Matching frictions, credit reallocation and macroeconomic activity: how harmful are financial crises?

Emanuele Ciola, Edoardo Gaffeo e Mauro Gallegati
Guidelines for authors
Papers may be written in Italian or in English. Faculty members of the Department must submit to one of the editors in pdf format. Management papers should be submitted to R. Gabriele. Economics Papers should be submitted to L. Andreozzi. External members should indicate an internal faculty member that acts as a referee of the paper.

Typesetting rules:
1. papers must contain a first page with title, authors with emails and affiliations, abstract, keywords and codes. Page numbering starts from the first page;
2. a template is available upon request from the managing editors.
Matching frictions, credit reallocation and macroeconomic activity: how harmful are financial crises?*

Emanuele Ciola †‡  Edoardo Gaffeo §¶  Mauro Gallegati †

February 19, 2018

Abstract

This paper develops a macroeconomic model of real-financial market interactions in which the credit and the business cycles reinforce each other according to a bidirectional causal relationship. We do so in the context of a computational agent-based framework, where the channelling of funds from savers to investors occurring through intermediaries is affected by information frictions. Since banks compete in both the deposit and the loan markets, the whole dynamics is driven by endogenous fluctuations in the size of the intermediaries balance sheet. We use the model to show that financial crisis are particularly harmful when hitting in phase with a real recession, and that when this occurs the loss in real output is permanent.

Keywords: Agent-based model; matching frictions; banking; financial crises

JEL classification: E32; E37; G01

*We thank, without implications, the participants to the conference "Finance and Economic Growth in the Aftermath of the Crisis" for their helpful comments.

†Polytechnic University of Marche
‡University Jaume I
§University of Trento
¶Corresponding author: Edoardo Gaffeo, Department of Economics and Management, University of Trento, Via Inama 5, 38122 Trento, Italy. E-mail: edoardo.gaffeo@unitn.it.
1 Introduction

Credit to non-financial businesses and households is notoriously pro-cyclical (Covas & Haan 2011, Schularick & Taylor 2012), a fact that in the last three decades has attracted a huge amount of theoretical work meant to explore the channels linking financial flows to real macroeconomic activity (Foglia et al. 2011). The received pre-2007 approach was centered on the idea that the causality runs from changes in the real sector to movements in financial flows, whereas financial frictions impinging on the borrowers’ capacity amplify the macroeconomic impact of exogenous shocks to productivity or preferences (Bernanke & Gertler 1989, Greenwald & Stiglitz 1993, Kiyotaki & Moore 1997). In turn, the bulk of the research emerged after the global financial crisis has offered theories pointing toward an inversion of the causality nexus. In the models surveyed in e.g. Brunnermeier et al. (2013) and Christiano & Ikeda (2011), a shrinkage in the total amount of funds channelled from lenders to borrowers derives from disruptions in financial markets due to shocks affecting either banks’ capital or liquidity. Recessions are therefore the outcome of a cut in spending and hiring by borrowers generated by a supply-induced credit tightening.

In this paper we present an agent-based computational model in which the flowing of funds from savers to investors is intermediated by a stream of banks competing in fully decentralized markets for deposits and loans, where the only friction affecting agents is that they are imperfectly aware of the economic opportunities they potentially face. In our setting, the transmission channel between the financial and the real sectors turns out to be bidirectional, while cyclical fluctuations emerge endogenously as intermediaries adjust the size of their balance sheet to the varying competitive conditions affecting the asset and liability sides, respectively. In particular, banks compete on prices either to attract demand deposits from households and to offer long-term - but freely severable - credit contracts to firms, so that the net interest margin represents a key state variable indicating the viability of the banking sector while transforming maturities. Due to search costs in locating profitable opportunities, the economy is affected by matching and allocation imperfections that co-evolve endogenously, giving rise to large reallocations of credit among firms on a regular basis. The dynamics of credit creation and destruction are related to the relative strength of economic activity, however. This is due to the ensuing complementarity between the number of active bank-firm relationships and the savers’ incentive to provide
funds to intermediaries, which returns a pro-cyclical average productivity. The time paths we obtain through simulations show that the statistical features of the gross flows of credit, as well as its aggregate volume, are in line with those observed in real data. Interestingly enough, this result is obtained in a simplified framework completely abstracting from agency frictions, aggregate disturbances to primitive parameters, time-varying risk taking due to capital regulation or institutional arrangements like a deposit guarantee or a LOLR authority.

Having established the general features of the relationship linking the credit and the business cycles, we go on to investigate how a financial crisis – here modelled as a funding run leading to the disruption of one or more banks – affects the macroeconomic performance. We find that the interplay of credit creation and destruction associated to a financial turmoil generates downturns that are deeper and more prolonged, that the subsequent rebound can be insufficient to regain pre-crisis output levels, and that these effects are more pronounced when the financial crisis coincides with a “real” downturn. Thus, our model presents two implications corroborated by the empirical evidence: 1) financial crisis can lead to permanent output losses (Boysen-Hogrefe et al. 2016); 2) banking crises operate as a factor limiting aggregate activity distinct from standard recessions (Dell’Ariccia et al. 2008).

While close in spirit to the stream of research dealing with equilibrium search in credit markets (Diamond 1990, Den Haan et al. 2003, Wasmer & Weil 2004, Becsi et al. 2013), our model is firmly rooted in the tradition of the agent-based literature exploring the emergence of macroeconomic features from the localized interactions of heterogeneous agents employing decentralized matching and bargaining protocols (Fagiolo et al. 2004, Gaffeo et al. 2008, 2015, Riccetti et al. 2015, Guerini et al. 2018). This approach offers us two key advantages. First, we can extend the macroeconomic-oriented analysis of credit allocation from a static multiple equilibria framework to a dynamic one, where credit mismatches develop from feedback effects as agents interact in distinct but interrelated customer markets (Gottfries 1991). Second, beyond overcoming the need to postulate an exogenously-given matching technology, we provide accurate sensitivity analyses to analyse how macroeconomic outcomes are affected by banks’ behavior in extending credit.

