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Ricardo Alberto MARQUES PEREIRA
Dipartimento di Informatica e Studi Aziendali
Università degli Studi di Trento
Via Inama 5, TN 38122 Trento ITALIA
Tel +39-0461-282147 Fax +39-0461-282124
E-mail: ricalb.marper@unitn.it

Timing of Investment and Dynamic Pricing in Privatized Sectors*

Sandro Brusco§ Ornella Tarola¶ Sandro Trento‖

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Abstract

In equipment-intensive sectors – such as water utilities, power generation, gas – billions of dollars are spent in capital equipment. We discuss and characterize the optimal policy of a profit-maximizing firm and compare it with the optimal policy of a welfare-maximizing planner. When there is no technical progress, the duration of the plant is longer for a private firm. With technical progress, we show that duration tends to increase when the installed capacity increases over time, while it tends to decrease when technical progress reduces operating costs. Under some conditions we also show that when capacity expands over time the duration of the plant is shorter for a public firm than for a private firm.

JEL Classification: D21, L21, L23.
Keywords*:* Dynamic investment, privatization.

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§Department of Economics, Stony Brook University, Stony Brook, NY 11794. E-mail: sandro.brusco@stonybrook.edu

¶DAES, University of Rome 'La Sapienza', Piazzale Aldo Moro 5. Phone number: +39-06-49910253, fax number +39-06-49910231. E-mail: ornella.tarola@uniroma1.it

‖Dipartimento di informatica e studi aziendali, Facoltà di Economia, Università di Trento, via Inama, 5 – I-38100 Trento. E-mail: sandro.trento@economia.unitn.it.

1 Introduction

In equipment–intensive sectors – e.g. most utilities – large amounts are spent each year in new capital. Investment is directed either at replacing existing equipment or expanding capacity, benefiting at the same time from the technological improvements embedded in new capital. When deciding the level of investment at any moment in time the main trade-off is between taking immediate advantage of the new capacity or postponing the investment, thus maintaining flexibility in order to exploit future technological progress.

Many public utilities – traditionally in charge of operating service – have been privatized. Incentives for a private firm are obviously different from the ones of a public firm. It is reasonable, for example, to expect a more aggressive use of the pricing policies[1]. Due to the extent of these privatizations, a large empirical literature has analyzed utilities' performance under private ownership. The picture that emerges is mixed[2]. Firms moving from a public to private ownership go from being highly unprofitable to meeting strong efficiency requirements. A significant increase in output and service quality has also been reported (see, among others, Andres et al. [1], Bortolotti et al. [6], Brown et al. [7], Boubakri and Cosset [2]). The evidence on the investment pattern in the maintenance and expansion of utility networks is instead rather mixed, being mainly related to the institutional environment where privatization takes place, utilities' inefficiency under public ownership and the chance to introduce new technologies (Cambini and Rondi [8], Estache and Rossi [15], Gassner et al. [18]).

In the theoretical literature, a problem which partially resembles the joint determination of investment and price over time has been considered by inventory theory. Recent developments in this field have focussed on the problem of coordinating price and inventory replenishment under various assumptions (see e.g. Federgruen and Heching [16], Karakul [19], and Webster and Weng [27]). Yano and Gilbert [26] and Elmaghraby and Keskinocak [14] provide surveys on pricing and inventory control.

We analyze a similar problem under the assumption that the instan-

[1] See for example Chen and Simchi–Levi [10], McGill and van Ryzin [21], and Cook [11].

[2] Privatizations were implemented in sectors and countries with vastly different institutional, political and economic environments. The procedures adopted have also been quite diverse. Thus, it is not surprising that a mixed picture on utilities' performance has emerged. See Megginson and Netter [22] for a survey of empirical studies on privatization. Further analysis can be found in in Bortolotti et al. [5] and Cambini and Rondi [9].

taneous demand function is stationary and that physical deterioration of equipment reduces production capacity over time. We first determine the optimal pricing and investment policy under the assumption that there is no technical progress[3] and we next extend the analysis to the case in which technical progress increases the productivity of capital over time. It is worth noting that taking into account the impact of technological progress on production capacity, this paper brings together the inventory problem literature and the expansion capacity literature, initiated by Manne [20] and developed afterward by D'Aspremont, Gabszewicz and Vial [12], Gabszewicz and Vial [17], and more recently by Demichelis and Tarola [13] and Tarola [23].

In order to capture the lumpy nature of equipment investment, we assume that firms are restricted to investments of fixed size. The optimal investment policy can then be described as a countable sequence of points at which the investments are made. Furthermore, the optimal price pattern between two dates at which new plants are installed falls into two categories: either the capacity constraint is not binding, so that the instantaneous monopoly price is used, or the capacity constraint is binding and the price is higher than the monopoly price.

In the case in which there is no technical progress we show that a profit-maximizing firm replaces its production capacity less frequently than a welfare-maximizing firm would do. Even in the static case a welfare-maximizing firm produces more than a profit-maximizing firm. In the dynamic case this effect is magnified, since the more frequent rate of plant replacement implies that on average capacity constraints are less binding for the welfare-maximizing firm than for the profit-maximizing firm. It is also worth noting that, when the cost function is convex, this implies that in general the marginal cost at which the firm produces is higher when welfare, rather than profit, is maximized. This, it should be noticed, is an effect that holds even when public firms achieve productive efficiency. Of course cost will be even higher if there is productive inefficiency, which has been the main focus of attention in the privatization process.

When there is technical progress the results depend on the particular way in which the progress occurs. We show that when the installed capacity expands over time, the duration of the plant increases and, under some conditions (low installation cost and sufficiently large capacity), it remains true that public firms have a shorter duration than private firms. The case in

[3] Our problem is also related to the macroeconomic literature on models with vintage capital. For recent developments see Van Hilten [25], Boucekkine, Germain and Licandro [3], Boucekkine, del Rio and Licandro [4].

which technical progress leads to lower operating costs is more complicated, although under some conditions it is possible to show that plant duration decreases over time.

