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Monetary policy, de-anchoring of inflation expectations, and the "new normal"*

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Abstract
Persistently low inflation rates in the Euro Area raise the question whether inflation is still credibly anchored to the Euro-system's medium term target of below, but close to 2%. The purpose of this paper is twofold. First, we investigate why agents’ expectations that over the business cycle inflation will remain in line with the target begin to falter. Our hypothesis is that agents form expectations in terms of their confidence in the "normal regime", which is updated observing the state of the economy. Second, we study how the de-anchoring of expectations interacts with monetary policy determining whether the central bank is still able to achieve its target - and hence re-anchor inflation expectations - or whether the system drifts away towards depressed states of low inflation and output. Two are our main findings. The first is that, facing unfavourable shocks, if inflation expectations "fall faster" than the policy rate, and the zero lower bound is reached without correcting the shock, the system converges to a new steady state - the "new normal" - with permanent negative gaps. The second is that a more aggressive monetary policy is ineffective both at the ZLB and above the ZLB, when the shock is large and/or when the reactivity of inflation expectations is high enough. This last finding seems to support the necessity, in those conditions, to abandon conventional monetary policy and to switch to an aggressive reflationary policy that prevents the entrenchment of deflationary expectations.

Keywords: Monetary Policy, Zero Lower Bound, New Normal, Inflation Expectations
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“Inflation expectations are well anchored until they are not”
(Orphanides, 2015)

1. Introduction

Following the global financial crisis, inflation dynamics in advanced economies has surprised many economists. On the one hand, just after the outbreak of the crisis there has been a period of missing disinflation: based on historical data and given the depth of the recession, inflation should have declined much more than it actually did (IMF, 2013; Murphy, 2014). On the other hand, inflation has more recently surprised to the downside. Since the end of 2011, the Euro Area, the United Kingdom and the United States have been experiencing a sustained deflation, a downward drift of the inflation path away from the central bank's target, which has become more pronounced since 2013. Even greater surprise has been created by the fact that this process has been taking place in parallel with unprecedented easing of monetary conditions to the limit of the zero lower bound (ZLB) of policy interest rates. This concomitance has impaired explanations based on worldwide "structural flattening" of the Phillips Curves, and has raised concern among central bankers about their impotency to govern the dynamics of the price level (Iakova, 2007; Kuttner and Robinson, 2010; ECB, 2014).

As the President of the European Central Bank (ECB) Mario Draghi has recently stated “[...] The most fundamental question facing all major central banks today is this: can our price stability mandate still be delivered? Across advanced economies inflation is low and has been low for some time. And in several of those economies, long-term inflation expectations, based on market prices, remain below our numerical definitions of price stability. That has led some to question whether it makes sense for central banks to pursue expansionary policies to meet their inflation objectives. Are they fighting a futile battle against forces beyond their control?” (Draghi, 2016, p. 1)
Thus, low and declining inflation rates for such a long period of time, particularly, though not only, in the Euro Area, raise the question of whether inflation is still credibly anchored to the medium-term inflation target pursued by central banks. This question has come to be known as the de-anchoring problem of inflation expectations, and this problem plays a key role in the communication strategy of today’s conventional and non-conventional monetary policies (Draghi 2014a; 2014b; 2016; Yellen, 2014; Eggertsson and Woodford, 2004).

Independent empirical analyses support concern with the de-anchoring problem. In the empirical literature, the dominant criterion to test the anchoring of inflation expectations is based on the idea that firmly anchored expectations should be insensitive to the announcement of macroeconomic news (Gürkaynak et al., 2010). Survey-based analyses\(^1\) in the Euro Area suggest that in recent years inflation expectations have shown some signs of de-anchoring, moving away from the ECB target even at the longer horizons. Lyziak and Paloviita (2016) find some evidence of increased sensitivity of longer-term inflation forecasts to shorter-term forecasts and to actual HICP inflation, and that the role of ECB inflation target for those expectations has diminished in the last years. Buono and Formai (2016) show that short-term inflation expectations by the Professional Forecasters interviewed by Consensus Economics started falling in 2012 and that its reduction implies a reduction by 0.3 percentage points of long-run expectations.

Market-based analysis\(^2\) largely confirm these results. By applying the multiple endogenous break point tests of Bai (1997), Nautz et al. (2017)

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\(^1\) Various European institutions on a regular basis conduct surveys involving both consumers and professional forecasters. For example, in the European Commission’s Consumer Survey on Inflation Expectations, consumers are asked monthly to indicate their expectations in terms of the direction of inflation development over the next year. The ECB Survey of Professional Forecasters (SPF) is conducted quarterly and includes a panel of more than 70 forecast experts.

\(^2\) Market based measure of inflation expectations avoid many problems of measures taken from inflation surveys. First of all, data frequency is not an issue since market data are easily available on a daily basis. In contrast to survey-based measures, market analysis are determined by market activities and actual trading behaviour. Therefore, market data can be considered to represent the market’s actual inflation expectations and should not be distorted by psychological factors (ECB, 2006). For an overview of survey-based and market-based measures, see Grothe and Meyler (2015).
finds that expectations in the Euro Area became less well anchored since September 2011. Using a news-regression approach to assess the sensitivity (or lack thereof) of expectations to the release of unexpected macroeconomic news, Fracasso and Probo (2017) finds evidence that the de-anchoring of expectations started in December 2011 and never reversed. Symptoms have also been detected by Natoli and Sigalotti (2017): studying the pass-through of inflation expectations they find that, since mid-2014, negative tail events affecting short-term inflation expectations have been increasingly channelled onto long-term ones, and that this phenomenon has ignited both downward revisions in expectations and upward shifts in uncertainty.3

This evidence of de-anchoring of inflation expectations challenges standard macro-theory of conventional monetary policy based on agents' long-run rational expectation (RE) that actual inflation will remain anchored to the central bank's target up to short-run deviations (Woodford, 2003, ch. 2).4 Parallely, it supports the concern of central banks about their ability to fulfil their inflation target once inflation expectations are de-anchored.