Our findings are also related to two other broad streams of literature. First, a large body of empirical research has persuasively shown that the dynamic process of credit reallocation across heterogeneous firms is an intense and pervasive phenomenon, that its magnitude and volatility dwarf that of
the total volume of credit, and that it plays a significant role in determining the strength of macroeconomic activity and real growth rates (Jayaratne & Strahan 1996, Herrera et al. 2011, Inklaar et al. 2015). The reshuffling of financial resources across firms due to the creation and destruction of bank debt is particularly compressed during episodes of financial turmoil (Contessi & Francis 2013, Hyun 2016), signalling that the interplay between resource misallocation and the endogenous worsening of financial frictions is at the heart of the deep and prolonged periods of low resource utilization following acute financial crises (Hall 2011, Bordo & Haubrich 2017).

We add to this essentially empirical literature by providing a theoretical justification for these stylized facts. In particular, we show that the intense credit reallocation observed in real data can be traced back not only to the way intermediaries compete in the market for loans, but also to the feedback exerted on it by the fact that banks also struggle to attract depositors as they set their passive interest rates.

Second, a rapidly growing body of work has stressed the role of financial intermediaries’ balance sheets both as shock amplifiers and as a source of fluctuations in economic activity (Boyarchenko et al. 2015, Brunnermeier & Koby 2016). Accordingly, using data for the USA Adrian & Shin (2010) show how fluctuations in the balance sheets of financial institutions possess forecasting power for future GDP growth, while the evidence provided by Adrian et al. (2010) suggests the presence of a causal chain running from the term spread, moving to the net interest margin of banks, advancing to lending volumes and finally reaching real growth. We complement this literature by highlighting a different channel linking fluctuations in the size of intermediaries’ balance sheets and fluctuations in real activity. In addition to the risk-taking and the leverage channels explored so far, we advance an explanation in which the net interest margin registered by banks evolves in cycles because of a “competition” channel driven by a complementarity between financial intermediation and aggregate investment.

The rest of the paper is organized as follows. Section 2 presents the model and provides a discussion of the initial conditions used in simulations. Section 3 takes stock of simulation results to highlight the existence of endogenous and co-evolving business and financial cycles. Section 4 extends the basic model by examining the effects of exogenously determined bank runs. Section 5 concludes and outlines directions for further research.
2 The model

The economy is composed of a fixed number of households \((h = 1, \ldots, n_h)\), intermediaries \((i = 1, \ldots, n_i)\) and firms \((f = 1, \ldots, n_f)\). Households are endowed with accumulable wealth, which we assume for simplicity as the only factor of production, while firms have no resources of their own and must raise funds on the credit market to start production. Families deposit their wealth in only one intermediary at a time as demand deposits and they can not establish direct connections with producing firms. Here we are implicitly assuming that investment evaluation and ex-post verification is costly and intermediaries are more efficient than households in performing these tasks. In addition to this, we assume that the financial sector can supply credit without sustaining any additional effort or expense. On the other hand, households can always search a new counterpart and they can move their wealth from an intermediary to another without constraints.

As in Den Haan et al. (2003) intermediaries supply credit to firms through a simple contract in which a fixed portion of the production \((P_{i,f})\) is left to the entrepreneur as payment for the effort. From the point of view of the entrepreneur this value can be interpreted both as a measure of firm profitability \((P_{i,f})\) and of financing costs \((r_{i,f} = [f(k_{i,f}) - P_{i,f}] / k_{i,f})\), where \(f(\cdot)\) is the production function and \(k_{i,f}\) is the external finance supplied by the \(i\)-th intermediary to the \(f\)-th firm. The level of profit \((P_{i,f})\) and the loan size \((k_{i,f})\) are set at the beginning of the open-ended credit relation and they are not modified through time\(^1\). On the other hand, both the intermediary and the firm can close a credit contract in any moment. Specifically, the former will reduce its investment when it requires additional liquidity to pay back its depositors, while the latter will search for another intermediary whenever it observes in the market new contracts which pay a higher average profit \((P_t)\).

Finally, we assume that entrepreneurs have a reserve utility \((\theta)\) and sustain a cost \((x)\) each time they search for a new counterpart. This reserve utility can be interpreted as a home production level which is not included in the aggregate production calculations. As we will explain later on, entrepreneurs will thus have the incentive to demand credit and start a market activity only if the new firm with external financing will return an expected

\(^1\)Entrepreneur which are not borrowing have \(P_{i,f} = \theta\) and \(k_{i,f} = 0\), where \(\theta\) is the reserve utility which we will define later in the text.
level of profits higher than their reserve value. To further simplify the model, we assume also that both the home and the market activities require the same level of effort from the entrepreneur. In this way the only discriminants in entrepreneurs’ choices can be found in their reserve utility and in the search cost. In addition to this, we force intermediaries to finance the search cost of new lenders for the first period (see Table 1). In this way the value of the firm, and consequently of the credit relationship, increases through time, in line with the literature on relationship banking (see e.g. Boot (2000) for a review).

Notice that in this framework intermediaries react to the behaviour of households and firms. From the liabilities side, families can move their wealth across intermediaries without any constraint, while on the assets side entrepreneurs can shift from the current lender to a new one every time they observe a better financing opportunity in the market. As stated before, in this choice firms must take into account the search cost. In particular, an entrepreneur will look for a new counterpart if

$$ E[\text{search}] - E[\text{not search}] = \frac{P_t}{R_t} - x - \frac{P_{i,f}}{R_t} \geq 0 $$

(1)

where we assume that firms discount future cash flows considering the current market interest rate ($R_t$) as fixed and constant through time. In addition to this, entrepreneurs use average profits ($P_t$) as a proxy for evaluating new loans and do not take account of the probability of future rescissions of the contract.