The rest of the paper is organized as follows. The basic model without technical progress is described in section 2. Thus, the optimal policy is fully characterized. The model with technical progress in analyzed in section 3. Concluding remarks are in section 4, and an Appendix collects the proofs.

2 The Model without Technical Progress

A monopolist with an infinite horizon faces a continuous and strictly decreasing inverse demand function $p(q)$, where q is the instantaneous quantity. At each time t the amount that can be produced is limited by the production capacity $x(t)$. Producing a quantity $q \leq x(t)$ at time t has an instantaneous cost of $c(q)$ and it is impossible to produce quantities larger than $x(t)$.

After installation, capacity decreases continuously over time. The only way to increase production capacity is to replace the old equipment, building a new plant. In other words, the sequence of replacements may only consist of lumpy investments whose size is constant and equal to $\overline{x}$. Installing a new equipment of size $\overline{x}$ has a cost of k. For the moment we assume that no technical progress takes place, so that both the quantity $\overline{x}$ and the cost function $c(q)$ are fixed over time. If a new plant of size $\overline{x}$ is installed at time t_i and no further installation occurs, then the capacity at any time $t = t_i + s$ is given by the function

$$x(t) = \overline{x} g(s), \tag{1}$$

where the function $g(s)$ is continuous, positive, strictly decreasing and it satisfies $g(0) = 1$ and $\lim_{s \to +\infty} g(s) = 0$. Let

$$q^m = \arg\max_q \quad p(q)\, q - c(q)$$

be the monopoly quantity computed when no capacity constraints are present (if there are multiple quantities let q^m be the lowest such quantity). We assume $\overline{x} > q^m$, i.e. a newly installed plant has a capacity which is sufficient to produce the monopoly quantity.

The policy of the firm can be represented by a *quantity policy* $q(t)$ which specifies the quantity produced by the monopolist at each instant t, and by an investment policy $\mathbf{t} = (t_1, t_2, ...t_i...)$ specifying the dates at which a new plant of size $\overline{x}$ is built. The present value of profits is given by

$$V(\mathbf{t}, q(t)) = \int_0^\infty e^{-rt} (p(q(t)) q(t) - c(q(t))) \, dt - \sum_{i=0}^{\infty} k e^{-rt_i},$$

where $t_0 = 0$. A policy is *optimal* if it maximizes $V(\mathbf{t}, q(t))$ under the capacity constraint

$$q(t) \leq \overline{x} g(t - t_i) \qquad \text{each } t \in [t_i, t_{i+1}) \text{ and } i = 0, 1, 2, \ldots$$

In the next section we will discuss and compare the optimal policy for a profit-maximizing firm and for a welfare-maximizing regulator.

2.1 The Optimal Policy

2.1.1 Optimal Policy for a Profit–Maximizing Firm

We start observing that, since there are no intertemporal links among the quantities determined at different moments in time, at each instant $t \in [t_i, t_{i+1})$ the quantity can be chosen to solve

$$\max_q \quad p(q) q - c(q)$$

subject to

$$q \leq x(t - t_i).$$

The solution to the problem is simple. If at time t we have $q^m \leq x(t - t_i)$ then optimality requires $q(t) = q^m$. Thus, if we define s^* as the solution to

$$\overline{x} g(s) = q^m \qquad \Longrightarrow \qquad s^* = g^{-1}\left(\frac{q^m}{\overline{x}}\right),$$

i.e. s^* is the amount of time that it takes for the capacity to shrink to the level q^m, then the quantity produced will remain constant over the interval $[t_i, t_i + s^*]$. After that, the capacity constraint becomes binding and the optimal quantity has to decrease, which in turn implies that the price ends up being higher than the monopoly price. The optimal policy takes a particularly simple form when the profit function is increasing in q up to the point q^m. In that case the quantity produced is q^m until $t_i + s^*$ and then $q(t) = x(t - t_i)$ until replacement occurs.

Define $q(x)$ as the solution to

$$\max_q \quad p(q) q - c(q)$$

subject to

$$q \leq x,$$

and let

$$\pi(x) = p(q(x))\,q(x) - c(q(x))$$

be the highest instantaneous profit achievable when the capacity is x. Since the optimal quantity at each time t depends only on the capacity constraint, the maximization problem for the firm can be simplified by looking only at the replacement times $\{t_i\}_{i=1}^{\infty}$. The problem of the firm can then be written as

$$\max_{\{t_i\}_{i=1}^{\infty}} \qquad \sum_{i=0}^{\infty} e^{-rt_i} \left(\int_{t_i}^{t_{i+1}} e^{-r(t-t_i)} \pi\left(\overline{x} g\left(t - t_i\right)\right) dt - k \right) \tag{2}$$

with the initial condition $t_0 = 0$.

We will make the following assumption.

Assumption 1 $\int_0^{+\infty} e^{-rt} \pi\left(\overline{x} g(t)\right) dt > k$.

This is an extremely weak feasibility assumption. It says that a monopolistic firm is able to obtain strictly positive profits, once installment costs are taken into account. The assumption implies the following lemma.

Lemma 1 *Under Assumption 1 the optimal policy requires that replacement occurs in finite time.*

The logic of the result is simple. Since capacity goes to zero as t goes to infinity, if it is profitable to build a plant at time 0 then it must be profitable to build a new plant when the capacity is sufficiently depleted.

To understand the solution to problem (2) we start observing that, absent technological progress, the problem is stationary. At each time t_i at which a new plant is installed, the problem is identical to the one faced by the firm at time 0. This implies that we can restrict attention to policies in which the optimal duration Δ of a plant is constant. The next proposition further characterizes the solution.

Proposition 1 *There is a solution to problem (2) such that replacement occurs at a constant finite interval Δ, i.e. $t_{i+1} - t_i = \Delta$ for each i. The optimal duration of a cycle is given by the lowest solution to*

$$\frac{\pi\left(x\left(\Delta\right)\right)}{r} = \frac{\int_0^{\Delta} e^{-rs} \pi\left(x\left(s\right)\right) ds - k}{1 - e^{-r\Delta}}, \tag{3}$$

and the solution is such that $x(\Delta) < q^m$.