Benhabib et al. (2001) first warned that a feedback rule à la Taylor may generate multiple RE equilibria, one of which at the ZLB with perpetual "liquidity trap" at low inflation (below target) and output (below potential).5 Woodford (2003, Ch. 2, sec. 4) discusses "self-fulfilling inflations and deflations". He also finds that under an interest-rate feedback rule a multiplicity of equilibria may arise and that the RE equilibrium that fulfils

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3 This result is in line with the high sensitivity of medium-to-long term expectations to inflation surprises observed in Miccoli and Neri (2015).

4 Indeed, today's theory of conventional monetary policy is a modernized version of Wicksell's theory put forward in Interest and Prices (1898), where the problem of anchoring inflation expectations was identified as a consequence of the central bank controlling (or seeking to control) the nominal interest rate instead of the quantity of money (Woodford, 2003; Mazzocchi et al., 2014).

5 Of course, the problem is not low inflation per se, but that the economy settles down below potential output. From this point of view, the use of the Keynesian term "liquidity trap" to denote this equilibrium - apparently reintroduced by Krugman (1998) - seems appropriate. However, is should also be recalled that in Keynes's General Theory, and in the Keynesian tradition, the "liquidity trap" had different causes. Namely the inability of the central bank offering more money to lower the nominal interest rate, even above zero, owing to asset holders' expectations of falling asset prices leading to infinitely elastic supply of bonds in exchange for money. This original notion of liquidity trap is more relevant to what is now called unconventional monetary policy.
the central bank's inflation target is locally stable, but not globally unique. He also highlights that the drift towards self-fulfilling inflations and deflations has to be driven by de-anchored inflation expectations that, in the case of deflations, *combine with the ZLB of the interest rate*. In simple words, expected inflation should "fall faster" than the interest rate.\(^6\)

In the light of this account, the de-anchoring problem consists of *two phenomena* that we investigate in this paper. The first is why agents' expectations that inflation will remain in line with the central bank's target begin to falter. The second is how the de-anchoring of expectations interacts with monetary policy determining whether the central bank is still able to achieve the inflation target, and hence re-anchor inflation expectations, or whether the system drifts away towards negative inflation and output gaps.

Insights into the de-anchoring of inflation expectations can be found in different strands of literature that drop the RE hypothesis. In the first place, one may find studies which question the validity of the RE hypothesis empirically. For the Euro Area, using the European Commission survey, Forsells and Kenny (2004) find that inflation expectations appear unbiased, but they reject the hypothesis of orthogonality with respect to the available information. Evidence in favour of adaptive learning in inflation expectations is provided by Branch (2004) and Pfajfar and Santoro (2010). A more theoretical literature investigates processes of expectation formation and whether they converge to the unique RE equilibrium. Generally, this literature posits agents lacking full information or knowledge about the data generating process, and examines some form of learning mechanism thereof. Multiple equilibria typically emerge, among which self-fulfilling "liquidity traps" in conjunction with the ZLB on nominal interest rate. Some authors (e.g. Bullard and Mitra, 2002; Evans et al., 2008; Evans and McGough, 2017) show that, under least-square learning introduced by Marcet and Sargent (1989) and Evans and Honkapohja (2001), the "liquidity trap" exists, but it fails to be stochastically stable. Others, employing different learning mechanisms instead show that it can be stable (Arifovic et al., 2012; 2017; Busetti et al., 2014; De Grauwe and Ji, 2016).

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\(^6\) "Along such a path, interest rates are constantly being lowered in response to decline in inflation, but because *expected* future inflation falls at the same time, *real* interest rates are not reduced, and continue to be high enough to restrain demand despite the falling prices" (Woodford, 2003, p.126).
In this paper we present a model consisting of the standard New-Keynesian three equations used in this literature, augmented with an inflation-expectation mechanisms which endogenises the de-anchoring process, it is both consistent with a broad notion of "rational beliefs" (e.g. Kurz, 2011), and is amenable to simple parameterisation and control for comparative simulations. We draw on the key idea in Arifovic et al. (2017), namely that, after a shock to the natural interest rate opening negative inflation and output gaps, agents consider a regime switch scenario, from the "normal regime", where the economy will return to the zero-gaps equilibrium, to a "deflationary regime" where it no longer will. Agents' confidence in the normal regime shrinks vis-à-vis worse deflationary conditions. With respect to the bulk of the literature, we enlarge our analysis beyond the single issue of the existence and stability of depressed states of the economy at the ZLB. Starting from normal conditions consistent with the zero-gaps equilibrium, we introduce the origin of the de-anchoring problem and, by means of simulations, we investigate the global dynamic behaviour of the system. Key to whether the system settles down in a negative-gaps steady state or not is the interaction between the inflation-expectation mechanism and monetary policy, together with contour conditions given by structural parameters and the extent of the initial shock. Therefore, our study also offers some policy implications.

More in detail, section 2 introduces the model. The New Keynesian block of the model consists of the linearized equations of the output gap relative to potential, of the inflation gap relative to the central bank's inflation target, and of the nominal interest rate (policy rate for short) set by the central bank. Then we introduce the fourth equation for the inflation expectations.

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7 This was also Wicksell's original concern with interest-rate based monetary policy, in particular because he regarded the natural interest rate as a non-observable variable subject to various sources of modification (e.g. Boianovsky and Trautwein 2004, Mazzocchi et al. 2014). Since the beginning of the new century, a growing empirical literature has sought to estimate the natural rate and its changes over time, and has pointed out the consequent challenges to the conventional monetary policy (Orphanides and Williams, 2002; 2007; Mazzocchi, 2013b). The fall of the natural rate also plays a prominent role in the debate over the so-called "secular stagnation", in particular because it may entail a permanent interest-rate gap as the policy rate hits the zero lower bound (e.g. Teulings and Baldwin, eds. 2014).

