ before production takes place, but labour contracts are in nominal terms $W_t$. The nominal wage rate is obtained by way of indexation of the negotiated real rate $\omega_t$ to the expected price level $P_t^e = P_{t-1}^e(1 + \pi_t^e)$. Hence the nominal wage rate results

$$W_t = \omega_t P_{t-1}(1 + \pi_t^e)$$

The \textit{contractual} wage is set at the competitive full-employment equilibrium\textsuperscript{14}. Subsequently, firms choose $L_t$ for production, observing the actual wage rate $w_t$ given by the nominal rate deflated by the \textit{actual} price level $P_t^e$, namely $w_t \equiv \frac{W_t}{P_t^e}$. Since $P_t = P_{t-1}(1 + \pi_t)$,

$$w_t \equiv \omega_t \frac{1 + \pi_t^e}{1 + \pi_t}$$

For each period $t$, firms’ programmes consist of the choice of labour input $L_t^*$ for current production and the capital stock $K_t^{*+1}$ for the next one, in order to maximize their expected profit stream, given (3.1), (3.2) and the gross

\textsuperscript{12}This shortcut avoids another problem: if the model allows for fixed capital - instead of only circulating capital - it runs up against the problem on how to evaluate existing capital stock \textit{off} the intertemporal general equilibrium path. The solution of this trouble would require the integration of short-run and long-run macroeconomics and consequently the description of the term structure of interest rates. Yet, this task is beyond the scope of this paper. I owe this point to Robert Solow.

\textsuperscript{13}This representation of the labour market is akin to Keynes (1936, ch. 19). See also Hargreaves-Heap (1992).

\textsuperscript{14}This is only a simplifying assumption. Whether the equilibrium employment is equal to, or less than, the total labour force is immaterial here, or as implicit as the labour market in Woodford’s version of the NNS. Another characterization of labour market equilibrium may be obtained by way of “real rigidities” like efficiency wages. With this assumption it would seem natural to think that at the intertemporal general equilibrium, real wage rate still has some unutilized resources.
income distribution constraint, namely:

\[
\max_{L_t; K_t+1} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \Theta^{-s} (Y_{t+s} - w_{t+s}L_{t+s} - R_{t+s}K_{t+s+1}) \right] \tag{3.3}
\]

The FOCs for firms are thus the following:

\[
K^*_t = L^*_t \left( \frac{a}{R_{t+1}} \right) ^ {\frac{1}{1-a}} \tag{3.4}
\]

\[
L^*_t = K_t \left( 1 - \frac{a}{w_t} \right) ^ {\frac{1}{a}} \tag{3.5}
\]

where \( w_t \) is the real wage rate, and \( R_t \) is the real gross return to be paid on the capital stock operative at time \( t \) and purchased at time \( t-1 \).

The households hold claims to the capital stock of the economy, supply their whole labour force \( L_t \) inelastically, and choose a consumption plan \( \{C_{t+s}\}_{s=0}^{\infty} \) in order to maximize their utility under the following budget constraint:

\[
C_t + S_t = Y_t \tag{3.6}
\]

where \( S_t \) is net saving. In consideration of the assumptions concerning the capital accumulation technology, and of the definition of \( Y_t \), the households’ budget constraint (3.6) can also be written as:

\[
B_{t+1} = H_t + R_t B_t - C_t \tag{3.7}
\]

where \( H_t = w_t L_t \) is labour income and \( B_t \) is the outstanding real stock of bonds\(^{15}\). Consequently \( B_{t+1} - B_t = S_t \) is the net saving, thus if \( S_t > B_t \), then \( B_{t+1} > B_t \). Therefore, given a constant rate of time preference \( \theta > 0 \) and \( \Theta \equiv 1 + \theta \) the households’ intertemporal maximization problem is\(^{16}\):

\[
\max_{C_t} \mathbb{E}_t \left[ \sum_{s=0}^{\infty} \Theta^{-s} U(C_{t+s}) \right] \tag{3.8}
\]

subject to the iterated budget constraint (3.7):

\[
C_t + \sum_{s=1}^{\infty} \mathbb{E}_t \left[ \frac{C_{t+s}}{\prod_{s=1}^{\infty} R_{t+s}} + \frac{B_{t+s+1}}{\prod_{s=1}^{\infty} R_{t+s}} \right] = H_t + \sum_{s=1}^{\infty} \mathbb{E}_t \left[ \frac{H_{t+s}}{\prod_{s=1}^{\infty} R_{t+s}} + R_t B_t \right]
\]

where the transversality condition imposes:

\[
\lim_{s \to \infty} \frac{B_{t+s+1}}{\prod_{s=1}^{\infty} R_{t+s}} = 0
\]

\(^{15}\)By analogy with physical capital, bonds are indexed by their maturity, i.e. \( B_t \) are bonds purchased in \( t-1 \) with maturity in \( t \), etc.

\(^{16}\)where \( U'(C_t) > 0, U''(C_t) < 0 \).
The households’ FOC, as of time $t$, is thus characterized by:

$$U'(C_t) = E_t \left[ \frac{U'(C_{t+1})}{\Theta} R_{t+1} \right]$$

Preferences of the households are described by the following logarithmic utility function$^{17}$:

$$U(C_t) = \ln C_t$$

therefore the household’s optimal consumption is:

$$C_t = E_t \left[ \frac{C_{t+1}}{R_{t+1}} \Theta \right]$$ (3.9)

In order to isolate the macroeconomic effects of the interest-rate gaps, the analysis should start at a point in which the economy is in intertemporal general equilibrium. Therefore we can consider the (stationary) steady-state solution $SS$ for firms’ and households’ optimal plans (3.4), (3.5), (3.9), given full employment normalized to one, $L^{SS} = 1$ and $\pi_{t+1}^{\pi} = \pi_t^{\pi} = \pi^{SS} \geq 0$. Hence the return to capital will be $R_{t+1} = \Theta = R^{SS}$, so that for all $t$, $C_{t+1} = C_t = C^{SS}$. For the given production function (3.1), the constant real interest rate also yields a constant capital stock $K^{SS}$ and hence constant output and factor incomes, that is $F_K' = R^{SS}$, $B_t = K_{t+1} = K^{SS}$, $Y_t = Y^{SS} = H^{SS} + R^{SS} K^{SS}$. As long as optimal saving is equal to optimal investment, the real value of bonds with maturity at any time $t$ coincide with the operating capital stock, $B_t = K^{SS}$. Consequently, the resource constraint is satisfied for:

$$C^{SS} \left( 1 + \sum_{i=1}^{\infty} \frac{1}{R^{SS}} \right) = H^{SS} \left( 1 + \sum_{i=1}^{\infty} \frac{1}{R^{SS}} \right) + R^{SS} K^{SS}$$

Since $\lim_{i \to \infty} \sum_{i=1}^{\infty} \frac{1}{R^{SS}} = \frac{1}{1 + R^{SS}}$, we could obtain the optimal consumption, the optimal capital stock and the equilibrium output as follows:

$$C^{SS} + K^{SS} = Y^{SS}$$ (3.10)

$$K^{SS} = \left( \frac{a}{R^{SS}} \right)^{1-a}$$ (3.11)

$$Y^{SS} = K^{SS} a$$ (3.12)

The real interest rate $R^{SS}$ associated with the intertemporal general equilibrium is the so called natural rate of interest. This implies that $S_t' = I_t + K_t = I_t' = K^{SS}$, that is, in steady-state net saving and investment

---

$^{17}$This functional form has the advantages that the optimal consumption does not depend on the time horizon of the model.
are nil in all $t$, and the economy only replaces the optimal stock of capital $K_{SS}$. Finally, it should be $(1 + i_t) = R_{SS}(1 + \pi_{t+1}^e)$ for all $t$. Note that the system has thus three key elements: first, the rate of time preferences of households $\Theta = R_{SS}$; second, the market real rate of interest, $R_{t+1}$; third, the return to capital, $R^*_t$. In order to achieve the steady state we should have that $\Theta = R_{t+1} = R^*_{t+1} = R_{SS}$. Intuitively a variation of $R_{t+1}$ will influence (with some delay due to time lag or adjustment costs of capital) $R^*_{t+1}$, so we can assume that on average $R_{t+1} = R^*_{t+1}$. But as long as $R_{t+1} \neq \Theta = R_{SS}$ a gap remains. This point will be clearer in the subsequent sections.

3.2 Three-Gaps analysis

Let me suppose that, during any period $t$, the economy may be hit by a shock to the capital market. The shock may be real (a change in the determinants of thrift or productivity not matched by $i_t$) and imply a new natural interest rate, or it may be nominal (a disturbance to the nominal interest rate, i.e. $i_t$ is set or changed inconsistently with the natural rate) and generate a divergence between the market real rate $R_{t+1}$ and the existing natural rate $R_{SS}$. As will be seen, the only key variable in the problem at hand is the gap between the market and the natural rate, while which of the two has been shocked is immaterial here.

Before proceeding, it should be borne in mind that the problem is neither one of trades at the “right” rate, nor is it one of quantity rationing at a fixed rate (Hargreaves-Heap, 1992), but it is one of market-clearing trades at the “wrong” rate allowed for by bank intermediation. It is not a semantic distinction: in the case of rationing the market does not clear and, generally, the “short-side of the market” rule is invoked. In the latter case, the market may well clear, but the distinction between notional plans and actual trades is introduced. Notional plans are those that agents would realize if they could trade at the general equilibrium prices. Conversely, actual trades are those that agents realize upon observing spot market prices. Thus actual trades may differ from notional plans if one or more spot prices observed in one or more markets are not general equilibrium prices.

This scheme is appropriate to the role of the central bank as it pegs the nominal interest $i_t$ under limited information. Pegging implies that as long as $R_{t+1} \neq R_{SS}$ an excess investment (supply of bonds) or saving (demand for bonds) in the capital market arises. The central bank’s open-market operations that peg the nominal interest rate entail that the excesses of

\[\text{Abstracting from technical progress or technology shocks, this is a Sidrausky-type steady-state, where the key variable in the capital market is the rate of intertemporal consumer preferences } \Theta.\]

\[\text{For example, if } R_{t+1} > R_{SS} \text{ firms realize their planned investment whereas households are constrained to save less than planned.}\]
supply or demand of bonds are cleared by way of purchases or sales by the central bank. These allow excess investment/saving to be financed and allow households and firms to carry on their saving and investment actual plans, respectively. However, these are not the notional plans that would be undertaken with $R_{t+1} = R^{SS}_t$, and the inconsistency between the actual households’ consumption plans and firms’ capital-stock choices will spill over across markets and time. In other words, the following proposition holds:

**Proposition 1** Given $R_{t+1} \neq R^{SS}_t$ in any period $t$, although the bonds market clears, the ensuing levels of saving and investment are not consistent with the intertemporal general equilibrium values of output and at the general price level that would obtain at the natural rate of interest $R^{SS}_t$ both in $t$ and in the subsequent periods.