Notice that the RHS of equation (3) is the present value of profits when the plant is replaced at intervals of length Δ. This implies that the present value of profits is equal to the one that would be obtained with a constant stream of value $\pi(x(\Delta))$.

How does Δ depends on the parameters of the model? It turns out that the optimal length increases with the cost of installation. The impact of changes in the initial capacity $\overline{x}$ turns out to be more difficult to establish, as it depends on the rate at which the profit $\pi(\overline{x}g(t))$ changes over time.

Proposition 2 *The optimal length Δ is increasing in k. If $\frac{d\pi(\overline{x}g(t))}{dx}g(t)$ is decreasing in t then the optimal length is non-increasing in $\overline{x}$. If $\frac{d\pi(\overline{x}g(t))}{dx}g(t)$ is increasing in t then the optimal length is non-decreasing in $\overline{x}$.*

The intuition is very simple for the cost of installation. If k increases then new plants become more costly. At any given moment in time the trade-off is between obtaining the profit $\pi(\overline{x}g(t))$ at no additional cost or starting a new cycle at a cost k. If the cost of starting a new cycle goes up, then avoiding replacement becomes more attractive. Thus replacement will occur only when the instantaneous profit is lower. Since instantaneous profit decreases over time, this requires a longer length.

The intuition with respect to $\overline{x}$ goes as follows. First observe that $\frac{d\pi(\overline{x}g(t))}{dx}g(t)$ is the derivative of π with respect to $\overline{x}$. A marginal increase in $\overline{x}$ increases capacity, and therefore weakly increases profits, at each point at which the plant is in place. If $\frac{d\pi(\overline{x}g(t))}{dx}g(t)$ is increasing, this means that the marginal increase is larger when t is large. Thus, while both the current profit $\pi(\overline{x}g(t))$ and the present value of profits obtained installing a new plant increase, the former increases more than the latter. This makes it optimal to wait a little bit longer to replace the plant. The opposite occurs when $\frac{d\pi(\overline{x}g(t))}{dx}g(t)$ is decreasing. Observe that

$$\frac{d}{dt}\left[\frac{d\pi(\overline{x}g(t))}{dx}g(t)\right] = \frac{d^2\pi(\overline{x}g(t))}{d^2x}\overline{x}g^2(t) + \frac{d\pi(\overline{x}g(t))}{dx}g'(t).$$

Since $\frac{d\pi}{dx} \geq 0$ and $g'(t) < 0$, a sufficient condition for $\frac{d\pi(\overline{x}g(t))}{dx}g(t)$ to be decreasing is $\frac{d^2\pi}{d^2x} \leq 0$. This is the case, for example, when the revenue function $p(q)q$ is concave and the cost function is convex. In that case the optimal policy is to set $q = x$ when $x \leq q^m$ and $q = q^m$ otherwise, and it is easy to check that $\frac{d^2\pi}{d^2x} \leq 0$ for each x.

2.1.2 Optimal Policy for a Welfare-Maximizing Regulator

Suppose now that the firm is controlled by a regulator whose goal is to maximize social welfare. At each instant t the regulator solves

$$\max_q \quad \int_0^q p(y)\,dy - c(q)$$

$$\text{s.t.} \quad q \leq x(t)\,.$$

Let q^w be the quantity that maximizes social welfare when there is no capacity constraint. As in the previous case, if $x(t) \geq q^w$ then clearly the optimal policy is to set the quantity at q^w, attaining the maximum welfare level. If $x(t) < q^w$ then let $w(x(t))$ be the welfare attainable. The same analysis performed for the profit maximizing case can now be applied, with the only difference that instead of $\pi(x(t))$ we should use $w(x(t))$. We state this result without proof.

Proposition 3 *Social welfare is maximized by renewing the plant at a constant interval Δ, where Δ is the lowest solution to*

$$\frac{w(x(\Delta))}{r} = \frac{\int_0^{\Delta} e^{-rs} w(x(s))\,ds - k}{1 - e^{-r\Delta}}. \tag{4}$$

The optimal interval Δ increases in k. It increases in $\overline{x}$ if $\frac{dw(\overline{x}g(t))}{dx} g(t)$ is increasing, while it decreases in $\overline{x}$ if $\frac{dw(\overline{x}g(t))}{dx} g(t)$ is decreasing.

Let Δ^m be the lowest solution to equation (3) and Δ^w the lowest solution to (4). What is the relation between the two optimal durations? Will a profit-maximizing firm renew plants more or less often than a welfare-maximizing regulator? The next proposition establishes that the optimal duration is shorter for a regulator.

Proposition 4 $\Delta^w < \Delta^m$, *i.e. the welfare-maximizing regulator invests in new plants more frequently than a profit-maximizing firm.*

The intuition is that the relevant objective function decreases faster over time (i.e. as capacity shrinks) for the regulator, since consumer surplus is taken into account. If a profit-maximizing firm wants to renew the plant when capacity reaches the value $x(\Delta^w)$, it must be that the instantaneous value of the profit equals the value of the profit stream (net of investment cost) obtained by changing the plant. Since this ignores the change in consumer surplus, the point of indifference must be reached by the regulator at a higher level of capacity, which in turn implies a shorter time for renewal.

What does this imply in terms of productive efficiency? If we consider a static model, the welfare-maximing firm produces more than the profit-maximizing firm. If the cost function $c(q)$ is convex this implies that in general the marginal cost at which the public firm produces is higher. Since we assume that the only thing that deteriorates over time is the capacity, while $c(q)$ does not change, Proposition (4) implies that this effect will be amplified in a dynamic model in which the time of replacement is optimally chosen. Since the welfare maximizing regulator renews the plant more often, capacity will on average be larger than for a profit maximizing firm. This increases the difference between the quantities produced by a regulator and a profit maximizer, which in turn increases the difference between marginal costs. Of course the opposite will occur when the cost function $c(q)$ is concave: in that case it is the welfare-maximizing firm which ends up producing at a lower marginal cost, and the effect is enhanced by dynamic consideration.