At the beginning of the world families have an homogeneous quantity of wealth $d_{h,0} = W_0$ ($\forall h = 1, \ldots, n_h$) which they deposit in a bank selected at random. We define the subset of households connected to the $i$-th intermediary at time $t$ as $H_{i,t}$, hence for $t = 0$ we have $\Pr(h \in H_{i,0}) = U(1, n_i)$, where $U(\cdot)$ is an uniform distribution.

Intermediaries set an initial contractual profit $P_{i,0} = \theta + xR_0$, fix a minimum investment dimension $\bar{k}_{i,0}$, evaluate the resources available for investments and generate a credit supply schedule\footnote{Firms which have just started a new credit relation ($t = 1$) must satisfy the condition: $E[\text{search}] - E[\text{not search}] = \frac{P_t}{R_t} - \frac{P_{i,f}}{R_t} + xR_t \geq 0$.}.

We define minimum investment dimension as the lowest value of capital which produces a non negative interest rate: $\bar{k}_{i,t} = \{ k \mid f(k) - P_{i,t} = 0 \}$. We

\footnote{We set the initial contractual profit $P_{i,0} = \theta + xR_0$ to start the model with a value corresponding to the long term zero profit condition $\frac{P_{i,0}}{R_0} = \frac{\theta}{R_0} + x$.}
**Table 1: Firm value**

<table>
<thead>
<tr>
<th>Time</th>
<th>$t = 0$</th>
<th>$t = 1$</th>
<th>$t = 2, \ldots, \infty$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash flow</td>
<td>$P_{old} + x - x$</td>
<td>$P_{new} - (1 + R)x$</td>
<td>$P_{new}$</td>
</tr>
<tr>
<td>Firm value</td>
<td>$V_0 = P_{old} + \frac{P_{new}}{R_0} - x$</td>
<td>$V_1 = (1 + R_1) \left( \frac{P_{new}}{R_1} - x \right)$</td>
<td>$V_2 = (1 + R_2) \frac{P_{new}}{R_2}$</td>
</tr>
</tbody>
</table>

*Note:* The firm starts a new credit relation at time $t = 0$ and considers the current interest rate ($R_t$) as fixed and constant through time. Because of bounded rationality, the entrepreneur does not take account of the probability of future rescissions of the contract.

**Figure 1: Flow of funds and agents’ behaviour**
set this constraint to avoid negative returns, which could increase excessively
the volatility of simulations and, because of this, could occasionally produce
degenerate equilibria in which only one intermediary remains in the market\(^4\).
From a theoretical point of view, however, this assumption allows us also to
define deposits as truly risk-free, in the sense that the capital quota is always
repaid.

The credit supply of every intermediary is then computed according to a
stochastic allocation process. The quantity of available resources is simply
given by the sum of the initial deposits \(\hat{K}_{i,0} = \sum_{h \in H_{i,0}} d_{h,0}\), while the size
of each new loan is extracted from the distribution \(k_{j}^{new} \sim U(\hat{k}_{i,0}, \hat{K}_{i,0})\), until the condition \(\hat{K}_{i,0} = \sum_{j \in I_{i}^{new}} (k_{j}^{new} + x)\) is satisfied\(^5\).

Lastly, firms demand for funds and start production. Given that at
this stage all the intermediaries offer the same contractual profit, we allo-
cate credit across entrepreneurs at random. In other words, we extract the
composition of the initial credit network with probabilities
\(\Pr(f \in F_{i,0}) = U(1, n_i) \cdot U(1, n_f)\), where \(F_{i,t}\) is the set of firms connected with the \(i\)-th in-
termediary at time \(t\).

Once the system starts the economy evolves recursively following the
subsequent steps in each iteration:

1. Entrepreneurs start production, retain profits and return to lenders the
   interest rate:
   \[
   r_{i,f} = \frac{f(k_{i,f}) - P_i}{k_{i,f}} \quad (2)
   \]
   The average productivity of the economy is then defined as:
   \[
   z_t = \frac{\sum_{f=1}^{n_f} f(k_{i,f})}{\sum_{f=1}^{n_f} k_{i,f}} \quad (3)
   \]

2. Intermediaries collect the stream of income produced by their invest-
   ments and calculate the active interest rate:
   \[
   r_{i,t}^{a} = \frac{\sum_{f \in F_{i,t-1}} r_{i,f} \cdot k_{i,f} + \sum_{f \in F_{i,t-1} \wedge f \notin F_{i,t-2}} r_{i,f} \cdot x}{\sum_{f \in F_{i,t-1}} k_{i,f}} \quad (4)
   \]

\(^4\)Degenerate equilibria usually are the effects of an excessive reaction of the system in
the early phases of the simulation. A possible solution to this problem would have been to
use a larger number of agents to avoid initial jumps and thus have a smoother transition,
but from a computational point of view the system would have become excessively costly.

\(^5\)We define \(I_{i}^{new}\) as the set of new loans supplied by the \(i\)-th intermediary.
and its counterpart on deposits:

\[ r_{i,t}^d = \frac{\sum_{f \in F_{i,t-1}} \rho_{i,f} \cdot k_{i,f} + \sum_{f \in F_{i,t-1} \setminus f \notin F_{i,t-2}} \rho_{i,f} \cdot x}{\sum_{h \in H_{i,t-1}} d_{h,t-1}} \]

where the return obtained from the financing of the search cost is excluded only for new loans \( (f \in F_{i,t-1} \setminus f \notin F_{i,t-2}) \).