The proof of this proposition is a well-known implication of Walras Law. Actual consumption is given by (3.9) whereas actual investment of firms is given by (3.4). Ceteris paribus, with respect to the intertemporal general equilibrium, $R_{t+1} \neq R^{SS}_t$ consumption to the present ($R_{t+1} < R^{SS}_t$) or to the future ($R_{t+1} > R^{SS}_t$), while investment in $t$ - and the capital stock available in $t+1$ - are increased/reduced respectively. Consequently, there is a unique relationship between interest-rate gaps and saving-investment gaps, namely if $R_{t+1} > R^{SS}_t$ then $S'_t > I'_t$ and if $R_{t+1} < R^{SS}_t$ then $S'_t < I'_t$.

In Table 1 I briefly recall the disequilibrium relations in the goods market associated with $R_{t+1} \neq R^{SS}_t$ in the bond market. Now let us consider the case $R_{t+1} < R^{SS}_t$. In this case, the central bank clears the excess supply of bonds so that firms are allowed to invest more (by adding physical capital to their net worth) than households are actually ready to save (adding more bonds to their wealth). The consequence of this discrepancy is an excess demand in period $t$ and an excess supply in period $t+1$, corresponding to a production capacity that is not matched by planned consumption. If we abide by the principle that markets always clear, we should understand how these intratemporal and intertemporal inconsistencies among notional plans can be brought into equilibrium. To this effect, I put forward the following proposition:

**Proposition 2** Given $R_{t+1} \neq R^{SS}_t$ in any period $t$, there exists one single
sequence of output and price realizations in \( t \) and onwards that clears the output market. By implication of Proposition 1, these realizations cannot be the same that would obtain with \( R_{t+1} = R^{SS} \).

The full proof is provided in Appendix. The intuition of the proof is as follows: given \( R_{t+1} \neq R^{SS} \), what is the sequence of real income realizations that would "force" the households’ consumption plan to be consistent with the capital path chosen by firms? The solution technique, drawn from Smith and Wickens (2006) and Tamborini (2010a), consists of plugging each period budget constraint (3.7) into households Euler equation (3.9). Note that as long as \( R_{t+1} \neq R^{SS} \), then \( B_{t+s} \neq K_{t+s} \), \( s = 1, \ldots, \infty \), that is, the value of real bonds over time is inconsistent with that of real capital. Hence, to correct for this “wrong accounting” of real resources, the “forcing constraint” \( B_{t+s} = K_{t+s} \), \( s = 1, \ldots, \infty \), should also be added to the planning problem. As is proved in Appendix, the answer to the above question is the following:

\[
Y_t = E_t Y_{t+1} \frac{R^{SS}}{R_{t+1}} + K_{t+1} \left( 1 - \frac{R^{SS}}{R_{t+1}} \right) \tag{3.13}
\]

Then, in order to gauge the rate of deviation from the intertemporal general equilibrium output path, both sides are divided by \( Y^{SS} \). Denoting \( \hat{Y}_t \equiv \frac{Y_t}{Y^{SS}} \), \( \hat{R}_t \equiv \frac{R_{t+1}}{R^{SS}} \), the output and interest-rate gaps, respectively, the following sequence of output gaps is obtained

\[
\hat{Y}_{t+1} = \hat{R}_t^\frac{n-\alpha}{\alpha} \tag{3.14}
\]
\[
\hat{Y}_t \approx \hat{R}_t^\frac{1}{1-\alpha} \tag{3.15}
\]

As explained above, these output gaps are necessary to force the consumption path of households to be consistent with the capital path chosen by firms, given \( R_{t+1} \neq R^{SS} \) and output market clearing at all dates.

To understand the reasons underlying these results, consider again the case in which \( R_{t+1} < R^{SS} \) or \( \hat{R}_t < 1 \). As we have seen, households in period \( t \) reckon a real value of bonds that is smaller than the real value of capital that bonds are supposed to represent. Hence, the economy needs a real correction of resource accounting: to this effect, output (real incomes accruing to households) should be greater along the whole consumption path of households, namely \( \hat{Y}_{t+s} > 0 \) (with \( s = 0, \ldots, \infty \)).

Parallely, in Appendix I show that in order for profit-maximizing firms to increase output with respect to potential, the inflation rate, too, should be (unexpectedly) different than the normal rate embedded in nominal wage contracts according to

\[
\tilde{\Pi}_t = \left( \frac{\hat{Y}_t}{K_t} \right)^\frac{\alpha}{1-\alpha} \tag{3.16}
\]

17
where $\tilde{\Pi}_{t+1} \equiv \frac{1 + \pi_e^t}{1 - \pi_t}$ and $\tilde{K}_t = \frac{K_t}{K^{SS}}$. Recall that the contractual nominal wage rate in force in each period is set ex-ante according to $W_t = \omega_t P_{t-1} (1 + \pi_e^t)$, where $\omega$ is the full-employment real wage. In order to produce $Y_t \neq Y^{SS}$, firms should face an actual real wage, $w_t = \omega_t \frac{1 + \pi_e^t}{1 + \pi_t}$, realigned with the ensuing marginal product of labour. For instance, in order to produce more, firms should either face a positive “inflation surprise” that lowers the actual real wage or have greater capital or both.

To sum up, I have shown that in the presence of an interest-rate gap, and the underlying investment-saving imbalance, goods and labour market-clearing over time can only take place by way of a sequence of output gaps and inflation surprises with respect to their intertemporal general equilibrium values. Analyzing expressions (3.14)-(3.15) and (3.16) in more detail, they present two main feature that distinguish them from the standard NNS model and that result directly from the underlying out-of-equilibrium nature of the model. The first concerns output gaps. Notably, this result is reminiscent of Keynes’s claim that investment-saving imbalances are corrected through changes in real incomes. As can be seen, however, the main implication is a sequence of intertemporal and interconnected output gaps each depending on the current interest-rate gap. This may be called a “feed-forward effect” of interest-rate gaps and it is related to the fact that I have retained the assumption that agents are forward-looking. By contrast, in the NNS framework each period’s output gap only depends on the contemporaneous interest rate gap, so that the feed-forward effect is absent.

The second feature to be noted concerns inflation gaps. The price level changes as much as is necessary to equate demand and supply, given the expected inflation rate. Hence, price stickiness is not an issue. This result is instead reminiscent of Wicksell’s claim that investment-saving imbalances give rise to inflationary phenomena. In this framework they reflect unexpected inflation, whereas in the NNS model they reflect anticipated inflation by price-setting firms. What is crucial is that unexpected inflation is an integral part of the process in the present model, in the sense that, as long as there exist interest-rate gaps - and hence output gaps - the economy must be off whatever price level path was expected by agents. As a matter of logic of the rational-expectations hypothesis, inflation expectations can, at best, be elaborated by agents consistently with their notional plans (i.e. the inflation rate that would result if the plans based on saving and investment were in fact the intertemporal general equilibrium plans) but these turn out to be unfeasible in the output market. Hence also the related expectation of the inflation rate will be falsified.

More precisely, any output gap at time $t$ depends on the expected output gap and the present interest rate gap which, ultimately, depends on the future interest rate gaps. Of course, different microeconomic assumptions may lead to different results and different combinations of price-quantity changes, but this is an empirical matter.
3.3 A log-linear version

To set the stage for further analyses and facilitate comparison with the NNS framework, I present now a log-linear version of the previous model. As usual log-variables are denoted with small-case letter of the corresponding symbol. Let me first analyze the output gaps (3.14)-(3.15). The main implication of an interest rate gap is a sequence of intertemporal output gaps, each depending on the current interest rate gap. In the Appendix I show that, since $\hat{Y}_t$ and $\hat{Y}_{t+1}$ share the same common factor, $\hat{R}_{t+1}$, it is in general possible to express them in a single reduced form displaying (spurious) autocorrelation. Hence a new “Investment and Saving” IS function is obtained:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t$$  \hspace{1cm} (3.17)$$

where $\hat{y}_t \equiv \ln \hat{Y}_t$, $\hat{i}_t = i_t - \pi_{t+1}^e - \hat{r}^{SS}$ and where $\rho$ and $\alpha$ are two parameters such that:

$$\alpha = \frac{a - \rho}{1 - a}$$

There is a clear analogy with the IS in the NNS model. Yet there are also substantial differences. As a consequence of the intertemporal “feed-forward effect” of interest-rate gaps, which is not in the NNS model, this generates time series of output gaps that, ex-post, display two main features: first, a dependence on the lagged values of interest-rate gaps, and second, some degree of spurious serial correlation or “inertia” between $\hat{y}_{t+1}$ and $\hat{y}_t$ measured by the parameter $\rho$. Notably, a dynamic structure like (3.17) is consistent with recurrent empirical estimates of IS equations (Laubach and Williams, 2003; Caresma et al., 2005; Orphanides and Williams, 2002a; 2006). These empirical regularities are not easily accommodated in the NNS framework unless it is filled with additional ad hoc frictions, usually interpreted as the outcome of backward-looking behavior of agents (Aghion et al., 2004; Mankiw and Reis, 2003; Sims, 2003). Here the correlations result directly from forward-looking agents and the fact that intertemporal disequilibrium connect each period’s variables in a way that is not captured by the NNS framework\(^{22}\). In the present model, the usual findings that $0 < \rho < 1$ and $\alpha > 0$, imply that $\rho < a$.