8 Kurz (2011) illustrates the class of rational beliefs concerning state transitions of the economy.
Facing a negative shock to inflation and output, agents believe in the normal regime with confidence \( p \in [0, 1] \) and in the switch to the deflationary regime with confidence \((1 - p)\) under the hypothesis that the current gaps persist. Thus, in this context, the long-run inflation (gap) expectation elaborated by agents in each period is the \((p; 1-p)\) mean of the equilibrium inflation (gap) in the two regimes. This affects the actual output and inflation gaps and hence the monetary policy response, normally a cut in the interest rate aimed at closing the output and inflation gaps. This induces a revision of agent's regime confidence and inflation expectations, and so on and so forth.

In order to examine the analytical properties of this kind of economic system, its long-run evolution by means of simulations, and hence to draw some policy implications, we have found it useful to posit a parsimonious mathematical formulation of the updating mechanism of beliefs.\(^9\) This is a logistic function such that the confidence \( p \) in the normal regime is nonlinearly decreasing (de-anchoring is increasing) in the extent of the observed output gap. The single key parameter \((\sigma)\) of this function is its reactivity to output gaps: larger \( \sigma \) induces larger drops of \( p \) for a given output gap. Hence we have precise identification and full control of the conditions that determine the extent of de-anchoring of inflation expectations, and whether they "fall faster" than the interest rate, in accordance with the empirical methodology that identifies de-anchoring with the reactivity of expectations to macroeconomic news (Gürkaynak et al., 2010).

In section 3 we analyse the properties of the system and present simulations. In the normal regime \((p = 1)\), the system displays the usual RE equilibrium with zero gaps. With \( p \) endogenous, however, the system becomes nonlinear, and dependent on the values of six parameters plus the initial natural-rate shock. Hence we examine its global dynamics by means of simulations. These are organised in a baseline case with parameter values consistent with the empirical literature, and then pairwise interactions of the reactivity parameter \( \sigma \) with other three: the extent of the initial shock, the output gap elasticity of inflation gaps \((\kappa)\) and the inflation gap parameter \((\gamma_{\pi})\) in the central bank’s rule.

\(^9\) See footnote 19 for an analogy between this function and Bayes rule.
Our main findings, summarised and discussed in section 4, are the following. Depending on the parameter values and the initial shock, the system dynamic behaviour is more complex and richer than in similar studies on the de-anchoring problem. The system displays three possible regions. One of convergence to the zero-gaps steady state and \( p = 1 \), i.e. the system remains in the normal regime. One of convergence to nonzero inflation and output gaps, and \( p < 1 \): the nonzero gaps and \( p < 1 \) are mutually consistent, and denote what we call a "new normal", i.e. a depressed state of the economy to which there corresponds a permanent, though finite in size, de-anchoring of inflation expectations from the central bank's target. One of divergence from any steady state. In the second and third region we have a clear representation of the phenomenon of inflation expectations that "fall faster" than the policy rate.

The system remains in the normal regime as long as the policy rate can accommodate the natural rate shock above the ZLB; yet this basin of attraction shrinks as \( \sigma \) increases. With the shock large enough, so that the policy rate hits the ZLB leaving a permanent excess interest-rate gap, the system settles down in the new normal, which is however delimited by a lower boundary value of \( \sigma \). A combination of high \( \sigma \) and large shock may push the system into the divergence region.

Controlling for structural price stickiness (\( \kappa \)), simulations show that, ceteris paribus, as \( \kappa \) increases (prices are more flexible and the PC is steeper) the convergence regions shrink, the reason being that a steeper PC amplifies the output gaps. With regard to the inflation parameter in the central banks' policy rule (\( \gamma_{\pi} \)), simulations show that a more aggressive (conventional) monetary policy, i.e. with larger \( \gamma_{\pi} \), is obviously ineffective at the ZLB, but it is also ineffective above the ZLB when adverse conditions push the system into the divergence region. This finding seems to support the necessity, in those conditions, to abandon conventional monetary policy and to switch to an aggressive reflationaly policy that prevents the entrenchment of deflationary expectations.

2. The model

2.1. The standard New Keynesian three equations

We draw on the standard New Keynesian three equations of the so-called output gap, the inflation gap or Phillips Curve and a Taylor Rule that determines the policy rate controlled by the central bank.
The output gap equation (OG) yields the logarithmic difference $y$ between the current output and the potential output as given by technology and endowment of factors.\(^\text{10}\) The OG equation, for any period $t$, is:

$$y_t = E_t y_{t+1} - \alpha (i_t - E_t \pi_{t+1} - r_t)$$

where $E_t$ is the usual expected value operator conditional on information at time $t$, $i_t$ is the nominal interest rate controlled by the central bank (policy rate for short), $\pi_t$ is the inflation rate, $r_t^*$ is the "natural" (real) interest rate, i.e. the interest rate corresponding to the general equilibrium of the economy at potential output.\(^\text{11}\)

It is assumed that the natural rate is a constant that at any time $t$ may however shift unexpectedly by a permanent amount $\rho_t$. Then, the term in parentheses can be read as the "interest rate gap" $\hat{i}_t$, i.e. the deviation of the policy rate from the natural rate and the expected inflation, with $\alpha$ measuring the (constant) elasticity of substitution of aggregate spending.

Inflation is driven by its own expectations and the output gap according to the so-called Phillips Curve (PC):

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t$$

where $\beta < 1$ is the time discount factor and $\kappa > 0$ is a parameter reflecting the degree of price stickiness in the goods market ($\kappa$ is decreasing in price stickiness).\(^\text{12}\)

Finally, the model is closed by the equation determining the policy rate, which is typically in the format of a Taylor Rule (TR), namely

$$i_t = (r_t^* + \pi^*) + \gamma_\pi (\pi_t - \pi^*) + \gamma_y y_t$$

where $\pi^* \geq 0$ is the inflation target, and $\tau$ is a time index that can be determined according to various specifications, e.g. "real time" $\tau = t$, forward

\(^{10}\) The potential output can be measured as the market clearing, Pareto efficient, level of output, or alternatively the maximal level of output obtainable in the presence of distortionary "real rigidities" (Woodford, 2003; Trautwein and Zouache, 2009). This difference is immaterial here.