The aggregate active and passive interest rates are then defined through the weighted averages:

\[ R_{a,t} = \frac{\sum_{i=1}^{n_i} \sum_{f \in F_{i,t-1}} \rho_{a_{i,f}} \cdot k_{f,t-1}}{\sum_{i=1}^{n_i} \sum_{f \in F_{i,t-1}} k_{f,t-1}} \]

\[ R_{d,t} = \frac{\sum_{i=1}^{n_i} \sum_{h \in H_{i,t-1}} \rho_{d_{i,t}} \cdot d_{h,t-1}}{\sum_{i=1}^{n_i} \sum_{h \in H_{i,t-1}} d_{h,t-1}} \]

3. Households observe the return obtained on deposits, consume and allocate capital. Specifically, families receive the interest rate paid by their corresponding intermediary, namely \( r_{h,t} = r_{i,t}^d \) with \( h \in H_{i,t} \), and maximize the intertemporal problem:

\[ \max E_t \left[ \sum_{s=0}^{\infty} \beta^s \log(c_{h,t+s}) \right] \]

s.t. \( d_{h,t+s} + c_{h,t+s} = (1 + r_{h,t+s})d_{h,t+s-1} \)

whose solutions are:

\[ c_{h,t} = (1 - \beta)(1 + r_{h,t})d_{h,t-1} \]

\[ d_{h,t} = \beta(1 + r_{h,t})d_{h,t-1} \]

Subsequently, households search at random a new intermediary such that \( r_{new,t}^d \geq r_{i,t}^d \), which in turn implies \( h \notin H_{i,t} \cap h \in H_{new,t} \).

Notice that the combination of this mechanism with the minimum investment condition ensures that families will always allocate their wealth to an active intermediary. In fact, banks without deposits will not have the resources to supply credit and will not pay any interest.
On the contrary, intermediaries with a positive balance sheet will return a positive, even if small, income stream to their depositors.

Therefore, in this framework non-performing banks will only exit from the economy. Indeed, households will slowly withdraw all their deposits from sub-optimal intermediaries, up to the point when all the resources in those banks will be completely depleted. From that moment on those intermediaries will no longer be selected by the families because of the zero interest rate. This condition is necessary to grant the emergence of an endogenous bargaining power among agents.

4. Intermediaries evaluate liquidity, modify the contractual profit on new loans and supply credit.

The term liquidity \( L_{i,t} \) indicates the amount of unallocated resources available for investments in each intermediary at the beginning of every period. This value is given by the difference between the current level of deposits and the portfolio of investments inherited from the preceding period:

\[
L_{i,t} = \sum_{h \in H_{i,t}} d_{h,t} - \sum_{f \in F_{i,t-1}} k_{i,f}
\]  

(12)

When an intermediary experiences an excessive withdraw of funds, namely when \( L_{i,t} < 0 \), it has to reduce lending to repay depositors. Therefore, a bank in this situation will evaluate the interest rate of each investment and will close the contracts with the lowest return, until the condition \( \sum_{f \in F_{i,t-1}} \tilde{k}_{i,f} + L_{i,t} \geq 0 \) is satisfied, where \( \tilde{k}_{i,f} \) are the credit lines with the minimum profitability.

Subsequently, intermediaries modify the contractual profit that they will pay on new loans. In particular they follow the rule:

\[
P_{i,t} = \begin{cases} 
P_{i,t-1} & \text{if } \sum_{h \in H_{i,t}} d_{h,t} \geq \sum_{h \in H_{i,t-1}} d_{h,t-1} \\
\left[1 + \gamma \cdot \mathbb{1} \cdot U(0, 1)\right] \cdot P_{i,t-1} & \text{otherwise}
\end{cases}
\]  

(13)

where \( \mathbb{1} = \begin{cases} +1 & \text{if } U_{i,t-1} \geq \bar{k}_{i,t-1} + x \\
-1 & \text{otherwise}
\end{cases} \)

When an intermediary observes an increase in the total amount of deposits it keeps the contractual profit at the level of the previous period.
In fact, it recognizes that it is paying an interest rate higher than the rest of the economy and that it is optimal for it to not modify the conditions which produced that result. Conversely, when it experiences a withdrawal of deposits, the intermediary must understand why it is not able to pay a sufficiently high return to its investors.

The main driver of this decision is the amount of unallocated capital \( (U_{i,t-1}) \) accrued in the preceding period. As stated earlier, in this framework it is always optimal for the single intermediary to invest all the available resources to obtain a non-negative interest rate. Hence, an excess of uninvested funds – defined here as a positive difference between the overall quantity of unallocated resources and the minimum investment dimension plus the financing of the search cost \( (U_{i,t-1} \geq \bar{k}_{i,t-1} + x) \) – indicates that the bank is not offering a sufficiently competitive contractual profit to the firms. In the opposite situation, the intermediary recognizes that it is leaving an excessively high quota of the production to the entrepreneurs and it is obtaining a sub-optimal return from its investments.

Finally, as in the initialization phase of the economy, intermediaries compute the minimum investment dimension from the new contractual profit \( (P_{i,t}) \) and extract the size of each new loan from the distribution \( k_{j}^{new} \sim U(\bar{k}_{i,t}, \bar{K}_{i,t}) \), where \( \bar{K}_{i,t} = \sum_{f \in F_{i,t-1}} \tilde{k}_{i,f} + L_{i,t} \).

5. Firms observe the average contractual profit and evaluate the opportunity cost of changing their counterparts.

The average contractual profit on new loans is defined:

\[
P_t = \frac{\sum_{i=1}^{n_{i}} \sum_{j \in I_{i}^{new}} P_{j,t}}{\sum_{i=1}^{n_{i}} 1}
\]

and is updated each time a firm borrows from a bank. Specifically, the order of firms acceding to the credit market is extracted at random and when a new loan is issued it is removed from the credit supply schedule. The convenience of searching a new intermediary is then obtained through equation (1) and, if positive, the entrepreneur will look for another counterpart such that:

\[
\frac{P_{z,t}}{R_t} - x \geq \frac{P_{i,f}}{R_t}
\]
which implies \( f \notin F_{i,t} \land f \in F_{z,t}, P_{i,f} = P_{z,t} \) and \( k_{i,f} = k_{j}^{\text{new}} \) with \( j \in I_{z}^{\text{new}} \).

6. Finally, when the credit supply is completely exhausted or there is no more demand for new loans, intermediaries evaluate the final amount of unallocated resources:

\[
U_{i,t} = \sum_{h \in H_{i,t}} d_{h,t} - \sum_{f \in F_{i,t}} k_{i,f} - \sum_{f \in F_{i,t} \land f \notin F_{i,t-1}} x
\tag{16}
\]

where they subtract also the financing of the search costs of new borrowers.