Let me now turn to the inflation rate consistent with $\hat{y}_{t+1}$. This can be obtained from the inflation equation (3.16) for $t+1$, which is in fact a “Aggregate Supply” equation (AS). As shown in Appendix, it can easily be log-linearized as follows:

$$\pi_{t+1} = \pi_{t+1}^e + \kappa \hat{y}_{t+1} + \nu \hat{i}_t$$  \hspace{1cm} (3.18)$$

\(^{22}\)This specification entails considerable differences in the dynamic properties of the economy.
where:

\[ \kappa = \frac{a}{1 - a} \quad \nu = \frac{a}{(1 - a)^2} \]

Parameter \( \kappa \) represents the inflation/output elasticity while \( \nu \) could be interpreted as the elasticity of the capital stock to the interest-rate gap. Again we can make a comparison between our AS equation and the so-called New-Keynesian Phillips Curve in the standard NNS model. Our AS indicates that the future inflation rate in \( t + 1 \) will depend on its expectation as of \( t \), on the future output gap, and on the current interest-rate gap. First of all, as explained above, as long as a non-zero interest-rate gap persists, an inflation surprise, \( \pi_{t+1} \neq \pi_{t}^{e} \), will arise. In the second place, also the adjustment dynamics of the capital stock induced by the interest-rate gap is crucial, a factor which is ignored by the NNS\(^{23}\). As indicated by the parameter \( \nu > 0 \), with \( \hat{i}_t \neq 0 \) there is positive/negative net investment and the capital stock will be above/below its steady state level thereby modifying the productive capacity of the economy. This means that an interest-rate gap affects not only aggregate demand, but also aggregate supply. This phenomenon is often referred to as saying that aggregate supply and demand “moves together” (Greenwald and Stiglitz, 1987). For instance, a negative interest-rate gap generates excess aggregate demand, but it will also spur productive capacity exerting a countervailing pressure on future inflation. The final outcome will depend of the relative movement of the two curves. As we shall see, this feature will significantly change the policy conclusions that can be drawn from the model.

### 3.4 A model check

It is now convenient to re-express the AS equation in terms of the inflation surprise \( \tilde{\pi}_{t+1} = \pi_{t+1} - \pi_{t+1}^{e} \), that is:

\[ \tilde{\pi}_{t+1} = \kappa \hat{y}_{t+1} + \nu \hat{i}_t \] 

Consequently, the IS-AS equations form a first-order difference, non-homogeneous system in the two gaps \([\hat{y}_{t+1}; \tilde{\pi}_{t+1}]\) with a given exogenous interest-rate gap \( \hat{i}_t \neq 0 \). This formulation is sufficient for a preliminary check of its dynamic properties. We have thus a non-homogeneous system:

\[
\begin{bmatrix}
\hat{y}_{t+1} \\
\tilde{\pi}_{t+1}
\end{bmatrix} =
\begin{bmatrix}
\rho & 0 \\
\kappa \rho & 0
\end{bmatrix}
\begin{bmatrix}
\hat{y}_t \\
\tilde{\pi}_t
\end{bmatrix} +
\begin{bmatrix}
-\alpha \\
\nu - \kappa \alpha
\end{bmatrix}
\hat{i}_t
\]

\(^{23}\)Of course, if the interest rate gap is zero, the capital stock is \( K_{t+1} = K_{SS}^{K} \) and thus we have zero net investment (only capital stock replacement). This conclusion holds only in a stationary steady-state analysis. Indeed, if we introduce a positive rate of growth of population (and possibly a technological progress), we should observe a positive net investment to keep constant the per-capita capital stock even if \( R_{t+1} = R_{SS}^{K} \).
For any initial value \( \hat{i}_t = i_0 \neq 0 \), it possesses the following steady-state solutions:

\[
\hat{y} = -\left( \frac{\alpha}{1 - \rho} \right) \hat{y}_0 \tag{3.20}
\]

\[
\tilde{\pi} = \left[ v - \frac{\kappa \alpha}{1 - \rho} \right] \hat{y}_0 \tag{3.21}
\]

That is to say:

**Proposition 3** A permanent interest-rate gap determines permanent output and inflation gaps. Conversely, the output and inflation gaps are nil only if the interest-rate gap is also nil.

**Proposition 4** If \( \rho \in [0, 1] \) output and inflation converge to - and remain locked in - the steady-state values, with both output and inflation being inefficiently high or low, and being inconsistent with their intertemporal general equilibrium expected values.

These propositions capture the essence of a cumulative processes as disequilibrium phenomena both on the nominal and real side of the economy (Leijonhufvud, 1981, p.136). If \( i_0 \neq 0 \) and if the initial inflation rate is nil, then the price level is set on the path given by (3.21) where it grows/declines indefinitely at a constant rate\(^{24}\). As long as the interest-rate gap is not closed, changes in the price level persist. This raises the problem of how expectations are possibly revised, and how the revision mechanism impinges upon the dynamic process. This problem will be reconsidered in section [3.5.2]. Instead, the real disequilibrium shows that deflation *per se* cannot be the solution to the problem originating from a saving-investment imbalance as long as the interest-rate gap is not closed. Indeed a real misallocation requires a real resource adjustment, irrespective of the degree of flexibility of prices\(^{25}\).

Let me say a few words on the sign of the two gaps. Equation (3.20) and (3.21) can be rewritten as:

\[
\hat{y} = \frac{a - \rho}{(a - 1)(1 - \rho)} \hat{y}_0 \tag{3.22}
\]

\[
\tilde{\pi} = \frac{a}{(a - 1)(\rho - 1)} \hat{y}_0 \tag{3.23}
\]

\(^{24}\)Cumulative processes of the price level in the Wicksellian literature are often associated with accelerating inflation rates. This possibility, however, is closely related by Wicksell to the mechanism of expectations formation: *as long as the change in prices (…) is believed to be temporary, it will in fact remain permanent*" (Wicksell, 1922, XII, n.1).

\(^{25}\)This aspect was underscored, but not denied, by Wicksell whereas it was brought into full light by Keynes with his theory of effective demand. Nevertheless, contrary to interpretations of the Old and New Synthesis, Keynes did not ignore that (unexpected) deflation (3.21) was the counterpart to the real adjustment process of supply and demand (Keynes, 1936, ch.19).
The coefficient of $\hat{i}_0$ in the first equation is negative as long as $a > \rho$ whereas it is always positive in the second equation. Therefore given the structural values of the parameters, if $\hat{i}_0 < 0$ we will have a positive output gap ($\hat{y} > 0$) and a negative inflation gap ($\tilde{\pi} < 0$). This last result seems to run contrary to the standard macroeconomics models used nowadays. It depends neither on the structural construction of the model nor on its microfoundation, rather it is due to the choice of the Cobb-Douglas production function and the underlying hypothesis of constant returns to scale. This assumption determines an excessive elasticity of the capital stock to the interest rate, measured by the parameter $\nu$, which amplifies the adjustment of the aggregate supply. Intuitively, if $\hat{i}_0 < 0$, we have a big increase of the capital stock and the AS curve will move much more than the AD. Hence, the dynamics will conclude with a negative inflation gap. Conversely, if the reactivity of the capital stock to the interest rate is low (as the empirical evidence seems to suggest), the AS will move slightly and the system will end with a positive inflation gap. In both cases the co-movements of the two curves lead to a pronounced change in the output gap without causing appreciable inflation gaps. These results seem to correspond to a stage described by Keynes when he emphasized that aggregate demand and aggregate supply could contract/expand in a multiplier process at an (nearly) unchanged price level. This conclusion significantly alters certain policy provisions provided by the NNS.

### 3.5 Extensions

#### 3.5.1 Adjustment costs of capital

So far we have assumed that the capital stock is always equal to the one obtained from the FOCs, namely $K_{t+1} = K_{t+1}^*$. This does not represent a problem in standard RBC models because technology shocks have a large impact on the real interest rate, consequently the response of investment mimics its empirical counterpart well. However this assumption presents some theoretical and empirical problems in sticky prices model of NNS framework. Indeed, as Ellison and Scott (2000) pointed out, models with endogenous capital stock imply unrealistically high volatility of investment. This depends on the fact that changes in the nominal interest rate translate one-to-one into changes in the real rates leading to the excessively high volatility of all endogenous variables. This problem also arises in the basic framework I presented in the previous sections: it is sufficient to look at equation (3.4) to note that a change in market interest rate is fully translated on investment and thus on the capital stock of the next period, causing

\footnote{In any case we should note that a negative inflation gap is present also in Casares and McCulham (2000) and Ellison and Scott (2000).}
\footnote{It is simple to prove that with increasing return to scale, $a + b > 1$, the elasticity of the capital stock to the interest rate gap would be less. Same thing if $a$ is small enough.}
large and immediate adjustments. The usual shortcut to have more realistic movements of capital and investment is to posit some form of adjustment costs which are able to "buffer" the capital stock adjustment. An example of such costs are those of installing new capital and training workers to operate new machines (Eisner and Strotz, 1963; Lucas, 1967)\textsuperscript{28}. We follow the same path. Let me suppose that in each period firms face the following adjustment-cost function:

\begin{equation}
Z_t = \mu_1(K_{t+1} - K_{t+1}^*)^2 + \mu_2(K_{t+1} - K_t)^2
\end{equation}

We could interpret \(\mu_1\) as unitary \textit{disequilibrium costs}, i.e. the cost to have a capital stock different from the optimal one (given the interest rate). Vice-versa, we could interpret \(\mu_2\) as the unitary \textit{change costs} of the capital stock from one period to another. Given \(K_{t+1}^*\) and \(K_t\), the firm seeks to minimize \(Z_t\) as follows:

\begin{equation}
\min_{K_{t+1}} Z_t = E_t \left\{ \sum_{s=0}^{\infty} \left[ \mu_1 \left( K_{t+s+1} - K_{t+s+1}^* \right)^2 + \mu_2 \left( K_{t+s+1} - K_{t+s} \right)^2 \right] \right\}
\end{equation}

and thus:

\begin{equation}
K_{t+1} = (1 - \psi)K_{t+1}^* + \psi K_t
\end{equation}

where \(\psi = \frac{\mu_2}{\mu_1 + \mu_2}\). Let me briefly discuss the two limit cases. First, if the disequilibrium costs are equal to zero we have that \(K_{t+1} = K_t\), i.e. the capital stock does not change over time and remains anchored to the steady-state level \(K^{SS}\) (Tamborini et al., 2014). Second, if the change costs are equal to zero we have that \(K_{t+1} = K_{t+1}^*\), i.e. the actual capital stock is always equal to the optimal one, given the interest rate. In this way it is as if there were no adjustment costs of capital. This last case brings us back to the basic framework I developed in section [3.1].