\(^{11}\) In the standard Keynesian model, the natural interest rate equates the rate of time preference of households with marginal return to capital.

\(^{12}\) According to the Calvo pricing mechanism embedded into the standard New Keynesian Phillips Curve,

$$\kappa = (1 - \phi)(1 - \phi \beta) \phi^{-1}$$

where $\phi$ is the probability of prices being unchanged (the fraction of firms not changing their price) after a change in aggregate demand. Clearly, $\phi = 1$, $\kappa = 0$, represent the Old Keynesian fixed-price economy where the Phillips Curve is horizontal, and the steady-state inflation is zero, whereas $\phi = 0$, $\kappa \to \infty$, represents the New Classical flex-price economy where the Phillips Curve is vertical, and the steady-state inflation is undetermined.
looking \( \tau = t+n \ (n = 1, \ldots) \), lagged \( \tau = t-n \ (n = 1, \ldots) \). For reasons that will be explained below, we adopt the lagged specification with \( n = 1 \).

The three equations OG-PC-TR form a consistent system that determines the equilibrium values of the three endogenous variables, corresponding to the steady-state values of the OG and PC equations (\( \bar{y}, \bar{\pi} \)). The steady-state inflation implied by the PC is the key input to the central bank’s determination of the inflation target, which in turn provides the anchor for inflation expectations. The steady-state inflation is:

\[
\bar{\pi} = \frac{\kappa}{1-\beta} \bar{y}
\]

hence it is nil when the output gap is nil, so that \( \bar{\pi} = 0 \) is the central bank’s inflation target,\(^{13}\) and the PC is also the output gap equation. The equilibrium of the economy is \((\bar{y} = 0, \bar{\pi} = 0, \bar{\tau} = 0)\). Clarida et al. (1999, 2000), Woodford (2003, Ch. 2), and others have also shown that under suitable conditions in the TR equation (crucially \( \gamma_n > 1 \)), the system also ensures convergence to the steady state. Thus, in the presence of random shocks to the system, the RE of inflation and output should be consistent with, and remain "anchored" to, the zero-gaps (stochastic) equilibrium of the economy (being also called "long-run expectations", e.g. Woodford 2003, Ch. 2).\(^{14}\)

### 2.2 De-anchoring inflation expectations

We now address the point of the formation of long-run inflation expectations more closely. Given the PC equation, we have jumped to the steady-state solution, but it is also necessary that we work out the single steps, and we do so by forward iteration in accordance with the model-consistency principle (also Woodford, 2003, pp. 91-ff.). Therefore, we carry equation (2) one period forward, so that

\[
E_t \pi_{t+1} = \beta E_t \pi_{t+2} + \kappa E_t \gamma_{t+1}
\]

Substituting into (2) we obtain

---

13 The central bank may wish a nonzero inflation target if, for welfare reasons, it also targets \( \bar{y} > 0 \), insofar as the potential output with distortions falls short of the Pareto efficient potential output (Woodford 2003, Ch. 2).

14 In common DSGE applications of the New Keynesian model, the output and inflation expectational terms refer to one-period ahead values ("short-run expectations") because shocks are assumed to be autoregressive. Yet this assumption is immaterial theoretically. On this point see also Mazzocchi et al. (2014).
\[ \pi_t = \beta^2 E_t \pi_{t+2} + \beta \kappa E_t \gamma_{t+1} + \kappa y_t \]

After \(N\) iterations, the equation of the current inflation – the inflation gap for the central bank – results to be

\[ \pi_t = \beta^N E_t \pi_{t+N} + \kappa \sum_{n=1}^{N} \beta^n E_t \gamma_{t+n} + \kappa y_t \]  

In the second term, we have to track the series \(E_t \gamma_{t+n}\) for \(n = 1, \ldots, N\). Let us assume that, conditional on information available in \(t\), and information orthogonality, agents expect the series to be stationary, i.e. \(E_t \gamma_{t+n} = E_t \gamma_{t+1}\) for all \(n\). Hence, the key issue is the output-gap expectation for \(t+1\) as of \(t\).

What is the rational expectation for \(y_{t+1}\)? There is no unambiguous answer because the evolution of the output gaps depends on the central bank’s ability to control the system. Indeed, if agents derive \(E_t \gamma_{t+1}\) consistently with the full model solution, then \(E_t \gamma_{t+1} = 0\) as explained above. Yet this solution *presumes* that the central bank exerts stochastic control on the system, and this in turn *presumes* the RE hypothesis. In other words, the central bank succeeds in anchoring the expectations if the agents expect it to succeed, and vice versa. Hence we face the classic circularity of the fixed-point solution of models with the RE hypothesis (e.g. Farmer, 1993), which in our context means that the problem of anchoring the expectations is resolved by assumption.

To deal with this issue, we follow a different route, which starts from the premise that the central bank succeeds in anchoring agents' expectations insofar as they have reason to believe in the central bank’s success – we may call this a *rational belief* (Kurz, 2011). Minimal requirements of a rational belief are that 1) it is logically consistent (e.g. in accordance with probabilistic metrics), 2) it is consistent with the data generating process (though not necessarily the “true” process), 3) it is corroborated by experience (Bayesian updating is an example). Hence, it should not be falsified systematically, and it should be verified with some nonzero probability. On the other hand, a nontrivial treatment of rational beliefs is that a chance exists that they are falsified.

Starting from a natural rate shock with a negative output gap at any point in time \(y_p\), we draw on Arifovic et al. (2017) idea of regime switch. That is to say, agents consider the possibility of a switch from the "normal regime", where the economy will return to the zero-gaps equilibrium, to a "deflationary regime" where the output gap will remain constant at the observed value. If agents believe in the normal regime with confidence \(p \in\)
[0, 1], and in the deflationary regime with confidence \((1 - p)\), a consistent rational belief about the future output gap is the \((p; 1 - p)\) mean value of the equilibrium gaps in the two regimes, i.e.:

\[
E_t\gamma_{t+1} = (1 - p)\gamma_t
\]

where \(p > 1\) can be interpreted as an indicator of the de-anchoring of expectations.