As we will explain later in the text, to fully understand the circumstances which generate endogenous credit cycles, it is necessary to focus both on the dynamic of the interest rate differential and on the definitions of liquidity and unallocated capital.

Starting from the margin of intermediation, by definition the balance sheet equation of each intermediary implies the following economy-wide identity:

\[
\sum_{i=1}^{n_{i}} \sum_{h \in H_{i,t-1}} d_{h,t-1} = \sum_{i=1}^{n_{i}} U_{i,t-1} + \sum_{i=1}^{n_{i}} \sum_{f \in F_{i,t-1}} k_{f,t-1}
\tag{17}
\]

In accordance to this, the aggregate interest rate differential is a function of the overall level of unallocated capital and of the return obtained from investments:

\[
R^{a}_{t} - R^{d}_{t} = \frac{\sum_{i=1}^{n_{i}} U_{i,t-1}}{\sum_{i=1}^{n_{i}} U_{i,t-1} + \sum_{i=1}^{n_{i}} \sum_{f \in F_{i,t-1}} k_{f,t-1}} R^{a}_{t}
\tag{18}
\]

Hence, both a rise in contractual profits and a decline in firms productivity, by reducing the active interest rate, shrink the margin of intermediation. In a similar manner, a contraction in the level of unallocated capital decrease the interest rate differential.

With regards of the equations for liquidity (12) and unallocated capital (16), it may appear that they represent the same economic dimension. To better understand the differences between those measures and their economic intuition, they can be rewritten in terms of their past values. Specifically, by substituting (12) in (16) we find:

\[
U_{i,t} = \sum_{f \in F_{i,t-1}} k_{i,f} - \sum_{f \in F_{i,t}} k_{i,f} + L_{i,t}
\tag{19}
\]
where we removed for simplicity the financing of search costs of new borrowers. Similarly, by taking (16) with one period lag and by substituting it in (12) we obtain:

\[ L_{i,t} = \sum_{h \in H_{i,t}} d_{h,t} - \sum_{h \in H_{i,t-1}} d_{h,t-1} + U_{i,t-1} \]  \hspace{1cm} (20)

The new versions of the two equations put under light the main drivers of the underlying economic dynamic. Specifically, with regards of the unallocated capital \((U_{i,t})\), it emerges that, given the level of liquidity, the overall amount of uninvested funds left in the banks is determined by entrepreneurs’ movements across intermediaries \((F_{i,t-1} \rightarrow F_{i,t})\). Conversely, the resources available for supplying new credit \((L_{i,t})\) are the result of households’ investment decisions \((H_{i,t-1} \rightarrow H_{i,t})\). In other words, movements in the level of unallocated capital \((U_{i,t})\) capture the changes in the asset side of the intermediaries balance sheets, while adjustments in the liquidity \((L_{i,t})\) provide a proxy for the evolution of banks liabilities.

3 Simulation results

In this section we will discuss the results obtained from the simulation of the model. Specifically, we will focus on the effects of the parameters on the determination of the pseudo steady-state level of the economy and on the dynamic of the endogenous cyclicality of the system.

3.1 Initial setting of parameters and sensitivity analysis

As in all agent-based model, in this framework parameters play a major role in the definition of the results and in the emergence of endogenous dynamics. We hold particular attention on the effects on the system of the firms reserve utility \(\theta\) and of the intermediaries contractual adjustment factor \(\gamma\). Indeed, as we will see further in the text, these two parameters modify not only the shape of the cycles and the response of the economy to a bank run, but also the pseudo steady-state level of the system.

Conversely, with regards to the search cost \(x\), simulations have shown that modifications to this value produce only second order effects. Indeed, from the point of view of the single entrepreneur this search cost is relatively small compared to the expected value of the firm (see Table 1). This fact
emerges also from conditions (1) and (15): to search and accept a new credit line the difference between the current and the new contractual profit must be higher than the product $xR_t$. Given that in the pseudo steady-state the interest rate is close to zero, this dampens the effects of the search cost. On the other hand, banks and, in turn, depositors are indifferent to the size of this cost because they finance it and obtain a positive return from this investment. In the light of this result, we chose to not explore further this aspect and to fix its value to $x = 0.1$.

We set a sufficiently high number of households ($n_h = 2000$) and firms ($n_f = 2000$), while for the intermediaries ($n_i = 50$) we decide to limit their quantity, to keep the computational complexity of the model at an acceptable level. Indeed, if on the one hand this choice produces slightly irregular initial transition phases, on the other one it returns reasonable results in the long run.

Households parameters are set inside a realistic range. In particular, we choose a discount factor $\beta = 0.95$ and an initial capital $W_0 = 0.1$. Finally, with regards to the production function, we use a common functional form with a decreasing marginal productivity: $f(k) = k^\alpha$ where $\alpha = 0.3$.

We study the pseudo steady-state level of the system on a range of the firms reserve utility $\theta = [0.05, 1.0]$ and of the intermediaries contractual adjustment factor $\gamma = [0.05, 1.0]$.

Notice that we set an upper bound for the entrepreneurs reserve utility equal to one because of the intrinsic characteristic of the interest rate function. In particular, the return of the single investment is defined as:

$$r = \frac{k^\alpha - P}{k}$$  \hspace{1cm} (21)

This function presents a point of maximum, whose solution is:

$$k^{max} = \left( \frac{P}{1 - \alpha} \right)^{1/\alpha} \Rightarrow r^{max} = \alpha\left( \frac{P}{1 - a} \right)^{(1-1/\alpha)}$$  \hspace{1cm} (22)

Given the discount factor $\beta = 0.95$ and the production function parameter $\alpha = 0.3$, the maximum value of the firm profit compatible with the households steady-state condition ($r = 1/\beta - 1$) is equal to:

$$P^\beta = (1 - \alpha)\left( \frac{1 - \beta}{\beta \alpha} \right)^{a-1} = 1.4759$$  \hspace{1cm} (23)

A contractual profit over this threshold will always return an interest rate lower than the households discount factor. Therefore, in that situation there
will be an overall withdrawal of resources by families and, in the long run, a collapse of the economy to an equilibrium without production and consumption. In addition to this, firms' reserve utility posit a lower constraint on the intermediaries decision regarding the contractual profit. Indeed, values below this zero utility threshold will never meet entrepreneurs demand. In summary, to avoid a degenerate equilibrium and to leave enough decision space to the intermediaries in their job of choosing the optimal contractual profit, we decide to limit the study of the dynamic of the system up to this value of the firms reserve utility.