Let me indicate with \(\hat{k}_{t+1} = \log \frac{K_{t+1}}{K^{SS}_t}\). From \(K_{t+1} = K^{SS}(1 + \hat{k}_{t+1})\), \(K_{t+1}^* = K^{SS}(1 + \hat{k}_{t+1}^*)\) and \(K_t = K^{SS}(1 + k_t)\). Log-linearizing around the steady-state (Uhlig, 1999) we get:

\begin{equation}
K^{SS}(1 + \hat{k}_{t+1}) = (1 - \psi)K^{SS}(1 + \hat{k}_{t+1}^*) + \psi K^{SS}(1 + \hat{k}_t)
\end{equation}

dividing both sides for \(K^{SS}\) and iterating the terms over time we get:

\begin{equation}
\hat{k}_{t+1} = (1 - \psi)\hat{k}_{t+1}^* + \sum_{j=1}^{\infty} (1 - \psi)^j \hat{k}_{t+1-j}
\end{equation}

\textsuperscript{28}Usually the key assumption of the NNS model with endogenous investment is that firms face costs of adjustment which are a convex function of the rate of change of their capital stock (Casares and McCallum, 2000; Woodford, 2003)\textsuperscript{29}. These assumption implies that it is costly for a firm to increase or decrease its capital stock and that the marginal adjustment cost is increasing in the size of the adjustment.
Since \( \hat{k}_t^* = \hat{k}_{t-1}^* = \cdots = \hat{k}_{t+1-j}^* \) (with \( j = 1, \ldots, \infty \)) are predetermined variables we get:

\[
\hat{k}_{t+1} = (1 - \psi)\hat{k}_{t+1}^* + \sum_{j=1}^{\infty} (1 - \psi)^j \hat{k}_t^*
\]

For \( 0 \leq \psi \leq 1 \) we have that:

\[
\hat{k}_{t+1} = (1 - \psi)\hat{k}_{t+1}^* + \psi \hat{k}_t^*
\]  \hspace{1cm} (3.28)

In the Appendix I show that, introducing adjustment cost of capital, the model (3.17)-(3.19) becomes the following:

\[
\tilde{y}_{t+1} = \rho \tilde{y}_t - \alpha i_t - \beta i_{t-1} \quad \text{(3.29)}
\]

\[
\tilde{\pi}_{t+1} = \kappa \tilde{y}_{t+1} + \theta i_t + \sigma i_{t-1} \quad \text{(3.30)}
\]

where:

\[
\alpha = \rho - a \left[ 1 + \psi (\rho - 1) \right] \quad \beta = \frac{a \psi (\rho - 1)}{a - 1}
\]

\[
\kappa = \frac{a}{1 - a} \quad \theta = \frac{a (1 - \psi)}{(1 - a)^2} \quad \sigma = \frac{a \psi}{(1 - a)^2}
\]

Not surprisingly it can be seen that, for any initial value \( \hat{i}_t \neq 0 \) the system possesses the same steady-state as (3.22)-(3.23). In fact, the effect of adjustment costs tend to disappear over time and the capital stock gap will lead to its optimum level \( \hat{k}_{t+1}^* \) compatible with the interest rate gap \( \hat{i}_0 \). Nevertheless, adjustment costs have an effect on the convergence dynamics to steady state. If adjustment costs are small, the transient dynamics from a steady state to another is very rapid. Conversely, if the adjustment costs are large enough, the adjustment process may take several periods. All this may have very interesting implications: in fact, a permanent interest rate gap \( \hat{i}_0 < 0 \) may generate a more or less prolonged inflation phase followed by an outright deflation. I will address this issue again in the next sections.

### 3.5.2 Inflation expectations

What remains to be determined are the price level and the steady-state inflation rate. As is well-known, a competitive general equilibrium model like the present one leaves the price level and its changes undetermined. A consistent interest-rate theory of the general price level should describe how the nominal interest rate and the inflation expectations are determined, and the interaction between these two variables\(^{30}\). The simplest hypothesis\(^{31}\)

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\(^{30}\)Another possible solution is to insert a monetary equation that pins down the price level corresponding to the general-equilibrium output level (Balloussia et al., 2011).

\(^{31}\)For alternative hypotheses, such as rational expectations and interest-rate rules, see Mazzocchi (2012).
is to assume a simplified monetary system that consists of a central bank, representing the banking system as a whole. It operates by setting at the beginning of each period $t$ an official inflation target $\pi^*$ and exerting control on the nominal interest rate $i_t$ by trading bonds in the open market\textsuperscript{32}. The value at which the central bank set the inflation target is immaterial here. In perfect competition its value should be zero, while with different market structures - for example, imperfect competition - or in presence of real rigidities it should be derived from a welfare computation and it is usually greater than zero. Once the central bank has set $\pi^*$, it is ready to create or retire money (the counterpart of bonds) to the extent that is necessary to clear the market. Households and firms make up their expectations according to the announced inflation $\pi^*_{t+1} = \pi^*$. Then transactions take place and output and inflation for the period are realized.

The problem with this assumption is that, in the event of a persistent interest-rate gap $\hat{i}_t \neq 0$, the future path of the price level will no longer be the same as in the past. In modern parlance, in the course of the cumulative process, expectations of return to normality are systematically falsified. Whereas modern macroeconomists tend to rule this problem out of analysis by focusing exclusively on states of the economy where expectations are statistically correct\textsuperscript{34} - the so-called short-run rational expectations - older mainstreams (see Lindahl, 1939; Lundberg, 1930; Myrdal, 1927) were not concerned with jumps from one equilibrium to the next, and they introduced the hypothesis of learning in the cumulative process that shifts expectations from static to adaptive to forward-looking and eventually rational in the sense of self-fulfilling. The full exploration of such adaptive expectations dynamics is beyond the scope of this paper. Nevertheless there is a literature about heterogenous beliefs in financial market that provides routes to endogenize expectations in terms of rational reactions to the relative success of different strategies (Brock and Hommes, 1997; 1998). Without going too far with the analysis, it may be interesting to examine the case in which the switching of strategies would depend on the costs of sustaining the long-run rational expectations $\pi^*$ as compared to the information costs of forming the short-run rational expectations $E_t \pi_{t+1}$. The first cost could be represented as the cost $\Delta_1$ of having an inflation expectation other than $E_t \pi_{t+1}$, while

\textsuperscript{32}In reality, central banks generally control short-term interest rates only, whereas investment is largely financed with long-term bonds. For an analysis of the transmission of monetary policy through the term structure of interest rates see Turner (2011).

\textsuperscript{33}This is a difference with respect to the traditional NNS model, in which we observed an “un-anchored” one-period expected inflation $\pi^*_{t+1} = E_t \pi_{t+1}$. It can be shown that this difference does not entail major theoretical implications. This assumption (Tamborini, 2006) is both convenient and consistent with the usual treatment of policy games where the central bank moves first, and the problem is the conditions such that the target is also the rational expectation of the inflation rate, regardless of transitory inflation rates (Evans and Honkapohja, 2001).

\textsuperscript{34}More precisely $E_t(\pi_{t+1} - \pi^*_{t+1}) = 0$ and thus $\pi^*_{t+1} = E_t \pi_{t+1} = \pi_{t+1}$.
the second can be interpreted as the cost $\Delta_2$ of not adjusting expectation with respect to long-term one $\pi^*$. Therefore in each period $t$ agents faces the following cost function:

$$M_t = \Delta_1(\pi_{t+1}^e - E_t\pi_{t+1})^2 + \Delta_2(\pi_{t+1}^e - \pi^*)^2$$  \hspace{1cm} (3.31)

Minimizing with respect to $\pi_{t+1}^e$ and setting $\xi = \frac{\Delta_1}{\Delta_1 + \Delta_2}$ we get:

$$\pi_{t+1}^e = \xi E_t\pi_{t+1} + (1 - \xi)\pi^*$$  \hspace{1cm} (3.32)

To this effect, I replace this last relation in equations (3.19) and (3.17). As a result (see the proof in Appendix):

$$\hat{y}_{t+1} = \rho' \hat{y}_t - \alpha' \hat{i}_t - \beta' \hat{i}_{t-1}$$  \hspace{1cm} (3.33)

$$\hat{\pi}_{t+1} = \kappa' \hat{y}_{t+1} + \theta' \hat{i}_t + \sigma' \hat{i}_{t-1}$$  \hspace{1cm} (3.34)

where $\hat{\pi}_{t+1} = \pi_{t+1} - \pi^*$ and where:

$$\alpha' = \alpha \frac{1 - \xi}{1 - \xi(1 + \alpha \kappa - \theta)}$$

$$\beta' = \frac{\beta[1 - \xi(1 - \theta)] - \alpha \xi \sigma}{1 - \xi(1 + \alpha \kappa - \theta)}$$

$$\theta' = \frac{\theta}{1 - \xi(1 - \theta)}$$

$$\rho' = \rho \frac{1 - \xi(1 - \theta)}{1 - \xi(1 + \alpha \kappa - \theta)}$$

$$\kappa' = \frac{\kappa}{1 - \xi(1 - \theta)}$$

$$\sigma' = \frac{\sigma}{1 - \xi(1 - \theta)}$$

The steady-state solution for $[\hat{y}_{t+1}, \hat{\pi}_{t+1}]$ can simply be restated as follows

$$\hat{y} = -\left(\frac{\alpha' + \beta'}{1 - \rho'}\right) \hat{i}_0$$  \hspace{1cm} (3.35)

$$\hat{\pi} = \left[\theta' + \sigma' - \frac{\kappa'}{1 - \rho'}(\alpha' + \beta')\right] \hat{i}_0$$  \hspace{1cm} (3.36)

These new solutions are ambiguous as to their sign, magnitude and stability, not only because of what we said in the previous section, but also for the role played by the parameter $\xi$. Some preliminary considerations are now possible (proofs in Appendix). First, to keep the system stable $\xi$ should be bounded as follows:

$$\xi < \frac{1 - \rho}{\alpha \kappa + (\theta - 1)(\rho - 1)}$$  \hspace{1cm} (3.37)

If $\xi$ satisfies this condition, the system converges to $[\hat{y}; \hat{\pi}]$. Otherwise the system may take different trajectories, some of which may be explosive. Second, if $\xi$ satisfies the sign condition, the output gap $\hat{y}$ has always the normal negative relationship with interest rate gap $\hat{i}_0$, i.e. tightening monetary policy entails a drop in steady-state production compared to the potential level. Third, the inflation gap $\hat{\pi}$ has always positive/negative relationship with interest rate gap $\hat{i}_0$, whatever the value of $\xi$. The sign of the gap is
not due to inflation expectations but depends on the elasticity of the capital stock to the present and past interest rate ($\theta$ and $\sigma$). Forth, for structural values of the parameters the coefficient of $\hat{y}$ decreases with $\xi$ in absolute value, whereas the coefficient of $\hat{\pi}$ increases in absolute value. On the contrary, for empirical values of the parameters both the coefficients of $\hat{y}$ and $\hat{\pi}$ increases with $\xi$ in absolute value: short-run rational expectations amplify the deviation of the steady-state from the intertemporal equilibrium path. This last result illustrates the troublesome role of short-run rational expectations in cumulative processes. If we consider the case $i_0 < 0$, this produce a positive output gap $\hat{y} > 0$. As agents anticipate higher inflation, the market real interest rate $r_t = i_t - \hat{E}_{t+1}\pi_{t+1}$ is reduced further, increasing the gaps and so on. This expectation multiplier explains why short-run rational expectations are deviation amplifying. Fifth, the limit solution for $\xi \to 1$, is $[\hat{y}, \hat{\pi}] = [0, \hat{i}_0]$. The system “jumps” to an inflation gap equal to the interest-rate gap and forward-looking expectations are (self-)fulfilled. This case replicates the result of McCallum (1986). Using the so-called Fisher equation - namely $1 + r^{SS} = r^{SS}(1 + \hat{E}_{t+1}\pi_{t+1})$ - he argues that if expected inflation is to be determined by postulating an exogenous nominal interest rate, it should be high as long as the nominal interest rate is high relative to the natural rate, and vice-versa. This conclusion holds only in intertemporal general equilibrium (Tamborini et al., 2014).