Given this state of expectations, the PC becomes:

\[
\pi_t = \beta^N E_t\pi_{t+N} + \sum_{n=1}^{N} \beta^n (1 - p)\kappa\gamma_t + \kappa\gamma_t = \\
= \beta^N E_t\pi_{t+N} + \left(1 + (1 - p) \sum_{n=1}^{N} \beta^n \right)\kappa\gamma_t
\]

Given \(\beta < 1\),

\[
\lim_{N \to \infty} \beta^N E_t\pi_{t+N} = 0,
\]

and

\[
\lim_{N \to \infty} \sum_{n=1}^{N} \beta^n = \frac{\beta}{1 - \beta}.
\]

Therefore,

\[
(6) \quad \pi_t = \omega\kappa\gamma_t
\]

where \(\omega = 1 + (1 - p)\beta', \beta' = \beta(1 - \beta)^{-1}\).

Consequently, the PC turns out to be a linear function of the current output gap. Its slope has two main determinants: one is the structural parameter \(\kappa\) determined by price stickiness, the other is the weigh \(\omega\) of deflationary expectations, which increases as the confidence \(p\) in the normal regime shrinks.

We have thus a first clue about the effect of the de-anchoring of expectations that may shed light onto the deflation surprise observed after the Great Recession. A steeper PC translates the current negative output gap into a larger deflation gap, which is *amplified* by the implicit projection of the current state of the economy into the future. At the same time the system becomes more difficult to control.\(^{15}\)

We can now reformulate also the OG equation (1) according to our hypothesis of expectation formation. We have seen that the expectation of the equilibrium output gap is given by \(E_t\gamma_{t+1} = (1 - p)\gamma_t\), whereas the

\(^{15}\) This finding has some bearing upon the empirical debate over the "flattening" of the PC (e.g. Blanchard 2017). Its implication is that the empirical slope of the PC may be the result of (permanent) structural factors (\(\kappa\)) as well as of (transient) expectational factors (\(\omega\)). The steepening of the PC when deflation expectations take hold as described by our model (\(\omega\) rises as \(y\) falls) does not contradict the structural flattening of the PC (lower \(\kappa\)). Likewise, given \(\kappa\), if monetary policy restores confidence in the return towards the normal regime (\(\omega\) falls as \(y\) rises) the PC appears flatter than during the deflationary period.
concomitant inflation gap implied by the forward iteration procedure is
\[ E_t \pi_{t+1} = \omega \kappa E_{t+1} \pi_{t+1} \] (which is indeed the expectation of (6)). Substituting these values into (1) we obtain
\[ (7) \quad \theta = -\alpha \theta \]
\[ \theta = [\alpha \beta p^2 + (1 + \alpha (1 + 2\beta'))p - \alpha (1 + \beta')]^{-1} \]
Therefore, nonzero output gaps are linear functions of interest-rate gaps, and we shall consider these misalignments as the key shock to the system. The important content of this OG equation is that \( \partial \theta / \partial p < 0 \), i.e. the impact of the interest-rate gap, in absolute terms, is larger when the de-anchoring of inflation expectations is larger.\(^{16}\) Hence de-anchoring has a compound amplification effect, both on the inflation gap, for a given output gap, and on the output gap, for a given natural rate shock.

2.3. Endogenising the de-anchoring of inflation expectations

Having highlighted the effect of the de-anchoring of inflation expectations, we now wish to endogenise it. Pursuing the suggested idea of rational beliefs, i.e. beliefs driven by experience, we wish the normal-regime confidence \( p \) to be driven by the observed state of the system, in particular the output gap, given its role in the expectation formation process.\(^{17}\) Thus, we posit a relationship between \( p \) and \( y \), that we call de-anchoring function (DA), with the following properties. First, \( p \) is updated with a lag, i.e. \( p_t = h(y_{t-1}) \), which seems a realistic feature once account is taken of the time necessary to collect and process information. Second, updating is such that \( p \) is reduced proportionally to the absolute value of \( y \) (i.e. updating is symmetric for positive as well as negative output gaps). Third, we allow, and wish to control, for reactivity in updating (i.e. for a given output gap, \( p \) may be reduced more or less largely). A function that embodies these properties is the following logistic
\[ (8) \quad h(y_{t-1}) = \frac{ABC e^{-D_{y_{t-1}}}}{(Be^{-D_{y_{t-1}}} + 1)^2} \]

\(^{16}\) For \( p = 1 \), in the normal regime, \( \theta = 1 \), i.e. a positive interest-rate gap generates a negative output gap proportional to the elasticity of aggregate spending.

\(^{17}\) Central banks communicate their target and assess their policy stance in terms of output, inflation or both depending on their mandate (and communication style). Here we are concerned with the expectation formation of the private sector, and as shown by equation (4), this hinges on the forecast of future output gaps. Anyway, the inflation and output gaps are univocally related by way of equation (6).
In order to find the values of the parameters \((A, B, C, D)\) we impose the following conditions

- \(h(0) = 1\)
- \(\lim_{y \to \pm \infty} h_y = 0\)
- \(h'(0) = 0\)
- \(h''(D) = 0\), with \(D > 0\)

The first condition states that with zero output gap agents believe in the normal regime with certainty (de-anchoring is nil). The second states that as the output gap grows unboundedly, \(p\) tends to zero (de-anchoring is maximal).\(^{18}\) The third condition ensures that \(p\) is bounded at 1 when the output gap is zero. The fourth condition controls for reactivity of \(p\). The solution of the system given by the four conditions yields the following parameter values

\[
A = 4, \quad B = 1, \quad C = 1, \quad D = \sigma
\]

Therefore we can write

\[
(9) \quad p_t = \frac{4e^{-\sigma y_{t-1}}}{(e^{-\sigma y_{t-1}} + 1)^2}
\]

(Figure 1)

Figure 1 shows the plots of function (9) for increasing values of reactivity \(\sigma\); as can be seen, higher values of \(\sigma\) determine larger revision of \(p\) (more reactivity) for a given \(y\).\(^{19}\) The causes that determine the magnitude of \(\sigma\) go beyond the aims of this paper, but we shall explore the effects of different values of this parameter.\(^{20}\)

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\(^{18}\) Note that this assumption excludes that confidence falls to zero for any finite output gap. The meaning of this property for the system to have finite nonzero gaps equilibria will become clear in the simulations in section 3.