Here it is important to understand that we are ignoring the possibility that over time a plant becomes more inefficient (unit costs are higher for older plants) and that a public firm may not achieve productive efficiency. Both features are likely to be relevant in practice.

3 The Model with Technical Progress

Up to now we have assumed that technical progress is absent. This is clearly not a realistic assumption and it is therefore interesting to explore what happens when there is an expectation that the technology will improve over time.

There are many ways in which technical progress can affect the production function of the firm. Over time, plants can get better either because the cost of installation goes down, because the operating cost goes down or because the capacity goes up. The first aspect can be modeled by indexing to t the replacement cost, so that now $k(t_i)$ is the cost paid for replacing a plant at time t_i and $k(\cdot)$ is a decreasing function. Similarly, improvements in technology leading to a decrease in the cost of production can be modelled making the cost function dependent on the time of installation, so that $c(q, t_i)$ is the cost of producing quantity q when the plant has been installed at time t_i. Since over time the technology gets better, the function c is (at least weakly) decreasing in the second variable for any given q. At last, if technical progress expands the capacity at the time of installation, this can be modeled assuming that the newly installed capacity depends on the time of installation according to a function $\overline{x}(t_i)$. Thus the capacity available at time $t > t_i$ when a plant has been installed at t_i is $\overline{x}(t_i)\, g(t - t_i)$, where $\overline{x}(t_i)$ is increasing. Notice that the presence of technical progress does not change the basic fact that at each instant t the firm (or the regulator) can choose the quantity $q(t)$ to maximize the instantaneous objective function, since quantities have no intertemporal links across periods.

3.1 Profit Maximization

For a profit maximizing firm, define $\pi(s, t_i)$ as the value of the objective function obtained solving

$$\max_q \quad p(q)\, q - c(q, t_i)$$

subject to

$$q \leq \overline{x}(t_i)\, g(s)$$

where $s = t - t_i$ is the time passed since the current plant was installed. Given our assumptions, the function π is non-increasing in s and non-decreasing in t_i. We will maintain the assumption that at each date t_i the new installed capacity $\overline{x}(t_i)$ allows the firm to attain the monopoly profit $\pi^m(t_i)$ (the monopoly profit may depend on t_i if the cost function c depends on t_i).

The maximization problem now becomes

$$\max_{\{t_i\}_{i=1}^{\infty}} \qquad \sum_{i=0}^{\infty} \left(\int_{t_i}^{t_{i+1}} e^{-rt} \pi(t - t_i, t_i)\, dt - e^{-rt_i} k(t_i) \right)$$

with the initial condition $t_0 = 0$. A more general way to write the problem, which will turn out to be useful, is to write the objective function at any given arbitrary point t_i as

$$\widehat{V}(t_i, \mathbf{\Delta}) =$$

$$\sum_{q=1}^{\infty} e^{-r \sum_{j=0}^{q-1} \Delta_j} \left[\int_0^{\Delta_q} e^{-rs} \pi \left(s, t_i + \sum_{j=0}^{q-1} \Delta_j \right) ds - k \left(t_i + \sum_{j=0}^{q-1} \Delta_j \right) \right].$$

where $\mathbf{\Delta} = \{\Delta_j\}_{j=0}^{+\infty}$ and Δ_j is the planned duration of a plant installed at time t_{i+j-1} (we adopt the convention $\Delta_0 = 0$). More in general, define

$$V(t_i) = \sup_{\mathbf{\Delta}} \quad \widehat{V}(t_i, \mathbf{\Delta})$$

as the present discounted value that can be achieved when a new plant is installed with the technology available at time t_i. Notice that the following relationship

$$V(t_i) = \max_{\Delta \geq 0} \int_0^{\Delta} e^{-rs} \pi(s, t_i)\, ds - k(t_i) + e^{-r\Delta} V(t_i + \Delta) \qquad (5)$$

must hold at any time t_i at which an installation occurs. We will make the following assumption on the evolution over time of the technology.

Assumption 2 *For each s, the function $\pi(s, t_i)$ is non-decreasing and concave in t_i and bounded above by a real number $\overline{\pi}$ and below by 0. The function $k(t_i)$ is decreasing and convex in t_i and $\lim_{t\to\infty} k(t) = \underline{k} > 0$.*

The assumption states that, for any given duration of the plant, the firm can achieve (weakly) higher profits if the plant has been installed at a later date. This is uncontroversial, as it is basically the same as saying that there is technical progress over time (remember that demand is stationary); boundedness is also uncontroversial. The assumption that $\pi(s, t_i)$ is concave in t_i is more substantive, as it says that the rate of technical progress slows down over time. Similar considerations apply to the function $k(t_i)$.

Using boundedness of the profit function and the assumption on the long run behavior of the replacement cost we have the following Lemma.

Lemma 5 *Under assumption 2, there exists a value Δ^* such that each installed plant has a duration of at least Δ^*, i.e. under the optimal policy $t_{i+1} - t_i \geq \Delta^*$ for each i.*

Since the installation cost is bounded below by $\underline{k} > 0$, it cannot be optimal to let the time between two installation go to zero, since the value of the profit earned between the two periods would inevitably be lower than the installation cost. The Lemma allows us to use standard results in dynamic programming to obtain the following result.

Proposition 6 *The function $V(t_i)$ is increasing, differentiable and concave.*

The proposition implies that the optimal length of the plant at time t_i can be obtained from the first order condition of problem (5) i.e.:

$$\pi(\Delta, t_i) + V'(t_i + \Delta) = rV(t_i + \Delta). \qquad (6)$$

The concavity of V implies that the LHS is decreasing, while the RHS is increasing, so that the equation has a unique solution.

Without further information on the profit function, it is difficult to say anything on how the duration of the plant varies over time. If we differentiate both sides of (6) with respect to t_i, we obtain the following expression for $\frac{d\Delta}{dt_i}$:

$$\frac{d\Delta}{dt_i} = \frac{rV'(t_i + \Delta) - V''(t_i + \Delta) - \frac{\partial\pi(\Delta, t_i)}{\partial t_i}}{\frac{\partial\pi(\Delta, t_i)}{\partial\Delta} + V''(t_i + \Delta) - rV'(t_i + \Delta)} \tag{7}$$

The denominator of the RHS in (7) is negative but the sign of the numerator is undetermined.