4 Dynamic properties: a quantitative assessment

In this paragraph I wish to inspect more closely the dynamic properties of the model (3.33)-(3.34) with the help of quantification of its parameters and some simulations. To this end I find convenient to look at some NNS-estimates of these parameters that can be found in the leading literature.

The structural model underlying the framework represents a theoretical competitive, flex-price economy. It hinges on the primitive parameter set ($a$, $\rho$, $\xi$, $\psi$) from which we could derive ($\alpha'$; $\beta'$; $\kappa'$; $\theta'$; $\sigma'$). As far as the parameter set is concerned, we should consider that the NNS literature includes the sticky-price hypothesis, which leads to a different determination of the inflation-gap/output-gap elasticity, namely $\kappa$. The theoretical models as well as empirical evidence show that the combination of imperfect competition and sticky prices determines smaller elasticity of inflation gaps to output gaps than in the perfect competition and flex-price model we presented in the previous sections. Therefore I decide to organize the parametrization of the model around these two alternative hypothetical economies: the “theoretical” flex-price economy where $\kappa = \frac{a}{1-a}$ and the “empirical” sticky-price, for which $\kappa$ is taken from available estimates.

As to the remaining parameters of interest, we simply quantify the capital income share $a$ with the conventional value 0.4. Estimates of this param-
parameter have a long-standing controversial history (Felipe and Adams, 2005). In particular, the capital income share in developed countries seems to be increasing as a result of globalization. However, this conventional benchmark is still largely used and is probably not too unrealistic. Selected estimates of other parameters are organized in Table 2. The second key parameters are the elasticity of the capital stock to the present and past interest rate $\theta$ and $\sigma$. Theoretically, these two parameters depend on the capital-income share $a$ and the adjustment costs $\psi$: the higher are these two variables, the lower is the value of $\theta$. Among other things, the high theoretical value of $\theta$ is due to the assumption that the capital stock fully depreciates in each period and should ultimately be replaced every year. Thus $\theta$ can be interpreted as the responsiveness of investment to the interest rate, and not as the elasticity of the capital stock. However, empirically the value of $\theta$ is much lower, since the portion of capital which has not depreciated $(1 - \delta)K_t$ is substantially insensitive to changes of the interest rate. Unless otherwise stated, we shall thus assume $\theta = 0.1$ and $\sigma = 0.1$.

The last key parameter is $\xi$. It will be useful to consider the two extreme theoretical values of 0 (only long run expectations) and 1 (only short-run expectations). Since this last value implies a discontinuity that prevents computable simulations, it will by approximated by 0.9. Hence our parameter grid can eventually be generated by different “theoretical” and “empirical” combinations of $(\kappa, \xi)$ as in the Table 3.

The two extreme cases correspond, respectively, to an economy closer to the New-Neoclassical paradigm (NCM) of perfect flexible prices and short-run rational expectations ($\kappa = 0.7, \xi = 0.9$) and to an economy closer to the Old Neoclassical Synthesis (ONS) with sticky prices and static long-run expectations ($\kappa = 0.1, \xi = 0$). As we will see, the more the economy comes closer to the NCM paradigm, the more the model becomes anomalous and eventually unstable. Let me assume a negative interest rate gap of 100

<table>
<thead>
<tr>
<th>Paper</th>
<th>$a$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\xi$</th>
<th>$\theta$</th>
<th>$\sigma$</th>
<th>$\rho$</th>
<th>$K$ (sticky)</th>
<th>$K$ (flex)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Laubach-Williams (2003)</td>
<td>-</td>
<td>0.10</td>
<td>0.10</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.45</td>
<td>0.05</td>
<td>-</td>
</tr>
<tr>
<td>Garnier-Wilhelmsen (2005)</td>
<td>-</td>
<td>0.18</td>
<td>0.07</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.45</td>
<td>0.10</td>
<td>-</td>
</tr>
<tr>
<td>Rotemberg-Woodford (1997)</td>
<td>-</td>
<td>0.16</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>0.40</td>
<td>0.02</td>
<td>-</td>
</tr>
<tr>
<td>Orphanides-Williams (2002)</td>
<td>-</td>
<td>0.02</td>
<td>-</td>
<td>0.50</td>
<td>-</td>
<td>-</td>
<td>0.47</td>
<td>0.14</td>
<td>-</td>
</tr>
<tr>
<td>McCallum-Casares (2000)</td>
<td>-</td>
<td>0.21</td>
<td>0.12</td>
<td>-</td>
<td>0.13</td>
<td>-</td>
<td>0.38</td>
<td>0.11</td>
<td>-</td>
</tr>
<tr>
<td>Tamborini (2010)</td>
<td>0.40</td>
<td>0.15</td>
<td>-</td>
<td>0.50</td>
<td>-</td>
<td>0.10</td>
<td>0.33</td>
<td>0.10</td>
<td>0.67</td>
</tr>
<tr>
<td>This Model</td>
<td>0.40</td>
<td>0.20</td>
<td>0.10</td>
<td>0.50</td>
<td>0.10</td>
<td>0.10</td>
<td>0.40</td>
<td>0.10</td>
<td>0.70</td>
</tr>
</tbody>
</table>

Table 2 Available estimates of the model’s parameters
basis points, $\hat{i}_t = -100$. The ONS economy smoothly settles down to non-zero output and inflation gaps given by (3.35) and (3.36). The impact of interest rate shocks is very big both on output and inflation. This is due to the time-to-build hypothesis that assumes a lag between the installation of production capacity and its ability to be used. Therefore the change in interest rate causes a shift in aggregate demand while aggregate supply will adjust with a delay, in part also because of the presence of adjustment costs of capital. Too low nominal interest rate ends up with too high level of output but with an almost unchanged inflation rate (or with a slight deflation). The latter result is the combined effect of the adjustment of the capital stock and the low elasticity $\kappa$ (Figure 1).

The NCM economy for $\xi > 0.88$ is instead driven on an unstable, explosive path whereby both output and inflation gaps grow exponentially (see equation (3.37)). Although the simulation (Figure 2) does not impose a resources constraint to the output gap, we can assume that sooner or later it becomes binding. In this last case the simulation accounts for Wicksell’s well-known concern with the pure inflationary effects of dynamic expectations: “[A]s long as the change in prices is believed to be temporary, it will in fact remain permanent; as soon as it is considered to be permanent, it will become progressive, and when it is eventually seen progressive it will turn into an avalanche”. (Wicksell, 1922; p. XII, n.1). Of course, starting from too high nominal interest rate $\hat{i}=100$ would yield the specular results with bottomless real and nominal downward spiral for the Neoclassical economy. This latter result vindicates Keynes’s claim that flexible prices and wages - together with short-run dynamic expectations - would worsen the perverse effect of a saving-investment imbalances process triggered by a wrong interest rate (Keynes, 1936, ch. 12).

Finally, the economy portrayed by the NNS paradigm lies somewhere in between, as it can be identified with a combination of the ONS and NMC

<table>
<thead>
<tr>
<th>Weight of short-term rational expectations</th>
<th>Sticky-price parametrization ($k=0.1$)</th>
<th>Intermediate-case parametrization ($k=0.4$)</th>
<th>Flex-price parametrization ($k=0.7$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi=0$</td>
<td>$\alpha'=0.20$, $\beta'=0.10$</td>
<td>$\alpha'=0.20$, $\beta'=0.10$</td>
<td>$\alpha'=0.20$, $\beta'=0.10$</td>
</tr>
<tr>
<td></td>
<td>$\rho'=0.40$, $\theta'=0.10$</td>
<td>$\rho'=0.40$, $\theta'=0.10$</td>
<td>$\rho'=0.40$, $\theta'=0.20$</td>
</tr>
<tr>
<td></td>
<td>$\sigma'=0.10$, $\kappa'=0.10$</td>
<td>$\sigma'=0.05$, $\kappa'=0.40$</td>
<td>$\sigma'=0.10$, $\kappa'=0.70$</td>
</tr>
<tr>
<td>$\xi=0.5$</td>
<td>$\alpha'=0.19$, $\beta'=0.09$</td>
<td>$\alpha'=0.20$, $\beta'=0.09$</td>
<td>$\alpha'=0.21$, $\beta'=0.09$</td>
</tr>
<tr>
<td></td>
<td>$\rho'=0.40$, $\theta'=0.18$</td>
<td>$\rho'=0.43$, $\theta'=0.18$</td>
<td>$\rho'=0.46$, $\theta'=0.18$</td>
</tr>
<tr>
<td></td>
<td>$\sigma'=0.18$, $\kappa'=0.18$</td>
<td>$\sigma'=0.18$, $\kappa'=0.73$</td>
<td>$\sigma'=0.18$, $\kappa'=1.27$</td>
</tr>
<tr>
<td>$\xi=0.9$</td>
<td>$\alpha'=0.12$, $\beta'=0.06$</td>
<td>$\alpha'=0.17$, $\beta'=0.08$</td>
<td>$\alpha'=0.31$, $\beta'=0.02$</td>
</tr>
<tr>
<td></td>
<td>$\rho'=0.44$, $\theta'=0.53$</td>
<td>$\rho'=0.64$, $\theta'=0.53$</td>
<td>$\rho'=1.19$, $\theta'=0.53$</td>
</tr>
<tr>
<td></td>
<td>$\sigma'=0.53$, $\kappa'=0.53$</td>
<td>$\sigma'=0.53$, $\kappa'=2.10$</td>
<td>$\sigma'=0.53$, $\kappa'=3.68$</td>
</tr>
</tbody>
</table>