\(^{19}\) Note that this function is behaviorally consistent with Bayesian inference. Our \(p\) is in fact analogous to the posterior belief \(p(N|y_{t-1})\) where \(N\) is the hypothesis of normal regime conditional on the observed nonzero output gap \(y_{t-1}\). Let the prior \(p(N) = 1\). Then, according to Bayes’ principle, the posterior belief is driven by the likelihood \(L(y_{t-1} | N) / \phi(y_{t-1})\) to observe \(y_{t-1}\) if \(N\) holds relative to its unconditional occurrence. Our DA function implies that this likelihood is reduced proportionally to \(y_{t-1}\).

\(^{20}\) The reactivity \(\sigma\) might be determined by history and conceived as changing over time, for instance in relation to the performance, or reputation, of the central bank in pursuing its target. For instance, a newly created central bank, or one with low reputation may generate high reactivity. This may also improve or diminish for a variety of reasons, such as a financial crisis, a crisis of confidence in the governor,
We now have a four equation system consisting of OG (7), PC (6), TR (3) and DA (9). For symmetry with the private sector, we assume that the central bank, too, adjusts its policy rate with one lag after observing the output and inflation gaps, i.e. \( \tau = t-1 \).

The reduced form of the system forms a dynamic first-order nonlinear system governed by the parameters \((\alpha, \beta, \kappa, \gamma_\pi, \gamma_y, \sigma)\), where each endogenous variable \(v_i\) in the vector \(v_t = [y_t, \pi_t, \hat{y}_t, p_t]\) is determined by a specific function of general form \(v_i(y_{t-1}, \pi_{t-1}, \hat{y}_{t-1}, p_{t-1}; y_0, \pi_0, \hat{y}_0, p_0)\), i.e. the lagged values and the initial state (or shock) of the endogenous variables themselves. Analytically, the system admits a zero-gaps steady state of the endogenous macroeconomic variables and with full confidence in the normal regime \(\bar{v} = [0, 0, 0, 1]\). However, after a shock, the trajectory taken by the system, and hence its dynamic evolution, depend on the values of the parameters. We are going to present the dynamic behaviour of this system by means of the results of simulations after a deflationary shock, that we have assumed in the form of a fall in the natural rate with a consequent interest rate gap, all other initial conditions being in steady state.

### 3. Simulations

#### 3.1. A baseline case

To begin with, we present a baseline case, given the following values of the parameters:

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r = 2% ), ( \alpha = 0.2 )</td>
<td>empirical estimates of the natural interest rate, and the interest-rate elasticity of private demand in the EU and US (e.g. Garnier and Wilhelmsen 2005, Laubach and Williams J. C. 2003)</td>
<td></td>
</tr>
<tr>
<td>( \beta = 0.98 )</td>
<td>standard model value of the time discount factor(^{21})</td>
<td></td>
</tr>
<tr>
<td>( \kappa = 0.09 )</td>
<td>structural elasticity of the PC, implied by the Calvo equation, given ( \beta ) and 75% of non-adjusted prices (e.g. Smets and Vouters 2003, Luk and Vines 2015)</td>
<td></td>
</tr>
<tr>
<td>( \gamma_\pi = 1 ), ( \gamma_y = 0.5 )</td>
<td>standard model values of the Taylor Rule</td>
<td></td>
</tr>
<tr>
<td>( \sigma = 1 )</td>
<td>our own reference value of the reactivity of the ( p ) function (see Figure 1).(^{22})</td>
<td></td>
</tr>
</tbody>
</table>

\(^{21}\) In the standard New Keynesian model, \( r = \beta^{-1} - 1 \approx 2\% \).

---

or a political conflict between the monetary authority and the government (Bordo and Siklos, 2014). Notably, high reactivity makes the central bank’s task harder.
The initial state of the system is a zero-gaps steady state, namely \((y_0 = 0, \pi_0 = 0, \hat{i}_0 = 0, (i_0 = r_0 = 2\%), \rho_0 = 1)\). The initial shock, at \(t = 1\), is an unaccommodated fall in the natural rate of \(\rho_1 = -1\%\), which implies \(\hat{i}_1 = i_0 - (r_0 + \rho_1) = 1\%.\) Virtual time is scaled in quarters. Figure 2 plots the four endogenous variables in percent points (except \(p\) which is measured in decimal points) over 30 periods.

The system converges monotonically to a new zero-gaps steady state, after an initial deflationary phase. It is worth noting that the policy rate correctly converges to the new value of 1% of the natural rate at zero inflation, while the normal-regime confidence is virtually unaffected (for precision it falls to 0.996 after 6 periods, and then it recoils). The negative output gap is larger and more persistent than the deflationary one, but this is almost entirely due to the role of the New-Keynesian structural parameters, which generate a rather flat PC combined with a Taylor Rule less reactive to output gaps than to inflation gaps. Though we may say that \(\sigma = 1\) generates a DA function which is not extremely smooth (see Figure 1), the system shows substantial resilience. Observationally, it could hardly be distinguishable from a normal system.