To better understand the circumstances under which plant duration increases or decreases, suppose $k(t) = k$ (installation cost is constant), so that $k'(t) = 0$. Let $\left\{\Delta_j^*\right\}_{j=1}^{\infty}$ be the optimal policy at time t_i and observe:

$$V(t_i) = \sum_{q=1}^{\infty} e^{-r\sum_{j=0}^{q-1}\Delta_j^*} \left(\int_0^{\Delta_q^*} e^{-rs}\pi\left(s, t_i + \sum_{j=1}^{q-1}\Delta_j^*\right) ds - k \right).$$

The envelope theorem implies

$$\begin{aligned} V'(t_i) &= \sum_{q=1}^{\infty} e^{-r\sum_{j=0}^{q-1}\Delta_j^*} \left(\int_0^{\Delta_q^*} e^{-rs} \frac{\partial\pi\left(s, t_i + \sum_{j=1}^{q-1}\Delta_j^*\right)}{\partial t_i} ds \right) \\ &= \sum_{q=0}^{\infty} e^{-r(t_{i+q} - t_i)} \left(\int_0^{\Delta_q^*} e^{-rs} \frac{\partial\pi(s, t_{i+q})}{\partial t_{i+q}} ds \right) \end{aligned} \tag{8}$$

A sufficient condition for $\frac{d\Delta}{dt_i} \leq 0$ is $rV'(t_{i+1}) \geq \frac{\partial\pi(t_{i+1}, t_i)}{\partial t_i}$. As it can be seen from (8), $rV'(t_{i+1})$ is a weighted average of the future values of $\frac{\partial\pi}{\partial t_i}$, i.e. the sensitivity of profit to technical progress. Thus, how the optimal plant duration evolves depends on the future impact of technical progress compared to the current impact. To better understand the issue, let's consider two special subcases.

3.1.1 Technical Progress in Capacity Only

Suppose first that $c(q, t_i) = c(q)$ and $k(t_i) = k$, so that the only effect of technical progress is to expand the capacity installed. This means that the monopoly quantity q^m remains the same. In this case the duration of a plant is bounded below by the length of time needed for capacity to reach q^m, the monopoly quantity. The lower bound increases over time, providing a reason for a longer duration of the plant. Suppose that $x(t_i)$ becomes very large with respect to q^m as time increases. In that case most of the time will be spent producing q^m. When production is constant we have $\frac{\partial \pi}{\partial x} = 0$, which in turn implies that $\int_0^{\Delta_q^*} e^{-rs} \frac{\partial \pi(s,t_{i+q})}{\partial t_{i+q}} ds$ will be close to zero. On the other hand $\frac{\partial \pi(t_{i+1},t_i)}{\partial t_i}$ is strictly positive, since at time t_{i+1}, write before substituting the plant, the firm is capacity constrained. We conclude that in this case it is likely that $rV'(t_{i+1}) < \frac{\partial \pi(t_{i+1},t_i)}{\partial t_i}$. If the concavity of V is not too pronounced, so that V'' is close to zero, then we have $\frac{d\Delta}{dt_i} \geq 0$.

3.1.2 Technical Progress in Operating Cost Only

Suppose now that $\overline{x}$ is constant over time, while technical progress decreases the cost of production $c(q,t)$. In that case the reduction in cost increases over time the monopoly quantity $q^m(t)$, thus shortening the amount of time necessary for the capacity constraint to become binding. This implies that $\frac{\partial \pi(s,t_i)}{\partial t_i}$ becomes larger for higher values of t_i, as the firm bumps more quickly in the capacity constraints. We conclude that in this case it is likely that $rV'(t_{i+1}) \geq \frac{\partial \pi(t_{i+1},t_i)}{\partial t_i}$, a sufficient condition for a reduction of the plant duration over time.

3.2 The Welfare-Maximizing Firm

As in the previous section, the analysis can be applied with little modification to the case of a welfare-maximizing planner. For this case let $w(s,t_i)$ be the value of the objective function obtained solving

$$\max_q \quad \int_0^q p(y)\,dy - c(q,t_i)$$

subject to

$$q \leq \overline{x}(t_i)\, g(s).$$

The function w is non-increasing in s and non-decreasing in t_i. The maximization problem now becomes

$$\max_{\{t_i\}_{i=1}^{\infty}} \quad \sum_{i=0}^{\infty} \left(\int_{t_i}^{t_{i+1}} e^{-rt} w(t-t_i,t_i)\,dt - e^{-rt_i} k(t_i) \right)$$

with the initial condition $t_0 = 0$. Let $W(t_i)$ be the value function when a new plant is installed at t_i. The relationship

$$W(t_i) = \max_{\Delta \geq 0} \int_0^{\Delta} e^{-rs} w(s, t_i)\, ds - k(t_i) + e^{-r\Delta} W(t_i + \Delta) \tag{9}$$

must still hold. If we assume, similarly to what done for $\pi(s, t_i)$ in Assumption 2, that $w(s, t_i)$ is concave in t_i, then we can show (similarly to what done in Proposition 6) that $W(t_i)$ is increasing, differentiable and concave, so that the optimal length of the plant at time t_i can be obtained from the condition

$$w(\Delta, t_i) + W'(t_i + \Delta) = rW(t_i + \Delta). \tag{10}$$

As in the case of the profit-maximizing firm, we can't establish how the optimal duration of a plant varies over time without further assumptions. We have

$$\frac{d\Delta}{dt_i} = \frac{rW'(t_i + \Delta) - W''(t_i + \Delta) - \frac{\partial w(\Delta, t_i)}{\partial t_i}}{\frac{\partial w(\Delta, t_i)}{\partial \Delta} + W''(t_i + \Delta) - rW'(t_i + \Delta)} \tag{11}$$

The denominator is negative but sign of the numerator is undetermined. Following the same reasoning as for the case of a profit-maximizing firm, when $k(t) = k$ we have

$$W'(t_i) = \sum_{q=0}^{\infty} e^{-r(t_{i+q} - t_i)} \left(\int_0^{\Delta_q^*} e^{-rs} \frac{\partial w(s, t_{i+q})}{\partial t_{i+q}} ds \right)$$

and considerations similar to the one discussed for the previous case apply.