Finally, we constrain also the intermediaries adjustment factor to an upper bound equal to one. In fact, in this case the range of the percentage change of the banks contractual profit is equal to $\pm 100\%$. Given that, we consider this limit sufficiently high for our purposes.

The system converges to a positive level of production and the number of intermediaries always remains above the degenerate equilibrium in which only one bank could be left in the economy (Figures 2, 3 and 4). As expected, the value of the firms profits increases with the reserve utility. In accordance to this, the growth of the production quota left to the entrepreneur has a negative effect on the interest rate paid to the households and, in turn, on the accumulation of capital and on the overall level of production of the system (Figures 2 and 3). Conversely, the intermediaries adjustment factor has a positive effect on the stability of the economy (Figures 2 and 5): a stronger reactivity of the financial system anticipates the formation of the cycles and dampens their effects. However, this characteristics comes with a drawback. In those situations in which the entrepreneurs reserve utility and, in turn, the lower bound for the contractual profit are high, an excessive responsiveness of the intermediaries increases the number of times in which the firms contractual profit exceed the households steady-state condition defined in equation (23). This explains the low number of intermediaries and the reduced level of production in high reserve utility and high contractual adjustment factor regions.

To end with, we also notice that the level of unallocated capital in the system increases with the firms reserve utility and with the intermediaries adjustment factor (Figure 6). At the same time, the firms extra profit ($P - \theta - xR$) decrease along those directions (Figure 7). As we will explain in the next section, this feature of the system is connected to the proximity of these points to the no entry condition and to the maximum production limit.
Figure 2: System evolution
Figure 3: Expected value of production

Figure 4: Number of active intermediaries
Figure 5: Expected value of production over standard deviation

Figure 6: Expected value of unallocated capital over total capital
Figure 7: Expected value of excess profits: $E[P] - \theta - xE[R]$

3.2 Credit cycles

An emerging feature of the stylized economy introduced here is the presence of endogenous credit cycles. To study the mechanisms behind these movements and to capture all the properties of the system we chose to analyse an intermediate combination of parameters, namely $(\theta = 0.5 \wedge \gamma = 0.5)$.

During periods of economic growth the system experience an increase in the productivity of firms and in the average interest rate paid by the intermediaries to the households (Figures 9 and 10). At the same time, the volume of unallocated resources reaches its maximum level (Figure 12), the average contractual profit decreases (Figure 11) and there is an expansion in the number of new credit line and a simultaneous contraction in loans closure (Figure 13 and Figure 14). Conversely, during economic downturns the interest rate and the firms productivity collapse (Figures 9 and 10), contractual profits increase (Figure 11) and in the initial phase of the crisis the volume of firms switching between intermediaries hik es, with a slow process of absorption in subsequent periods (Figure 14). Finally, the credit market experiences a fast compression, with a rise in the quantity of intermediaries
reducing the number of loans and a decline in the amount of banks offering new lending contracts (Figure 13).

The main drivers of this dynamic are the existence of a strategic complementarity in the intermediaries’ behaviour and the presence of a strong inter-firm competition. All these aspects are then amplified by an allocative friction which modifies the productivity of the firms and widens the reactivity of the system.

To understand the logic behind these fluctuations it is useful to start from the beginning of an expansionary phase. In that situation interest rates are higher than the households discount factor and therefore families have the incentive to save. At the same time, the amount of unallocated capital fluctuates around zero. Because of the positive conjuncture, contractual profits start following a decreasing trend through a self-reinforcing mechanism: when an intermediary experiences a withdraw of funds it recognizes that all its resources were invested in the preceding period and that it is leaving an excessively high production quota to the firms. In the light of this evidence, the bank decreases the contractual profit on new loans. From the point of view of the entrepreneurs, only new entrants have thus the incentive to borrow at the new conditions, hence leading to a positive inflow of firms. This fact combined with a fixed amount of capital in the short run and a production function with decreasing marginal returns bring to an increase in the total productivity of the economy and to a rise in the total volume of production. In addition to this, the reduction of the contractual profit leads to a further growth in interest rates and in the households propensity to save.

The incentive for all the intermediaries to reduce contractual profits holds as long as the amount of unallocated resources is low. Therefore, when there is a decrease in the demand for new loans this policy stops being optimal and a reversal in the trend starts. This situation happens when the contractual profit approaches the zero profit condition (Figure 11). Notice that the turnaround in contractual profits does not start immediately: even though there is an excess of unallocated capital, the increase in the overall productivity of the system prevents interest rates from falling below the households discount factor (Figures 9 and 10). Because of this, families still have the incentive to save at a lower but positive growth rate and to avoid a sudden and unexpected withdraw of funds. In other words, as long as the margin of intermediation of the banks remains sufficiently wide, these have the opportunity and the incentive to keep supplying new credit to the firms at the current market condition.
From a theoretical point of view this dynamic should converge to a stable equilibrium: the combined effects of high firm productivity and excess of capital should offset each others in the long run, producing a margin of intermediation and an interest rate compatible with the households discount factor. But the existence of a strategic complementarity in the intermediaries behaviour and the presence of a strong inter-firm competition limit the permanence of the economy at this equilibrium. Indeed, when the margin of intermediation of the single bank starts to squeeze because of a reduction in firms productivity, this has the incentive to increase the contractual profit on new loans to attract new counterparts. From the point of view of the intermediary this choice has a positive effect on its profitability because it reduces the volume of unallocated resources, but at the level of the whole economy it produces a decrease in the overall productivity of the system. In fact, the new contractual profit attracts not only new entrants, but also incumbent entrepreneurs (Figure 14). Therefore, at the aggregate level the same amount of capital is distributed across a smaller number of firms, with a decrease in the total productivity of the system (Figure 10). This in turn reduces the interest rate paid to households (Figure 9), driving them to start a withdrawal of deposits. Finally, as in the growing phase of the cycle, this downward trend is self-reinforcing. Indeed, other intermediaries recognize that they are accumulating an excess of unallocated resources because of their uncompetitive contractual profit. Hence, their optimal policy is to increase the profit quota on new loans. This results in an upward trend in contractual profit (Figure 11), a reduction in interest rates and firms productivity (Figures 9 and 10) and to an overall withdrawal of resources by the households, which in turn produces a compression of the credit market (Figure 13). Finally, the crisis stops when all the unallocated resources are depleted and the banks start again to decrease the contractual profit.