3.2. The zero lower bound, and the "new normal"

We have also "tested" the system at the zero lower bound (ZLB) of the policy rate. To begin with, the four equations of the system can provide the analytical solution. In fact, for any permanent \(i \neq 0\), the system yields (multiple) solutions with \((\bar{y} \neq 0, \bar{\pi} = 0, \bar{\rho} < 1)\). These are equilibria in the sense that all the variables are mutually consistent. Of course, the configuration of the equilibria depends on the interest-rate gap and the parameters \((\alpha, \beta, \kappa, \sigma)\).

\[22\] It may be noted that the empirical work closest to our model, the already mentioned Gürkaynak et al. (2010), finds significant reactivity of long-term inflation expectations to various macroeconomic news in US, UK and Sweden. Following news about real GDP, the estimated reactivity varies between 0.3 in Sweden and 1.8 in the US. The correspondent range of our parameter \(\sigma\) is between 1.1 and 2.2.
To this end, we present a simulation where we have set the initial shock to the natural rate at $\rho_1 = -3\%$, which implies a negative value of $-1\%$, an initial interest-rate gap $\hat{i}_1 = 3\%$ and a permanent one $\hat{i} = 1\%$ at the ZLB. The simulation has been run by freezing the policy rate when hitting the ZLB. The plots are drawn in Figure 3.

First, the system fails to reach a new zero-gaps steady state. Output and inflation, however, do converge to a steady state with permanent negative gaps ($-0.2\%$ and $-0.03\%$ respectively). Parallely, the normal-regime confidence $p$, though very slightly affected, settles down below unity ($0.989$) which indicates a permanent de-anchoring of expectations. A larger shock would, ceteris paribus, produce larger negative gaps and loss of confidence.

This simulation exemplifies that in the deflationary equilibrium, $p$ may not be zero. This property results in a backstop to the de-anchoring process: as $y$ stops falling, also $p$ does. At what point this occurs, however, emerges endogenously. Our suggested interpretation is that this kind of equilibrium represents a state of affairs that is commonly dubbed the "new normal". This state displays lower confidence in the resilience of the system by a finite amount rather than triggering a continuous fall of confidence, which indeed would seem an extreme outcome, at least for central banks with solid reputation. To put it in other words, while agents, at the ZLB, have no reason to believe that the central bank can do anything else to correct the non-zero gaps, they also believe that, ceteris paribus, nothing can bring the system into an even worse condition.

To modify this equilibrium, a structural change is necessary, where the structure of the economy includes the policy rule(s) (e.g. a change in the monetary policy conduct or a shift from monetary to fiscal policy). Then the existing degree of de-anchoring may be regarded as the hurdle to be overcome in order to restore full confidence in the normal regime.

### 3.3. Exploring the space of the parameter values

We now present further simulations whereby we have explored the dynamic behaviour of the system in the space of parameter values.

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23 Analytically, this property depends on the assumption that $p$ falls to zero as $y$ tends to infinity (see footnote 18).
We have selected three parameters of interest ($\kappa$, $\gamma_\pi$, $\sigma$) allowed to change within a range of values while keeping the others constant at their baseline value, and three types of shocks $\rho$ to the natural interest rate, small ($-1\%$), medium ($-2\%$), large ($-3\%$). Note that the first two types can be fully accommodated by cutting the policy rate, whereas the third cannot owing to the ZLB.

The first set of simulations that we present test the resilience of the system when the parameters ($\kappa$, $\gamma_\pi$) are kept constant at their baseline value, while the parameter $\sigma$ and the extent of $\rho$ are allowed to increase. Recall that higher values of $\sigma$ entail larger fall of $p$ for any observed negative output gap, which, as previously explained, amplifies the impact of interest-rate shocks on the output and inflation gaps.

The second set of simulations focuses on $\sigma$ vis-à-vis, respectively, $\gamma_\pi$ and $\kappa$, while keeping the other constant at the baseline value.

The number of iterations (30 "quarters") has been chosen to mimic the typical medium-term horizon of business cycles. However, the number of iterations is inevitably arbitrary, and hence we have set the following criteria of evaluation of the system behaviour. First, the benchmark variable is the level of the output gap at the last iteration. Then, we have identified four areas that gauge the extent of divergence (convergence) of the system relative to the zero-gaps steady state, indexed from 0 to 3:

- **0** (blue) = *convergence*, the residual output gap is less than 10% of the initial shock
- **1** (light blue) = *tendential convergence*, the residual output gap is between 10% and 50% of the initial shock
- **2** (yellow) = *tendential divergence*, the residual output gap is between 50% and 100% of the initial shock
- **3** (red) = *divergence*, the residual output gap is larger than the initial shock.

**The $\sigma$-$\rho$ interaction**

Two questions are particularly relevant. How do greater values of $\sigma$ affect the system given the baseline (standard) values of $\gamma_\pi$ and $\kappa$? How does

\[\text{To discard casual convergence of the very last observation, we have also checked that from the 25th period onwards the values of the output gap do not exceed the value of the initial shock.}\]
the entity of the shock modify the picture? The answer is given by the simulation results depicted in Figure 4 in the $\sigma$-$\rho$ space. The range of values of $\sigma$ has been set $\sigma \in [0, 8]$ and the natural-rate shock in the full range $\rho_1 \in [0, -3\%]$. Recall that for $\rho_1 < -2\%$, the ZLB is binding. The white dots indicate the positions of the system in the baseline cases with $\sigma = 1$, $\rho_1 = [-1, -2, -3]$.

[Figure 4]

The first notable feature of the system is that the regions of convergence or tendential convergence shrink with greater values of $\sigma$ and/or $\rho$. A small shock ($\rho_1 \geq -1\%$) allows the central bank to control the system up to relatively large values of $\sigma$ (about 6). With a medium shock (up to $-2\%$), the central bank is still able to control the system only up to a value of $\sigma$ of about 2.5. With a large shock (up to $-3\%$), the threshold value of $\sigma$ is even lower. Interestingly, the frontier of convergence is a well-defined convex relationship between $\sigma$ and $\rho$.

The second notable feature is the tendential convergence region (light blue) for large shocks, i.e. under the ZLB regime. This region embeds the states of the system that we have dubbed "new normal" in the baseline case with ZLB. The system stabilises itself "close" to, but below the zero-gaps steady state. As shown above analytically, in these states the confidence in the normal regime remains "close" to, but below 1. However, these states are attainable only for small values of $\sigma$.