3.3 Comparing Plant Duration

For the case in which there is no technical progress we have shown that the optimal plant duration is shorter for the welfare-maximizing firm. The basic force driving this result is that expanding production yields more benefits when social welfare, rather than profit maximization, is the objective. When technical progress is present it becomes difficult to provide analytical results, but the same insights apply, at least when capacity expansion is the main result of technical progress.

To better grasp the point, consider the case in which operating costs remain the same over time, i.e. $c(q)$ does not change, and the only technical progress is that installed capacity $\overline{x}(t_i)$ expands over time. Let q^m and q^w be the profit-maximizing and the welfare-maximizing quantities; observe that they are constant over time and we typically have $q^w > q^m$. A profit maximizing firm will never renew a plant if $x(t) \geq q^m$ and the welfare-maximizing firm will not renew the plant if $x(t) \geq q^w$. Thus, if a plant has been installed at a given time t_i, the optimal duration $\Delta^m(t_i)$ for the profit maximizing firm is such that $x(\Delta^m(t_i)) \leq q^m$ while the optimal duration for the profit maximizing firm $\Delta^w(t_i)$ satisfies $x(\Delta^w(t_i)) \leq q^w$. This implies that $x(\Delta^w(t_i)) \geq q^m$ is a sufficient conditions for $\Delta^w < \Delta^m$. This will surely be the case when the installed capacity $\overline{x}$ is sufficiently large and the installation cost is sufficiently low. To see this point, consider the limit case in which, after t_i, the next plant to be installed has infinite capacity, so that no new installations will be necessary. Thus, the instantaneous social welfare after installation is $w^* = \int_0^{q^w} p(s)\, ds - c(q^w)$ at each time. Dynamic maximization of social welfare requires solving

$$\int_0^{\Delta} e^{-rs} w(s, t_i)\, ds - e^{-r\Delta} k + e^{-r\Delta} \frac{w^*}{r}$$

and the first order condition is

$$w(\Delta, t_i) + rk = w^*.$$

As k converges to zero the solution converges to a value Δ^w such that $x(\Delta^w) = q^w$, which implies $\Delta^w < \Delta^m$.

The general case is more complicated. For example, when $c(q,t)$ changes a profit-maximizing firm may decide to renew the plant even if capacity exceeds the *current* monopoly quantity. The point is that the cost function of the lastly installed plant, $c(q, t_{i-1})$, may be much higher that what can be obtained with a new plant, say $c(q, t_i)$, and as a consequence the monopoly quantity $q(t_{i-1})$ can be much lower that the monopoly quantity $q(t_i)$. Thus, in the same way that without further assumptions it was impossible to say whether plant duration increases or decreases over time, it is impossible to establish whether plant duration is higher for a public or a private firm.

4 Conclusions

In this paper, we have considered the optimal investment timing and pricing policies of a privatized utility and have compared it with that of a welfare-maximizing regulator, both with and without technical progress. For the case in which technical progress is absent we show that the optimal plant duration for a public firm is shorter than for a private firm. When technical progress is introduced the results depend on the particular form taken by technical progress.

Our model is deterministic. Incorporating uncertainty, both on demand and on the evolution of the technology, seems to be an interesting extension. Another useful extension is relaxing the assumption of stationarity of demand; at least in certain industries there are commonly held expectations of expansion or contraction of demand over time and the interplay of demand change with technical progress may yield interesting results. Finally, it would be interesting to analyze the case in which the public firm is not welfare-maximizing, for example because it does not achieve productive efficiency.

Appendix

Proof of Lemma 1. Define

$$M=\int_0^{+\infty} e^{-rt}\pi\left(x\left(t\right)\right)dt-k.$$

By Assumption 1, $M>0$. We first show that it cannot be optimal never to replace the plant. In fact, suppose that the plant is replaced only once at time T. The value of the objective function under this policy is

$$\begin{aligned} V\left(T\right) &= \int_0^{T} e^{-rt}\pi\left(x\left(t\right)\right)dt-k+e^{-rT}\left(\int_T^{+\infty} e^{-r(t-T)}\pi\left(x\left(t-T\right)\right)dt-k\right) \\ &= \int_0^{T} e^{-rt}\pi\left(x\left(t\right)\right)dt-k+e^{-rT}M \end{aligned}$$

Thus, the difference between the policy of replacing the equipment at T only and the policy of never replacing the equipment is

$$V\left(T\right)-M=e^{-rT}\left(M-\int_T^{+\infty} e^{-r(t-T)}\pi\left(x\left(t\right)\right)dt\right).$$

Since $\lim_{t\to\infty}\pi\left(x\left(t\right)\right)=0$, there is a value T such that

$$\int_T^{+\infty} e^{-r(t-T)}\pi\left(x\left(t\right)\right)dt<M,$$

thus implying that never substituting the plant is suboptimal. ■

Proof of Proposition 1. To see that the maximum value of the objective function V^* can be attained with a policy such that the plant is replaced at a constant interval Δ^*, observe that for the optimal policy $\{t_i\}_{i=0}^{+\infty}$ it must be true that

$$V^*=\int_0^{t_1} e^{-rt}\pi\left(x\left(t\right)\right)dt-k+e^{-rt_1}V^*.$$

This follows from the fact that, once the plant is dismissed at t_1 the situation is exactly the same as at time zero. Letting $t_1=\Delta^*$ this implies

$$V^*=\frac{\int_0^{\Delta^*} e^{-rt}\pi\left(x\left(t\right)\right)dt-k}{1-e^{-r\Delta^*}}. \tag{12}$$

Now notice that when the plant is replaced at a constant interval Δ, the value of the objective function is

$$V(\Delta) = \sum_{n=0}^{\infty} \int_{n\Delta}^{(n+1)\Delta} e^{-rt} \pi(x(t - n\Delta))\, dt$$

Since

$$\int_{n\Delta}^{(n+1)\Delta} e^{-rt} \pi(x(t - n\Delta))\, dt = e^{-rn\Delta} \int_{0}^{\Delta} e^{-rt} \pi(x(t))\, dt$$

for each n, it follows

$$V(\Delta) = \frac{\int_0^\Delta e^{-rs} \pi(x(t))\, dt - k}{1 - e^{-r\Delta}}. \tag{13}$$

Thus, in order to achieve the maximum value V^* given in (12) it is sufficient to replace the plant at constant intervals of length Δ^*.