Table 3 Parameter grid
cases ($\kappa = 0.4, \xi = 0.5$). A negative interest rate gap will lead to a positive output gap and a slight inflation (Figure 3). The effect on inflation will depend mainly on the parameter $\theta$. If $\theta$ is big enough it is not excluded that the final result will not take the form of inflation, although latent inflationary pressures would normally exist, rather it could be an outright deflation. Analyses in a Wicksellian vein of recent episodes of over-investment, such as the U.S. “New Economy” bubble in the late 1990s and the housing and mortgages boom in the last few years, point out the missing inflation puzzle as a critical element in the picture, one that has probably played a role in driving monetary policy onto a wrong track. The evidence seems to contradict the conventional wisdom (Bordo et al., 2000; Howitt, 2012) according to which saving-investment imbalances will not develop in a low and stable inflation environment. Indeed the combination of strong economic growth and low inflation can lead to overly optimistic expectations about the future which could generate increases in credit markets significantly beyond those justified by the original improvement in productivity. Yet, a self-reinforcing boom can emerge supporting, at least for a while, the optimistic expectations. While the stronger demand can put upward pressure on inflation, this pressure can be masked or totally offset by the improvement to the supply side of the economy (Figure 4).

5 Conclusions

The core of the NNS is mostly presented as a three-equation model that combines an intertemporal IS relation with an aggregate-supply (AS) function in terms of a New Keynesian Phillips curve and a Taylor rule for monetary policy. Most macroeconomists nowadays consider it as a general theory of inflation and output dynamics that is capable of generating highly stylized and yet empirically plausible models for policy evaluation. Particular interest has been elicited by Woodford’s argument that the NNS policy implications are a modern restatement and refinement of the basic tenets of Wicksell’s theory of monetary policy, as in Interest and Prices (1898b). One extension of NNS model includes explicit investment dynamics that show how monetary policy can have supply-side effects by way of affecting the productive capacity of the economy.

The NNS framework is - both in its neo-Wicksellian and other versions - precariously based on combinations of continuous intertemporal equilibrium under rational expectations with specific concepts of wage and price stickiness, and other assumptions. The qualifying features of old macroeconomic mainstreams are remarkably absent. While banks play a key role as intermediaries between savers and investors in the real world, the underlying frictions - relative to the Walrasian equilibrium benchmark - are considered only in some of the latest contributions of the NNS (Goodfriend and McCa-
lum, 2007; Curda and Woodford, 2009; Christiano et al., 2010). Likewise, the problems of limited information that produce the dynamics of money creation, income and prices - i.e. the so-called “cumulative process” - are largely excluded from the NNS perspective. These limitations do not allow NNS models to deal with the kind of spillovers and feedbacks that seem to characterized modern economies.

In this paper I have presented a dynamic model in which cycles are driven by saving-investment imbalances. Interest-rate gaps in any period \( t \) give rise to an intertemporal spillover effect, i.e. a sequence of output and inflation gaps. This is a major difference with the NNS model where interest-rate gaps are associated with contemporaneous output gaps only and therefore future output gaps only depend on future interest-rate gaps. Moreover interest-rate gaps produce nominal as well as real effects (gaps) even in a competitive, flex-price economy: this is essentially a Keynesian result, which again marks a major difference with the NNS model - where real effects are only ascribed to sticky prices - but also with Wicksell himself, who did not consider - though did not deny either - real effects\(^{35}\). Lastly, (excess) inflation or deflation are disequilibrium phenomena in three distinct, but interconnected, meanings: a) excess investment or saving are being accommodated at the “wrong” real interest rate, b) the goods market clears at the “wrong” levels of output and inflation, c) the expected inflation rate is “wrong” with respect to the actual inflation rate.

The focus of the alternative framework presented in this paper is mainly on the fact that the natural rate of interest is volatile and that it is not easily transmitted to the capital market. Since the natural rate of interest consists of the marginal efficiency of capital and core inflation, these requirements should apply to both components or at least one. In developed countries with relatively stable and predictable inflation, the candidate to trouble-making remains the marginal efficiency of capital, and in this respect the inflexibility of the nominal market rate of interest determined by the asymmetric information, the heterogeneity of firms, and other New Keynesian explanations that may have a role to play (Greenwald and Stiglitz, 2003; Messori, 1996).

As long as the system has a “nominal anchor” - for example, a given core inflation rate in which agents have reason to believe - the system will converge to a different steady-state equilibrium with output and inflation to be inefficiently high/low with respect to the intertemporal general equilib-

\(^{35}\)Wicksell made incidental mention of the real side of the “cumulative process” (Wicksell, 1898b, p.77). It was Keynes, with his principle of effective demand, who understood that as long as the market real interest rate is “wrong” (e.g. too high) output should take care of adjusting excess saving no matter how deep deflation may be (Keynes, 1936, ch.19). Later, Lindahl (1939), drawing on Wicksell’s theory, included unemployment in his analysis, foreshadowing the modern distinction between cyclical and structural unemployment (Boianovsky and Trautwein, 2006a).
ium. The extent and magnitude of these gaps will depend, among other things, on the presence or absence of nominal rigidities and on the agents’ expectations. Approaching the Neoclassical case with rational expectations and perfect price flexibility, the system becomes more unstable and volatile and, in some cases, it may take divergent trajectories some of which may be explosive. Conversely, if the system has the typical old-Keynesian characteristics - i.e. static long-run expectations and sticky prices - it becomes more stable and predictable.

In general, we saw that saving-investment imbalances could build up also in a low inflation environment. The main reason may be that as long as firms over-invests, the stock of physical capital and thus the productive capacity increases. As a result output grows, excess demand is offset over time and inflation is damped. This type of prediction is like the one made by Casares and McCallum (2000), where the output gap is very sensitive to interest rate, whereas the opposite can be said of inflation. This result can have very important implications in terms of monetary policy. Indeed the authorities may not be able to identify the financial imbalances sufficiently early and with the required degree of comfort to take remedial action. As Leijonhufvud (2007) recently said, the traditional inflation targeting strategy (Svensson, 2010) pursued by many central banks around the world not only will not protect by itself against financial instability, but it might mislead into pursuing a policy that could actively damage financial stability. Recent episodes in the US seem to confirm this view. Probably there has been over-optimism in the NNS paradigm as if it were “End of History” of central banking (Tamborini, 2010a).

Although we have not explicitly treated the problem of monetary policy, we can draw some preliminary insights. In a context of imperfect information the dramatic distinction between “rules” and “discretion” is perhaps a semantic question. The critical elements that eventually determine whether a rule is good or bad are not the parameters but the crucial piece of information about the natural rate of interest: none of the traditional rules produces good results if the central bank is misinformed about this variable. If informational problems with a volatile marginal efficiency of capital are the crux, then interest-rate mechanisms relying upon timely and precise knowledge of the natural rate of interest are inapplicable (Orphanides and Williams, 2002b; 2002a).

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A Appendix

Interest rate gaps and output gaps

Here I provide the proof of Proposition 2, and the allocations that result if, starting in the steady state, the market real interest rate at time $t$, $R_{t+1}$, differs from the natural rate $R^{SS}$. Both rates are assumed to remain constant thereafter.

To begin with, let me recall that the intertemporal general equilibrium is a steady state characterized by fully employed labour force $L^{SS} = 1$, an expected inflation rate $\pi^e = \pi^{SS} \geq 0$, households’ time discount factor equal to the gross real return to the natural interest rate $\Theta = R^{SS}$, constant consumption $C^{SS}$, constant real stock of bonds representative of capital stock $B^{SS} = K^{SS}$, households’ real income given by labour and capital incomes $Y^{SS} = H^{SS} + R^{SS}K^{SS}$. Note that, as a consequence, gross saving and gross investment equal $K^{SS}$, i.e. $S_t' = I_t' = K^{SS}$, and once account is taken of capital replacement, net investment and saving are equal and nil.

Turning now to a period $t$ in which, ceteris paribus, $R_{t+1} \neq R^{SS}$, I first examine households’ optimal consumption path:

$$ C_t = E_t \left[ \frac{C_{t+1}}{R_{t+1}} R^{SS} \right] \quad (A.1) $$

Therefore, from the main text we know that:

$$ S_t' = H_t + R_t K_t - C_t \quad (A.2) $$

Now let me see the notional investment of firms, that is:

$$ I_t' = K_{t+1} = \left( \frac{a}{R_{t+1}} \right)^{\frac{1}{1-a}} \quad (A.3) $$

Following the same procedure as in Smith and Wickens (2006), I plug each period budget constraint (A.2) into households Euler equation (A.1):

$$ H_t + R_t K_t - B_{t+1} = E_t \left[ \frac{H_{t+1} + R_{t+1} B_{t+1} - B_{t+2} R^{SS}}{R_{t+1}} \right] \quad (A.4) $$

The saving-investment inconsistency leads to $B_{t+s} \neq K_{t+s}$, for $s = 1, \ldots, \infty$, where the real value of the stock of bonds purchased by households differs from the actual stock of capital goods purchased by firms at each point in time. By contrast, as long as $R_{t+1} \neq R^{SS}$ the actual consumption path consistent with $B_{t+s} = K_{t+s}$ should satisfy:

$$ Y_t - K_{t+1} = E_t \left[ (Y_{t+1} - K_{t+2}) \frac{R^{SS}}{R_{t+1}} \right] \quad (A.5) $$

where $Y_t = H_t + R_t K_t$ and $Y_{t+1} = H_{t+1} + R_{t+1} K_{t+1}$. 

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Given that $K_{t+1}$ is predetermined in $t$ and that $R_{t+1}$ remains constant with no other shock occurring, we have $E_tK_{t+2} = K_{t+1}$ and

$$Y_t = E_tY_{t+1} \frac{R^{SS}}{R_{t+1}} + K_{t+1} \left(1 - \frac{R^{SS}}{R_{t+1}}\right)$$  \hspace{1cm} (A.6)$$

which is equation (3.13) in the main text.