The endogenous cyclicity of the system is thus due to the interaction between the behaviours of intermediaries and firms. The absence of one of these two elements would in fact have prevented the beginning of the downward phase of the cycle.

In addition to this, from this study emerges that the level of unallocated resources of the intermediaries and the difference between the contractual profit and the reserve utility of firms are strictly connected to the phase of the cycle. This explains why different sets of parameters, which are characterized by dissimilar cyclicalities, have distinct levels of unallocated funds. In particular, economic systems featuring low volatility have also on aver-
age higher volumes of uninvested capital and smaller differences between entrepreneurs’ contractual profits and their reserve utilities. Indeed, in those situations the interaction between firms and intermediaries maintains the fluctuation of the system closer to the zero profit condition. In fact, in that phase of the cycle the level of unallocated capital is at its maximum level and the extra profits of the firms are nearly zero.

4 Bank run and persistent slump

In the preceding section we have shown that most of the movements in the system are the results of the interaction between firms and intermediaries. Households simply react to changes in the interest rate generated by the behaviour of these agents. In this section we will study the effects of a bank run, namely a sudden and unexpected withdrawal of resources by depositors. The idea is to understand the role played by banks creditors in the formation and in the destruction of lending relations. In addition to this, we will investigate whether the overall economic condition influences the outcome of this event.

The simplest way to model a bank run is to constrain the deposit decisions of the households. Specifically, we prevent a subset of families from depositing their capital after they have withdraw it. We perform this experiment

Figure 8: Production cycle
Figure 9: Average interest rate of the intermediaries

Figure 10: Average interest rate on loans and firms productivity
Figure 11: Average contractual profit

Figure 12: Average unallocated capital over total capital
Figure 13: Intermediaries behaviour: ratio over total active intermediaries (Baxter-King filter)

Figure 14: Firms behaviour: ratio over total active firms (Baxter-King filter)
through two different methodologies:

**Targeted Bank Run (TBR):** The subset of households belongs to a restricted group of intermediaries. The idea is to completely withdraw all the resources from a given number of banks and to interrupt all their lending activity.

**General Bank Run (GBR):** Families are selected at random across all the intermediaries. In this case, banks do not suffer from a complete depletion of their deposits and do not close all their credit lines.

In both experiments the volume of unallocated resources are approximately the same. In particular, we chose to withdraw an amount of deposits equal to the average ratio between liquidity and capital. Indeed, the idea is to conduct a shock whose dimensions are equivalent to the normal fluctuations of the system. For the TBR experiment we pick at random a subset of banks whose liabilities represent this capital quota, while for the GBR we chose a group of households which hold in overall the same amount of resources. To obtain a sufficiently high number of results we conduct 150 bank run of each type on a simulation of 10000 periods starting from the same random seed. The timing of the shock is then selected casually from a timespan between 4000 and 7000 periods. Finally, the experiment is repeated over four different combination of parameters: \( \gamma = 0.35 \land \theta = 0.35 \), \( \gamma = 0.35 \land \theta = 0.7 \), \( \gamma = 0.7 \land \theta = 0.35 \) and \( \gamma = 0.7 \land \theta = 0.7 \).

It clearly appears that the GBR has a smaller and only temporary effect on the economy (Figures 15, 16, 17 and 18). The reason behind this outcome is to be found in the destruction of credit lines. Indeed, by construction the only difference in the two methodologies is the allocation of the bank run: while in the GBR it is spread across all the intermediaries, in the TBR it is concentrated on a small subset of them. Therefore, the diffusion of the shock throughout banks dampens its effects and reduces the number of credit lines which are destroyed. This is due to the fact that, on average, there is always a positive, even if small, amount of unallocated capital in the intermediaries balance sheet. In the GBR treatment, banks have thus the opportunity to draw resources from that entry instead of closing a credit line. Conversely, in the TBR all the loans connected to a specific intermediary are closed.

This fact explains why shocks show conflicting outcomes for different sets of parameters. In fact, systems in which the average level of unallocated funds is low are subject to deeper and persistent recessions (Figures 15 and
Conversely, economies characterized by a higher level of uninvested capital are able not only to manage properly the shock, but also to improve their steady-state performance (Figures 16 and 18). Indeed, in these cases firms which suffer an unexpected closure of their credit lines can immediately borrow from other intermediaries. Therefore, the economy recovers faster and it is also able in the short run to allocate most of the exceeding capital inherited from the preceding period. In this way, in the long run this indirect rationalization of investments rises the overall productivity of the system, with an increase in both the production level and the profitability of firms. On the contrary, in an economy with a low level of unallocated capital the intermediaries are not able to cover all the additional demand for funds. Furthermore, in subsequent periods – namely, when the resources are reinvested in the economy by the households – banks start to decrease the contractual profit they offer in order to improve their profitability by exploiting this excess of demand. Therefore, the overall results in the short run are a reduction of the credit demand and a persistent accumulation of unallocated capital. Due to this fact, households have thus the incentive to withdraw and consume this excess of resources, and the economy converges in the long run to a lower steady-state equilibrium.