Since we can restrict attention to policies in which the plant is replaced at a constant interval Δ, the problem boils down to

$$\max_{\Delta \geq 0} \quad V(\Delta)$$

where $V(\Delta)$ is given by (13). The function V is continuous and differentiable, and we can write

$$V'(\Delta) = \frac{re^{-r\Delta}}{(1 - e^{-r\Delta})} \left(\frac{\pi(x(\Delta))}{r} - \frac{\int_0^\Delta e^{-rs} \pi(x(s))\, ds - k}{1 - e^{-r\Delta}} \right). \tag{14}$$

Since $\lim_{\Delta \to 0^+} V(\Delta) = -\infty$, $\lim_{\Delta \to +\infty} V(\Delta) = M$ and from Lemma 1 we know that some replacement must occur, $V(\Delta)$ must be maximized at some finite value Δ. At the optimal point it must be $V'(\Delta) = 0$. From (14) this is equivalent to

$$\frac{\int_0^\Delta e^{-rs} \pi(x(s))\, ds - k}{1 - e^{-r\Delta}} = \frac{\pi(x(\Delta))}{r}. \tag{15}$$

Notice that this implies that at all points such that $V'(\Delta) = 0$ we have $V(\Delta) = \frac{\pi^*(x(\Delta))}{r}$. Since $\pi^*(x(\Delta))$ is decreasing in Δ, the highest value of $V(\Delta)$ is obtained at the lowest value of Δ such that (15) is satisfied.

At last, notice that $x(\Delta) \geq q^m$ implies $\pi(x(\Delta)) = \pi^m$, making it impossible to have equation (15) satisfied. Thus, it must be the case that $x(\Delta) < q^m$. ■

Proof of Proposition 2. Using the expression of $V(\Delta)$ given in (13) we have

$$\frac{\partial^2 V}{\partial \Delta \partial k} = \frac{re^{-r\Delta}}{(1-e^{-r\Delta})^2} > 0.$$

Thus, V satisfies increasing differences in Δ and k, which in turn implies that the optimal value of Δ is increasing in k.

Similarly, notice that, writing

$$V(\Delta, \overline{x}) = \frac{\int_0^{\Delta} e^{-rs}\pi(\overline{x}g(s))\,ds - k}{1-e^{-r\Delta}}$$

we have

$$\frac{\partial V(\Delta, \overline{x})}{\partial \Delta \partial \overline{x}} =$$

$$\frac{re^{-r\Delta}}{(1-e^{-r\Delta})^2}\left(\int_0^{\Delta} e^{-rs}\left(\frac{d\pi(\overline{x}g(\Delta))}{dx}g(\Delta) - \frac{d\pi(\overline{x}g(s))}{dx}g(s)\right)ds\right)$$

Thus, if $\frac{d\pi(\overline{x}g(s))}{dx}g(s)$ is increasing in s we have $\frac{\partial V(\Delta,\overline{x})}{\partial\Delta\partial\overline{x}} \geq 0$. On the other hand, if $\frac{d\pi(\overline{x}g(s))}{dx}g(s)$ is decreasing in s we have $\frac{\partial V(\Delta,\overline{x})}{\partial\Delta\partial\overline{x}} \leq 0$. In the first case the optimal length Δ is increasing in $\overline{x}$, while in the second case it is decreasing. ■

Proof of Proposition 4. Define

$$y(x) = w(x) - \pi(x).$$

Notice that in general $y(x)$ is not immediately interpretable as consumer surplus, since at some values of x the quantities $q^m(x)$ and $q^w(x)$ may differ. We now show that at $x = x(\Delta^m)$ we have

$$\frac{w(x(\Delta^m))}{r} < \frac{\int_0^{\Delta^m} e^{\cdot rt}w(x(t))\,dt - k}{1-e^{-r\Delta^m}}, \tag{16}$$

thus implying $x(\Delta^w) > x(\Delta^m)$ and therefore $\Delta^w < \Delta^m$.

Observe that, by definition of the function $y(x)$:

$$\frac{w(x(t))}{r} = \frac{y(x(t))}{r} + \frac{\pi(x(t))}{r}. \tag{17}$$

Multiplying both sides by e^{-rt}, integrating over $[0,\Delta^m]$ and dividing by $1-e^{-r\Delta^m}$ we obtain

$$\frac{\int_0^{\Delta^m} e^{-rt} w\left(x\left(t\right)\right) dt - k}{1-e^{-r\Delta^m}} = \frac{\int_0^{\Delta^m} e^{-rt} \pi\left(x\left(t\right)\right) dt - k}{1-e^{-r\Delta^m}} + \frac{\int_0^{\Delta^m} e^{-rt} y\left(x\left(t\right)\right) dt}{1-e^{-r\Delta^m}},$$

and evaluating (17) at $t=\Delta^m$ we obtain

$$\frac{w\left(x\left(\Delta^m\right)\right)}{r} = \frac{y\left(x\left(\Delta^m\right)\right)}{r} + \frac{\pi\left(x\left(\Delta^m\right)\right)}{r}.$$