Now let me re-express equation (A.6) in terms of gaps by dividing both sides by $Y^{SS}$, denoting $\hat{Y} \equiv \frac{Y_t}{Y^{SS}}$, and in terms of the interest-rate gap as of $t$ denoted by $\hat{R}_t \equiv \frac{R_{t+1}}{R^{SS}}$. Therefore:

$$\hat{Y}_t = E_t\hat{Y}_{t+1}\hat{R}_t^{-1} + \frac{K_{t+1}}{Y^{SS}}(1 - \hat{R}_t^{-1})$$  \hspace{1cm} (A.7)$$

Let me first develop the expectational term $E_t\hat{Y}_{t+1}$. This is in fact known with certainty as of $t$. Recall that $E_tY_{t+1} = (K_{t+1})^aE_tL^{1-a}_t$, $Y^{SS} = (K^{SS})^a(L^{SS})^{1-a}$. By the assumption of continuous full employment, $E_tL^{1-a}_t = L^{SS} = 1$. Hence $Y_{t+1}$ is predetermined by the capital stock $K_{t+1}$ chosen by firms in $t$, and therefore, $E_t\hat{Y}_{t+1} = \left(\frac{a}{R^{SS}}\right)^{\frac{1}{1-a}}$, $K^{SS} = \left(\frac{a}{R^{SS}}\right)^{\frac{1}{1-a}}$. We know that $K_{t+1} = \left(\frac{a}{R_{t+1}}\right)^{\frac{1}{1-a}}$, so that:

$$E_t\hat{Y}_{t+1} = \hat{R}_t^{-\frac{a}{1-a}}$$  \hspace{1cm} (A.8)$$

Using the same previous relationship we can develop the second right hand side term in (A.7), i.e. $\frac{K_{t+1}}{Y^{SS}} = \left(\frac{a}{R^{SS}}\right)^{\frac{1}{1-a}}$. Substituting this expression and $E_t\hat{Y}_{t+1}$ into (A.7), after some algebraic manipulations we obtain:

$$\hat{Y}_t = \hat{R}_t^{-\frac{1}{1-a}} \left[1 - a\left(\hat{R}_t^{-1} - \hat{R}^{SS-1}\right)\right]$$

If the rates are sufficiently small, the multiplicative term in parentheses is close to unity, so that:

$$\hat{Y}_t \approx \hat{R}_t^{-\frac{1}{1-a}}$$  \hspace{1cm} (A.9)$$

In order to obtain a log-linear transformation of deviations from the intertemporal general equilibrium steady-state as in standard NNS models, I will employ the usual Uhlig’s procedure (Uhlig, 1999) to (A.6). It will be seen that the result is equivalent to directly taking the logarithms of expressions (A.8)-(A.9)\textsuperscript{36}. Log-deviations are denoted by small-case letters.

\textsuperscript{36}Any variable $X_t$ can be expressed as a deviation factor $\hat{X}_t$ from its steady-state value $X^{SS}$, i.e. $X_t = X^{SS}\hat{X}_t$. The deviation rate is $\hat{X}_t - 1$ and, if “sufficiently small”, it can be approximated by the natural logarithm $\hat{x}_t = \ln X_t - \ln X^{SS}$, such that by approximation, $X_t \approx X^{SS}(1 + \hat{x}_t)$. Any variable $Z_t = F(X_t)$ can be approximated by $Z_t \approx F(X^{SS})(1 + \ln F(X_t))$.\textsuperscript{42}
Therefore:

\[ Y^{SS}(1 + \hat{y}_t) = E_t \left[ Y^{SSa}(1 + \hat{y}_{t+1})(1 - \hat{r}_t) + K^{SS}(1 + \hat{k}_{t+1})\hat{r}_t \right] \]

Dividing for \( Y^{SS} \):

\[ 1 + \hat{y}_t = E_t \left[ (1 + \hat{y}_{t+1})(1 - \hat{r}_t) + \phi(1 + \hat{k}_{t+1})\hat{r}_t \right] \]  \hspace{1cm} (A.10)

where \( \phi = \frac{K^{SS}}{Y^{SS}} = \left( \frac{a}{R^{SS}} \right) \). Now let me proceed by ignoring the second-order non-linear terms \( E_t\hat{y}_{t+1}\hat{r}_t, \phi\hat{r}_t, \phi\hat{k}_t\hat{r}_t \), so that we can use the following approximation\(^{37}\):

\[ \hat{y}_t \approx E_t\hat{y}_{t+1} - \hat{r}_t \]  \hspace{1cm} (A.11)

To compute \( E_t\hat{y}_{t+1} \) consider that:

\[ E_t \left[ Y^{SS}(1 + \hat{y}_{t+1}) \right] = K^{SSa} \left( 1 + a\hat{k}_{t+1} \right) \]

\[ \hat{y}_{t+1} = a\hat{k}_{t+1} \]  \hspace{1cm} (A.12)

Likewise, knowing that \( K_{t+1} = \left( \frac{a}{R_{t+1}} \right)^{\frac{1}{1-a}}, K^{SS} = \left( \frac{a}{R^{SS}} \right)^{\frac{1}{1-a}} \), we obtain:

\[ \hat{k}_{t+1} = -\frac{1}{1-a}\hat{r}_t \]

so that

\[ \hat{y}_{t+1} = -\frac{a}{1-a}\hat{r}_t \]

Substituting this expression into (A.11),

\[ \hat{y}_t \approx -\frac{1}{1-a}\hat{r}_t \]

I now show that this sequence of output gaps, being driven by the common factor \( \hat{r}_t \), can be expressed in a single linear combination behaving like a first-order autoregressive process. Let me denote with \( z_1 = \frac{a}{a-1} \) and \( z_2 = \frac{1}{a-1} \):

\[ \hat{y}_{t+1} = z_1\hat{r}_t \]

\[ \hat{y}_t = z_2\hat{r}_t \]

We can write:

\[ \hat{y}_{t+1} = \rho\hat{y}_t - \alpha\hat{r}_t \]  \hspace{1cm} (A.13)

\(^{37}\)The capital-output ratio in steady state is a decimal number. Indeed \( \phi = \frac{K^{SS}}{Y^{SS}} = \frac{a}{R^{SS}} = \frac{a}{\Theta}, \) where \( 0 < a < 1 \) and \( \Theta > 1 \).
for an appropriate linear combination of parameters where $\rho$ can be interpreted as a spurious correlation between $\hat{y}_{t+1}$ and $\hat{y}_t$. We have that:

$$\alpha = \frac{a - \rho}{1 - a}$$

We know that $\hat{r}_t = r_t - r^{SS}$. Thus, if $r_t = i_t - \pi^{e}_{t+1}$, then we could write $\hat{i}_t = i_t - \pi^{e}_{t+1} - r^{SS}$. That is:

$$\hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t$$

which is equation (3.17) in the main text.

**Inflation gaps**

Firstly, as a result of the assumed Cobb-Douglas technology profit-maximizing output in any $t$ can be expressed as:

$$Y_t = K_t \left( \frac{1 - a}{\omega_t} \right)^{\frac{1-a}{a}}$$

(A.14)

where $w_t = \frac{W_t}{P_t}$ is the current real wage rate, $W_t$ is the nominal one and $P_t$ is the output price.

Secondly, according to the labour market model in the main text, the nominal wage rate for $t+1$ is given by indexing the full-employment real wage rate $\omega_{t+1}$ with the expected inflation rate $\pi^{e}_{t+1}$, i.e. $W_{t+1} = \omega_{t+1}P_t(1+\pi^{e}_{t+1})$. Firms can still adjust output in $t+1$ up to the point where the ensuing marginal product of labour equates the actual real wage rate $w_{t+1} = \frac{W_{t+1}}{P_{t+1}}$, where $P_{t+1} = P_t(1+\pi_t)$. As a result,

$$Y_{t+1} = K_{t+1} \left( \frac{1 - a}{\omega_{t+1}} \right)^{\frac{1-a}{a}}$$

(A.15)

Ceteris paribus, profit-maximizing firms are ready to expand/contract output as long as $\pi_{t+1}$, being greater/smaller than $\pi^{e}_{t+1}$, reduces/increases the actual real wage rate with respect to $\omega_{t+1}$. Conversely, we can derive the Marshallian supply curve of firms, that is, the inflation gap $\Pi_{t+1} = \frac{1+\pi_{t+1}}{1+\pi^{e}_{t+1}}$ which supports a given output gap. Let me divide (A.15) for $Y^{SS}$:

$$\frac{Y_{t+1}}{Y^{SS}} = K_{t+1} \left( \frac{1 - a}{\omega^{*}} \right)^{\frac{1-a}{a}}$$

Setting $\hat{Y}_{t+1} = \frac{Y_{t+1}}{Y^{SS}}$ and $\hat{\Pi}_{t+1} = \frac{\Pi_{t+1}}{\Pi^{SS}}$ we have:

$$\hat{Y}_{t+1} = \hat{K}_{t+1} \left( \frac{1 - a}{\omega^{*}} \right)^{\frac{1-a}{a}} \left( \frac{1 - a}{\omega^{*}} \hat{\Pi}_{t+1} \right)^{\frac{1-a}{a}}$$

We have that $\rho \hat{y}_t - \alpha \hat{i}_t = z_1 \hat{r}_t$, thus $\alpha = z_2 \rho - z_1$. **
and therefore:

\[ \hat{Y}_{t+1} = \hat{K}_{t+1} \left( \tilde{\Pi}_{t+1} \right)^{1-a} \]

and thus:

\[ \tilde{\Pi}_{t+1} = \left( \frac{\hat{Y}_{t+1}}{\hat{K}_{t+1}} \right)^{\frac{a}{1-a}} \]

which is equation (3.16) in the main text. Log-linearizing the expression:

\[ \tilde{\pi}_{t+1} = \frac{a}{1-a} \hat{y}_{t+1} - \frac{a}{1-a} \hat{k}_{t+1} \]

which is equation (3.19) in the main text.