These results are confirmed through an analysis of recovery times, namely the time required for the economy to reach the same level of production that would have been obtained if there had not been the bank run. In particular, we regress the logarithm of the recovery time on the logarithm of the production level for the TBR at the moment of the shock (Table 2). Estimates confirm the preceding intuition: recovery times are highly significant for economies with low volumes of unallocated capital. Specifically, an economy which is hit by a bank run in a moment characterized by an elevated level of production and uninvested resources requires a shorter time to recover. Clearly, for economies with high levels of unallocated funds this results does not hold because of their intrinsic characteristics. To confirm this finding we analyse also the recovery time for the GBR and, as expected, the estimations produce completely different results (Table 3).

Finally, we observe that recovery times in the GBR and in the TBR are nearer in moments of high production (Figure 19). This finding is clearly in line with the preceding analysis of the shock: the dampening of the bank run is strictly connected to the overall level of unallocated capital of the system. In other words, it is the destruction of the credit lines which amplifies the shock: an economy in which the overall financial system can easier substitute
Table 2: OLS estimates of Targeted Bank Run data

<table>
<thead>
<tr>
<th>Model</th>
<th>Constant Coefficient</th>
<th>p-value</th>
<th>log(Production) Coefficient</th>
<th>p-value</th>
<th>Test for normality p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 0.35 \land \theta = 0.35$</td>
<td>20.4840</td>
<td>3.58e-011</td>
<td>-2.0113</td>
<td>2.73e-07</td>
<td>9.265e-013</td>
</tr>
<tr>
<td>$\gamma = 0.35 \land \theta = 0.7$</td>
<td>11.2568</td>
<td>0.0016</td>
<td>-0.8127</td>
<td>0.0992</td>
<td>0.31962</td>
</tr>
<tr>
<td>$\gamma = 0.7 \land \theta = 0.35$</td>
<td>18.7510</td>
<td>0.0008</td>
<td>-1.6399</td>
<td>0.0227</td>
<td>0.02440</td>
</tr>
<tr>
<td>$\gamma = 0.7 \land \theta = 0.7$</td>
<td>3.76114</td>
<td>0.1118</td>
<td>0.21200</td>
<td>0.5607</td>
<td>0.31590</td>
</tr>
</tbody>
</table>

Dependent variable: log(Recovery Time); Number of observations=150

Table 3: OLS estimates of General Bank Run data

<table>
<thead>
<tr>
<th>Model</th>
<th>Constant Coefficient</th>
<th>p-value</th>
<th>log(Production) Coefficient</th>
<th>p-value</th>
<th>Test for normality p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma = 0.35 \land \theta = 0.35$</td>
<td>-8.48398</td>
<td>0.0459</td>
<td>1.6110</td>
<td>0.0040</td>
<td>0.63438</td>
</tr>
<tr>
<td>$\gamma = 0.35 \land \theta = 0.7$</td>
<td>-16.2040</td>
<td>0.0919</td>
<td>2.7739</td>
<td>0.0398</td>
<td>0.53786</td>
</tr>
<tr>
<td>$\gamma = 0.7 \land \theta = 0.35$</td>
<td>12.6443</td>
<td>0.2549</td>
<td>-1.1434</td>
<td>0.4267</td>
<td>0.04615</td>
</tr>
<tr>
<td>$\gamma = 0.7 \land \theta = 0.7$</td>
<td>-6.5072</td>
<td>0.4295</td>
<td>1.4582</td>
<td>0.2533</td>
<td>0.07801</td>
</tr>
</tbody>
</table>

Dependent variable: log(Recovery Time); Number of observations=150

A defaulted intermediary in its job of loan issuance will suffer minor losses.

5 Concluding remarks

This paper has developed a simple agent-based model of bank-intermediated credit flows in which information imperfections give rise to search frictions and resource misallocation. We have shown that credit and business cycles arise endogenously and co-evolve. The key driving mechanism is a pure competition effect, as banks struggle to manage their net interest margin through the activation and termination of loan relationships at varying interest rates in order to attract deposits. By assuming the possibility that banks can be affected by funding runs, we have also studied the macroeconomic impact of financial crisis in terms of output losses and speed of recovery.

The model can be extended in a number of directions to consider important issues. First, we have so far ignored agency frictions. The probability a long-run loan contract is severed – and, therefore, the extent of credit reallo-
Figure 15: Impulse response function: percentage deviation from the baseline simulation ($\gamma = 0.35 \land \theta = 0.35$)
Figure 16: Impulse response function: percentage deviation from the baseline simulation \((\gamma = 0.35 \land \theta = 0.7)\)
Figure 17: Impulse response function: percentage deviation from the baseline simulation ($\gamma = 0.7 \land \theta = 0.35$)
Figure 18: Impulse response function: percentage deviation from the baseline simulation ($\gamma = 0.7 \land \theta = 0.7$)
Figure 19: Recovery time and production level (log scale)
cation - is in general related to the effort choice of the borrower in servicing his debt, as well as on the risk appetite of the lender. Incorporating moral hazard problems associated to the level of net worth of agents are therefore likely to sensibly improve the explanatory capability of our model.

For reasons of tractability we have abstracted from the labor market or any other kind of physical production factors. As shown e.g. in Wasmer & Weil (2004), general equilibrium feedbacks between the markets for finance and productive inputs can magnify the response of the economy to exogenous shocks through a financial accelerator mechanism. Such an improvement could allow us to complement the story we offer here with an analysis of the way disruptions in financial markets affect unemployment and economic activity, and vice-versa, by exploring the issues of how credit misallocation interacts with labor misallocation (Delli Gatti et al. 2012) and income distribution (Dosi et al. 2013).

Finally, an interesting extension would be that of studying the role of the central bank in smoothing the credit cycle and responding to liquidity crisis by acting as a lender-of-last-resort.
References