Thus, inequality (16) is equivalent to

$$\frac{y\left(x\left(\Delta^m\right)\right)}{r} + \frac{\pi\left(x\left(\Delta^m\right)\right)}{r} < \frac{\int_0^{\Delta^m} e^{-rt} \pi\left(x\left(t\right)\right) dt - k}{1-e^{-r\Delta^m}} + \frac{\int_0^{\Delta^m} e^{-rt} y\left(x\left(t\right)\right) dt}{1-e^{-r\Delta^m}},$$

Since at Δ^m we have

$$\frac{\pi\left(x\left(\Delta^m\right)\right)}{r} = \frac{\int_0^{\Delta^m} e^{-rt} \pi\left(x\left(t\right)\right) dt - k}{1-e^{-r\Delta^m}},$$

the inequality is equivalent to

$$\frac{y\left(x\left(\Delta^m\right)\right)}{r} < \frac{\int_0^{\Delta^m} e^{-rt} y\left(x\left(t\right)\right) dt}{1-e^{-r\Delta^m}}$$

$$\Longrightarrow \qquad 0 < \int_0^{\Delta^m} e^{-rt}\left[y\left(x\left(t\right)\right) - y\left(x\left(\Delta^m\right)\right)\right] dt.$$

The inequality is satisfied if $y\left(x\right)$ is an increasing function of x, and therefore a decreasing function of Δ. Now observe that $y\left(x\right)$ can be written as

$$y\left(x\right) = \int_0^{q^w(x)} p\left(s\right) ds - c\left(q^w\left(x\right)\right) - p\left(q^m\left(x\right)\right) + c\left(q^m\left(x\right)\right), \qquad (18)$$

where $q^w\left(x\right)$ is the optimal instantaneous quantity of a welfare-maximizing regulator facing a capacity constraint x and $q^m\left(x\right)$ is the optimal quantity of a profit-maximizing firm. Thus:

$$y'\left(x\right) = \left(p\left(q^w\left(x\right)\right) - c'\left(q^w\left(x\right)\right)\right)\frac{dq^w}{dx} - \left(p'\left(q^m\left(x\right)\right) - c'\left(q^m\left(x\right)\right)\right)\frac{dq^m}{dx}.$$

It has to be $\frac{dq^w}{dx} \geq 0$ and $\frac{dq^m}{dx} \geq 0$. Furthermore notice that whenever $\frac{dq^m}{dx} > 0$ it has to be the case that $\frac{dq^m}{dx} = \frac{dq^w}{dx} = 1$ and $q^w\left(x\right) = q^m\left(x\right) = x$.

This is because in that case the capacity constraint is binding for the profit-maximizing firm, which implies $\frac{dq^m}{dx} = 1$. Furthermore, if it is binding for the profit-maximizing firm it must be the case that the marginal revenue is higher than the marginal cost. But this implies that the price is higher than the marginal cost, so the capacity constraint is binding for the welfare-maximizing regulator as well.

At the surplus-maximizing point we must have $p(q^w(x)) \geq c'(q^w(x))$. Thus, if $\frac{dq^m}{dx} = 0$ we have $y'(x) \geq 0$. Otherwise, using $\frac{dq^m}{dx} = \frac{dq^w}{dx} = 1$ and $q^w(x) = q^m(x) = x$ we can write $y'(x)$ as

$$y'(x) = p(x) - p'(x)$$

which is always positive. ■

Proof of Lemma 5. Let $\{t_i\}_{i=0}^{\infty}$ be the optimal policy and choose ε so that $\varepsilon\overline{\pi} < \underline{k}$. By contradiction, if there is no minimum duration then there must be i such that $t_{i+1} - t_i < \varepsilon$. Consider now the present value of the profit obtained between time t_{i-1} and time t_{i+1}. This is given by

$$A = \int_{t_{i-1}}^{t_i} e^{-rs}\pi(s, t_{i-1})\,ds - e^{-rt_{i-1}}k(t_{i-1}) + \int_{t_i}^{t_{i+1}} e^{-rs}\pi(s, t_i)\,ds - e^{-rt_i}k(t_i).$$

Consider now the alternative policy in which no investment occurs at period t_i. In that case the present value of the profit obtained between t_{i-1} and t_{i+1} is

$$B = \int_{t_{i-1}}^{t_{i+1}} e^{-rs}\pi(s, t_{i-1})\,ds - e^{-rt_{i-1}}k(t_{i-1}).$$

Thus

$$B - A = \int_{t_i}^{t_{i+1}} e^{-rs}\left[\pi(s, t_{i-1}) - \pi(s, t_i)\right] ds + e^{-rt_i}k(t_i) \geq$$

$$e^{-rt_i}\left(-\varepsilon\overline{\pi} + \underline{k}\right) > 0$$

where the first inequality comes from that fact that, given assumption 2, $\pi(s, t_{i-1}) - \pi(s, t_i) \geq -\overline{\pi}$ and $k(t_i) \geq \underline{k}$, while the second inequality comes from the definition of ε. We have therefore a contradiction and we conclude that there is a minimum duration Δ^* such that $t_{i+1} - t_i \geq \Delta^*$ for each i. ■

Proof of Proposition 6. Let Δ^* be the minimum plant duration identified in Lemma 5. Let $\mathcal{V}$ be the set of real-valued functions with domain $[0, +\infty)$.

Define the mapping $T : \mathcal{V} \to \mathcal{V}$ as

$$T(V)(t) = \max_{\Delta \geq \Delta^*} \int_0^\Delta e^{-rs} \pi(s,t)\, ds - k(t) + e^{-r\Delta} V(t+\Delta)$$

Mapping T is a contraction mapping with modulus $e^{-r\Delta^*}$, since

$$T(V+a)(t) = \max_{\Delta \geq \Delta^*} \int_0^\Delta e^{-rs} \pi(s,t)\, ds - k(t) + e^{-r\Delta} [V(t+\Delta) + a]$$

$$\leq \left[\max_{\Delta \geq \Delta^*} \int_0^\Delta e^{-rs} \pi(s,t)\, ds - k(t) + e^{-r\Delta} V(t+\Delta) \right] + e^{-r\Delta^*} a$$

Under assumption 2 the value function is a fixed point of the mapping T and it is increasing, differentiable and concave. ■

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