**Adjustment costs of capital**

Let me start from equations (A.10)-(A.12). Substituting equation (3.28) we have:

\[
\hat{y}_{t+1} = a((1 - \psi)\hat{k}_{t+1}^* + \psi\hat{k}_t^*) \\
1 + \hat{y}_t = E_t \left[ (1 + a((1 - \psi)\hat{k}_{t+1}^* + \psi\hat{k}_t^*))(1 - \hat{r}_t) \right] + \\
\quad + E_t \left[ \phi(1 + ((1 - \psi)\hat{k}_{t+1}^* + \psi\hat{k}_t^*))\hat{r}_t \right]
\]

Moreover we know that \( \hat{k}_{t+1} = -\frac{1}{1-a} \hat{r}_t \), thus:

\[
\hat{y}_{t+1} = \frac{a\hat{r}_{t-1}\psi}{a - 1} + \frac{a\hat{r}_t(1 - \psi)}{a - 1} \\
1 + \hat{y}_t = 1 + \hat{r}_{t-1}\hat{r}_t\phi\psi + \frac{a\hat{r}_{t-1}\hat{r}_t\psi}{a - 1} + \frac{a\psi\hat{r}_{t-1}}{a - 1} + \frac{\hat{r}_t^2\phi(1 - \psi)}{a - 1} + \frac{a\hat{r}_t^2(\psi - 1)}{a - 1} + \\
\quad + \hat{r}_t\phi + \frac{\hat{r}_t(a\psi - 1)}{1-a}
\]

where I dropped the expectation operator from the second equation since \( E_t\hat{k}_{t+1} = k_{t+1} \). The quadratic terms \( \hat{r}_t^2 \) and the product of the two decimal numbers \( \hat{r}_t \cdot \phi \) and \( \hat{r}_t^2 \cdot \phi \) are small log-deviations and therefore can be neglected. We get:

\[
\hat{y}_{t+1} = \frac{a(1 - \psi)}{a - 1} \hat{r}_t + \frac{a\psi}{a - 1} \hat{r}_{t-1} \\
\hat{y}_t \approx \frac{a\psi - 1}{1-a} \hat{r}_t + \frac{a\psi}{a - 1} \hat{r}_{t-1}
\]

Let me denote with \( z_1 = \frac{a(1 - \psi)}{a - 1} \), \( z_2 = \frac{a\psi}{a - 1} \), \( z_3 = \frac{a\psi - 1}{1-a} \) and \( z_4 = \frac{a\psi}{a - 1} \):

\[
\hat{y}_{t+1} = z_1\hat{r}_t + z_2\hat{r}_{t-1} \\
\hat{y}_t = z_3\hat{r}_t + z_4\hat{r}_{t-1}
\]
We can write:

\[ \hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{r}_t - \beta \hat{r}_{t-1} \] (A.16)

for an appropriate linear combination of parameters \(\alpha\) and \(\beta\) and where \(\rho\) can be interpreted as a spurious correlation between \(\hat{y}_{t+1}\) and \(\hat{y}_t\). We have that:

\[
\alpha = \frac{\rho - a[1 + \psi(\rho - 1)]}{a - 1}
\]

\[
\beta = \frac{a\psi(\rho - 1)}{a - 1}
\]

We know that \(\hat{r}_t = r_t - r^{SS}\). Thus, if \(r_t = i_t - \pi^e_t\), then we could write \(\hat{r}_t = i_t - \pi^e_t - r^{SS}\). That is:

\[ \hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t - \beta \hat{i}_{t-1} \]

which is equation (3.29) in the main text.

Let me consider now equations (3.18) and (3.28). Combining these two equations we get:

\[ \pi_{t+1} = \pi^e_{t+1} + \frac{a}{1-a} \hat{y}_{t+1} + \frac{a}{(1-a)^2} \left[(1-\psi)\hat{i}_t + \psi \hat{i}_{t-1}\right] \]

and setting \(\tilde{\pi}_{t+1} = \pi_{t+1} - \pi^e_{t+1}, \kappa = \frac{a}{1-a}, \theta = \frac{a(1-\psi)}{(1-a)^2}\) and \(\sigma = \frac{a\psi}{(1-a)^2}\) we get:

\[ \tilde{\pi}_{t+1} = \kappa \hat{y}_{t+1} + \theta \hat{i}_t + \sigma \hat{i}_{t-1} \]

which is equation (3.30) in the main text.

**Inflation expectations**

I can write equations (3.17) and (3.29) as follows:

\[ \hat{y}_{t+1} = \rho \hat{y}_t - \alpha (i_t - \pi^e_{t+1} - r^*) - \beta \hat{i}_{t-1} \] (A.17)

\[ \pi_{t+1} - \pi^e_{t+1} = \kappa \hat{y}_{t+1} + \theta (i_t - \pi^e_{t+1} - r^*) + \sigma \hat{i}_{t-1} \] (A.18)

Since \( E_t \pi_{t+1} = \pi_t \) (short run rational expectation hypothesis), I substitute \(\pi^e_{t+1} = \xi \pi_{t+1} + (1 - \xi)\pi^*\) in the (A.18) and I get:

\[ \pi_{t+1} - \xi \pi_{t+1} - (1 - \xi)\pi^* = \kappa \hat{y}_{t+1} + \theta (i_t - \pi^e_{t+1} - r^*) + \sigma \hat{i}_{t-1} \]

e then, by setting \(\tilde{\pi}_{t+1} = \pi_{t+1} - \pi^*, I have:

\[ \tilde{\pi}_{t+1} = \kappa' \hat{y}_{t+1} + \theta' \hat{i}_t + \sigma' \hat{i}_{t-1} \] (A.19)

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39We have that \(\rho \hat{y}_t - \alpha \hat{r}_t - \beta \hat{r}_{t-1} = z_1 \hat{r}_t + z_2 \hat{r}_{t-1}\), thus \(\alpha = z_3 \rho - z_1\) and \(\beta = z_4 \rho - z_2\).
where

\[ \kappa' = \frac{\kappa}{1 - \xi(1 - \theta)} \]
\[ \theta' = \frac{\theta}{1 - \xi(1 - \theta)} \]
\[ \sigma' = \frac{\sigma}{1 - \xi(1 - \theta)} \]

Let me reconsider equation (A.17). By similar substitutions I have:

\[ \hat{y}_{t+1} = \rho \hat{y}_t - \alpha_i t + \alpha \xi \pi_{t+1} + \alpha (1 - \xi) \pi^* + \alpha r^* - \beta \hat{i}_{t-1} \]

thus:

\[ \hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t + \alpha \xi (\pi_{t+1} - \pi^*) - \beta \hat{i}_{t-1} \]

substituting I get:

\[ \hat{y}_{t+1} = \rho \hat{y}_t - \alpha \hat{i}_t + \alpha \xi (\kappa' \hat{y}_{t+1} + \theta' \psi \hat{i}_t + \sigma' \hat{i}_{t-1}) - \beta \hat{i}_{t-1} \]

and inserting the values of \( \kappa' \), \( \theta' \) and \( \sigma' \) I get:

\[ \hat{y}_{t+1} = \rho' \hat{y}_t - \alpha' \hat{i}_t - \beta' \hat{i}_{t-1} \quad \text{(A.20)} \]

where:

\[ \rho' = \rho \frac{1 - \xi (1 - \theta)}{1 - \xi (1 + \alpha \kappa - \theta)} \]
\[ \alpha' = \alpha \frac{1 - \xi}{1 - \xi (1 + \alpha \kappa - \theta)} \]
\[ \beta' = \beta \frac{1 - \xi (1 - \theta) - \alpha \xi \sigma}{1 - \xi (1 + \alpha \kappa - \theta)} \]

The model will then consist of the equations (3.34) and (3.34) in the main text.

Let me consider now the steady-state conditions (3.35) and (3.36). The steady-state output-gap is given by:

\[ \hat{y} = \frac{\alpha' + \beta'}{1 - \rho'} i_0 = \frac{\alpha + \beta - \xi [\alpha (1 + \sigma) + \beta (1 - \theta)]}{1 - \rho - \xi [\alpha \kappa + (\theta - 1) (\rho - 1)]} i_0 \]

this quantity is greater than zero only if:

\[ \xi < \frac{\alpha + \beta}{\alpha (1 + \sigma) + \beta (1 - \theta)} \]
which is always satisfied for plausible values of the parameters. Let me compute the effect of $\xi$ on the output-gap:

$$\frac{\partial \hat{y}}{\partial \xi} = \alpha \left[ \frac{\kappa + (\alpha + \beta)(\rho - 1)}{(1 - \xi(1 - \theta))(\rho - 1) + \alpha \kappa \xi^2} \right]$$

which is positive when $(\alpha + \beta)\kappa > (\theta + \sigma)(1 - \rho)$ and negative otherwise. It is simply to note that for structural values of the parameters, $\hat{y}$ decreases when $\xi$ increases. On the contrary, empirically an increase in $\xi$ determines an increase in $\hat{y}$ (in absolute value).

Let me consider now equation (3.36). The steady-state is:

$$\hat{\pi} = \hat{\pi} = \left[ \theta' + \sigma' - \kappa' (\alpha' + \beta') \right] \hat{i}_0 = \frac{(\alpha + \beta)\kappa + (\theta + \sigma)(\rho - 1)}{(1 - \xi(1 - \theta))(\rho - 1) + \alpha \kappa \xi} \hat{i}_0$$

It is simply to check that this coefficient, whatever the value fo $\xi$, is always greater than zero for the structural values of the parameters, while it could be negative if $\theta$ and $\sigma$ are small. Moreover we have that:

$$\frac{\partial \hat{\pi}}{\partial \xi} = \frac{[(\alpha + \beta)\kappa + (\theta + \sigma)(\rho - 1)] \cdot [\alpha \kappa + (\theta - 1)(\rho - 1)]}{[(1 - \xi(1 - \theta))(\rho - 1) + \alpha \kappa \xi]^2}$$

An increase of $\xi$ determines an increase (in absolute value) of $\hat{\pi}$. In other words, forward-looking expectations are deviation-amplifying in steady state.

Finally it is easy to show that the system converges to $[\hat{y}; \hat{\pi}]$ only if $\xi$ satisfies the following condition:

$$\xi < \frac{1 - \rho}{\alpha \kappa + (\theta - 1)(\rho - 1)}$$

otherwise the system may take different trajectories, some of which may be explosive.
Figure 1 Old Neoclassical Synthesis case - $\kappa = 0.1$, $\xi = 0$

Figure 2 Neoclassical case - $\kappa = 0.7$, $\xi = 0.9$
Figure 3 Intermediate case - $\kappa = 0.4$, $\xi = 0.5$

Figure 4 High elasticity of capital stock to interest rate
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