**The $\sigma$-$\gamma_\pi$ interaction**

In the light of the previous results, the next relevant question is whether an aggressive Taylor Rule (larger $\gamma_\pi$) can counteract the effects of increasing $\sigma$. The ranges of the two parameters have been set $\sigma \in [0, 8]$ and $\gamma_\pi \in [0, 4]$. Figure 5 displays the graded divergence/convergence regions of the system in the space ($\sigma$-$\gamma_\pi$) after the three types of shocks to the natural rate.

[Figure 5]

Simulations are in line with the previous ones. With a small shock ($-1\%$), a standard value of $\gamma_\pi$ above 1 allows the central bank to control the system up to relatively large values of $\sigma$. Above this threshold, the system enters regions of tendential or outright divergence, typically with oscillatory dynamics. The extent of the shock systematically reduces the regions of convergence, so that the magnitude of $\sigma$ becomes more binding. With the large shock ($-3\%$), under the ZLB, the system again settles down in the
"new normal" (the light blue region) provided that $\sigma$ is sufficiently small (less than 2.5 approximately). Notably, the answer to the above question is that in the divergence regions, a more aggressive (conventional) monetary policy with larger $\gamma_\pi$ is ineffective, and it is obviously ineffective at the ZLB.

The $\sigma$-$\kappa$ interaction

Price stickiness is key to the New Keynesian framework in which our simulations are set. As said above, the usual estimations or calibration of the parameter $\kappa$ that measures the slope of the PC yield very small values. On the other hand, we have shown that the de-anchoring of expectations steepen the PC. The interaction between $\sigma$ and $\kappa$ is therefore an interesting issue to address. The value ranges of the two parameters have been set $\sigma \in [0, 8]$ and $\kappa \in [0, 4]$.

We have already seen that for standard values of $\kappa$ and $\gamma_\pi$, the system enjoys a large region of convergence up to relatively high values of $\sigma$. The noteworthy feature of the parameter $\kappa$ is that if it increases, the region of convergence shrinks, though this effect is relatively moderate. Medium and large shocks magnifies this phenomenon, especially at the ZLB. In other words, as prices become more flexible, the central bank loses control on the system, and is more exposed to de-anchoring. This may appear a counterintuitive result, since greater price flexibility is, generally, regarded as an unconditional benefit for the economy. However, it is worth recalling that the problem that price flexibility would undermine monetary policy based on the interest rate control was raised by Wicksell himself (Mazzocchi, 2013a; Mazzocchi et al., 2014).

4. Conclusions

Since its foundation the performance of the ECB has been viewed with general satisfaction. In line with the ECB’s commitment to price stability as its primary goal, its achievements in curbing inflation and anchoring

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25 According to the Calvo equation, $\kappa = 4$ corresponds (approximately) to less than 20% of firms not changing their price, i.e. an economy with almost full price flexibility
expectations contributed to the credibility of monetary policy in the Euro Area. However, the global financial crisis has proven a major strain to any monetary policy. Since 2012 inflation rates have been weakening rapidly despite the ongoing economic recovery. In particular, the low inflation rates in the Euro Area have raised the question whether inflation is still credibly anchored to the medium term target of below, but close to 2%. The empirical identification strategy of de-anchoring is that insofar as long-run inflation expectations remain unaffected by incoming economic surprises, economic agents apparently trust the ability of the central bank to maintain its inflation target (the normal regime). By contrast, if confidence in the normal regime weakens and inflation expectations are de-anchored, inflation and/or output gaps can lead to unwanted changes in expectations, away from the given inflation target. In other words, deviations from target are self-amplifying and make the system more difficult to control.

Our model confirms this interpretation and clarifies its implications. The first critical factor is the interaction between the reactivity of inflation expectations to output gaps and the dimension of shocks to the natural interest rate. The normal regime is resilient as long as the policy rate can accommodate shocks above the ZLB, and reactivity is sufficiently small. If reactivity is high, the system can take divergent paths, some of which may be explosive. Controlling for structural price stickiness, simulations show that, *ceteris paribus*, as stickiness decreases the convergence regions shrink, the reason being that a steeper PC amplifies the output gaps. If reactivity is such that expectations "fall faster" than the policy rate, which reaches the ZLB without correcting the interest-rate gap, the system converges to a new steady state with permanent negative gaps. This state of affairs represents the "new normal" in the sense that it reduces the confidence in the resilience of the system by a finite amount rather than triggering a continuous fall of confidence.

Our study also offers some policy considerations. More aggressive (conventional) monetary policy, i.e. with larger inflation coefficient, is obviously ineffective at the ZLB, but it is also ineffective above the ZLB when adverse conditions (large shocks and/or expectation reactivity) push the system into the divergence region. This finding seems to support the necessity, in those conditions, to abandon conventional monetary policy and to switch to an aggressive reflationalary policy that prevents the entrenchment of deflationary expectations.
Two main questions remain open to further research. One regards the DA function. Another reasonable characterization of reactivity may be the speed of updating, i.e. after how many periods of observed output gaps agents update $p$. This gives relevance to the persistence of output gaps, rather than amplitude, as a determinant of the normal-regime confidence. Another is whether and how non-conventional monetary policies (e.g. quantitative easing) may succeed in reverting the de-anchoring of inflation expectations when the conventional one fails.

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Figure 1. The DA function with increasing values of $\sigma$

Figure 2. Baseline case
Figure 3. The baseline case at the ZLB

Figure 4. The $\sigma$-$\rho$ interaction
Figure 5. The $\alpha$-$\gamma$ interaction

small shock ($\rho_1 = -1\%$)

medium shock ($\rho_1 = -2\%$)

large shock ($\rho_1 = -3\%$)
Figure 6. The $\sigma$-$\kappa$ interaction

small shock ($\rho_1 = -1\%$)  medium shock ($\rho_1 = -2\%$)  large shock ($\rho_1 = -3\%$)